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TURBOMACHINERY BLADE DESIGN USING THE NAVIER-STOKES EQUATIONS

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Abstract

This paper presents a design optimization method that couples a Navier-Stokes flow solver and numerical optimization algorithm. The design method improves the performance of a transonic turbine blade subject to specified design objective and constraints. The flow field prediction is based on the Navier-Stokes equations in order to represent the nonlinear, rotational and viscous physics of transonic flows. Sensitivity derivatives are obtained using finite differencing. Effects of different design variables on the performance of design optimization are evaluated. The method is demonstrated with several examples at transonic flow conditions.

Introduction

Computational fluid dynamics (CFD) has changed the aerodynamic design process. CFD was first employed in design in a cut-and-try manner by utilizing its flow analysis capability. That is, a specified configuration is first evaluated to get its aerodynamic performance characteristics, and then the geometry is modified to produce an improved performance. While this methodology may be effective if the designs fall within a known experimental database, it becomes cumbersome and difficult when the design progresses outside the domain of the known database. This type of design methods for turbomachinery system was discussed in Reference 1. Clearly, more automated design optimization technologies are desirable for the development of new turbomachinery components. As computational methods have advanced, CFD-based numerical optimization became a more viable design tool for advanced turbomachinery components.

A unique advantage of CFD is the capability of inverse design. Inverse design finds the blade geometry that produces the pressure or velocity distributions specified by a designer. Earlier inverse design methods were based on the potential equation due to its simplicity.[2,3] However, high-level physics has inherent difficulties in inverse formulation due to its multi-variable, multi-equation feature. Also, it is not easy to define the target pressure distributions that produce desired aerodynamic performance, and there is no guarantee that such geometry exists that produces the target pressures. Furthermore, turbomachinery blade performance is often judged by parameters such as kinetic energy and total pressure loss coefficients, instead of local blade-surface velocity or pressure distributions.

The use of numerical optimization eliminates some of the difficulties associated with inverse design. Numerical optimization provides a rational and directed search through the design space. CFD-based optimization is capable of modifying a geometry to improve a design while satisfying specified design constraints.[4,5] In this approach, a specified design parameter can be improved without degrading other aspects of the design. Unfortunately, numerical optimization requires intensive computations, particularly when the design involves nonlinear problems with many design variables and constraints. However, the efficiency of modern flow solvers, as well as the computational power available today, can alleviate this drawback.

The reliability of design results depends on the ability to accurately simulate the flow field. Hence the flow model used in a design process should be able to represent all the significant flow physics involved. Transonic turbomachinery flows contain a variety of complicated flow phenomena such as shock waves, shock-boundary-layer interactions, turbulent boundary layers and wakes. Several attempts have been made to utilize Euler or Navier-Stokes physics for turbomachinery design. References 6 and 7 used the Euler equations in their design methods. A recent study[8] demonstrated that

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Navier-Stokes designs are feasible for turbomachinery design. The objective of this study is to build a Navier-Stokes-based turbomachinery design tool using a constrained optimization and to evaluate its ability and efficiency in finding improved designs. The design goal is to decrease the loss coefficient without decreasing the mass flow rate, the blade loading, and the blade cross-sectional area.

Flow Analysis

The two-dimensional, unsteady, compressible Navier-Stokes equations are solved using a body-fitted, curvilinear coordinate system. The Baldwin-Lomax eddy viscosity model[9] is used for turbulence closure, and the transition point is fixed at fourteen percent of the chord. A finite volume method is employed for the spatial discretization. The flow variables are defined at cell centers, and centered differencing is used for the spatial derivatives. Second- and fourth-order artificial viscosities are added to enforce numerical stability.[10,11] The time integration is performed using an explicit, four-stage Runge-Kutta scheme. Local time stepping, variable-coefficient implicit residual smoothing, and a multigrid method are implemented to accelerate the convergence. Characteristic boundary conditions are imposed at the far-field boundary based on a one-dimensional eigenvalue analysis, and a no-slip, adiabatic-wall conditions are used on the blade surface.

The reliability of the Navier-Stokes code is first evaluated by comparing its analysis results with experimental data. All the analysis and design practices are performed using C-type grids with size of 257x33. The grid spacing adjacent to the blade surface is set to 0.01 percent of the chord length. Figure 1 shows the grid and the isentropic Mach number distributions for the VKI-LS82 turbine blade.[12] The computed isentropic Mach numbers are in good agreement with the experimental data.

Numerical Optimization

The optimization method employed in the present design study is based on a conjugate gradient algorithm.[13] The method searches for the optimum values of the design variables that minimize a specified objective function under a set of constraints. The general optimization problem can be written mathematically as follows:

$$\text{minimize} \quad F(\vec{X}) \tag{1}$$

$$\text{subject to} \quad g_j(\vec{X}) \le 0 \qquad j = 1 \text{ to } J$$

$$X_k^L \le X_k \le X_k^U \qquad k = 1 \text{ to } K$$

where F is the objective function, and g_j's are the constraints. J is the total number of constraints. The vector $\vec{X}$ represents design variables X_k with length K. The constraints imposed on design variables are called side constraints.

The optimization process starts with an initial guess of the design variables. The design is then updated using an iterative procedure given by

$$\vec{X}^{n+1} = \vec{X}^n + \beta^n \vec{S}^n \tag{2}$$

where the superscript n is the iteration number, the vector $\vec{S}$ is the search direction, and the scalar β is the step size to move into the search direction. The optimum step size is found using a one-dimensional search and interpolation. The process is iterated until it converges. The search direction must be both usable and feasible. A usable direction is the direction that reduces the objective function,

$$\nabla F(\vec{X}) \bullet \vec{S} \le 0, \tag{3}$$

and a feasible direction is the direction that satisfies the jth-constraint,

$$\nabla g_j(\vec{X}) \bullet \vec{S} \le 0. \tag{4}$$

When no constraints are active, the search direction is obtained using the Fletcher-Reeves conjugate direction method by imposing the orthogonality condition between search directions. A constraint becomes active when its value becomes zero within some numerical tolerance. When some constraints are active or violated, a sub-optimization process is used to find the search direction. The left-hand side of Eq. (3) is minimized subject to the active constraints given by Eq. (4). The gradient operator ∇ is the sensitivity of a function with respect to the design variables and is calculated using finite differencing.

Design Variables

In general, turbomachinery blade design involves different disciplines such as fluid mechanics, heat transfer, materials, acoustics, etc. For example, heat transfer is an important factor that determines the leading- and trailing-edge thicknesses of a turbine blade because turbines are operated at high temperature. Therefore, all significant physics should be considered in selecting design variables and constraints. However, increasing the number of design constraints and/or variables will increase the design cost. An appropriate choice of design variables may reduce the number of design constraints required while increasing design performance.

The blade geometry is modified by adding a smooth perturbation Δn to the initial geometry. The geometry perturbation normal to camber-line is defined as a linear combination of shape functions f_k:

$$\Delta n = \sum_{k=1}^{K} X_k f_k(x) \tag{5}$$

where x is the normalized position of the camber-line, and K stands for the number of shape functions to be used. The weighting coefficients, X_k, in the equation are the values of the design variables that are determined through the optimization process. The shape functions modify both upper and lower surfaces of the blade.

The performance of a design process is strongly influenced by the choice of shape functions because shape functions influence the convergence rate of the optimization process as well as the quality of design results. The present study examines the following four different shape functions shown in Figure 2.

Hicks-Henne Functions:

The sinusoidal shape functions are frequently used in airfoil optimization.

$$f_1(x) = x^{0.25}(1-x)\exp(-20x) \tag{6}$$
$$f_k(x) = \sin^3\left(\pi x^{e(k)}\right) \qquad \text{for } k>1$$

where

$$e(k) = \frac{\log(0.5)}{\log(x_k)}$$
$$x_k = 0.25,\ 0.5,\ 0.75,\ 0.85$$

Here x_k's are the locations of maximum height of corresponding shape functions.

Wagner Functions:

The Wagner functions provide large variations with high harmonics and may cause waviness in resulting designs.

$$f_1(x) = \frac{[\theta + \sin(\theta)]}{\pi} - \sin^2\left(\frac{\theta}{2}\right) \tag{7}$$
$$f_k(x) = \frac{\sin(k\theta)}{k\pi} + \frac{\sin[(k-1)\theta]}{\pi}, \quad \text{for } k>1$$

where

$$\theta = 2\sin^{-1}\left(\sqrt{x}\right)$$

Legendre Polynomials:

The Legendre polynomials are orthogonal functions that may be advantageous as shape functions.

$$f_1(x) = (1-x)^3\sqrt{x} \tag{8}$$
$$f_2(x) = (1-x)^3(2x)$$
$$f_3(x) = (1-x)^3\left(6x^2 - 6x\right)$$
$$f_4(x) = (1-x)^3\left(20x^3 - 30x^2 + 12x\right)$$
$$f_5(x) = (1-x)^3\left(70x^4 - 140x^3 + 90x^2 - 20x\right)$$

Patched Polynomials:

A cubic on one side of x_k is patched with another cubic on the other side to produce a smooth curve of second-order continuity. x_k is the location of maximum perturbation.

$$f_k(x) = 1 - \left(\frac{x - x_k}{x_k}\right)^2 \left(1 + \frac{A}{(1-x_k)^2}\frac{x}{x_k}\right)$$
$$\text{for } 0 \le x \le x_k \tag{9}$$

$$f_k(x) = 1 - \left(\frac{x - x_k}{1 - x_k}\right)^2 \left(1 + \frac{B}{(x_k)^2}\frac{1-x}{1-x_k}\right)$$
$$\text{for } x_k \le x \le 1$$

where

$$A = \max(0, 1-2x_k)$$
$$B = \max(0, 2x_k - 1)$$
$$x_k = 0.15,\ 0.25,\ 0.5,\ 0.75,\ 0.85$$

Results

The goal of the present design study is to generate a turbine blade geometry that produces the minimum loss coefficient at a given operating condition, while maintaining the blade loading, the mass flux rate, and the cross sectional area. The particular form of the current optimization problem can be stated as:

$$\begin{aligned} &\text{minimize} && C_{loss} = 1 - \frac{v_2^2}{v_{2is}^2} \\ &\text{subject to} && 1 - \frac{L}{L_0} \le 0 \qquad (10) \\ &&& 1 - \frac{M}{M_0} \le 0 \\ &&& 1 - \frac{A}{A_0} \le 0 \end{aligned}$$

where C_{loss}, L, M, and A are the loss coefficient, the blade loading, the mass flux rate, and the cross sectional area, respectively. The subscript 0 stands for the initial values. v_2 and v_{2is} are the exit velocity and the isentropic exit velocity, respectively.

All design practices are conducted with the VKI-LS82 turbine blade as the initial geometry at the exit Mach number 1.43 and Reynolds number one million. Results are given in Tables 1 to 4 and Figures 3 to 6 for shape functions of Hick-Henne functions, Wagner functions, Legendre polynomials, and patched polynomials, respectively. Figure 7 compares pressure contours between original and designed blades. Also compared are streamlines near the trailing-edge. All computations are conducted on an SGI Indigo workstation. In most cases, the design cycle is terminated after the first geometry update, indicating that the design process reaches a local minimum and cannot escape it. A more gain in the objective function is obtained with Wagner functions and patched polynomials. A caution should be taken in locating the maximum height of a shape function such that it will not significantly alter the leading- and trailing-edge radii. The tolerance level of side constraints should also be chosen not to allow a large change in leading- and trailing-edge shapes.

Concluding Remarks

The present paper demonstrates a design optimization method based on the Navier-Stokes equations for turbomachinery design. Preliminary results indicate that a considerable gain can be obtained in design objective while satisfying design constraints. However, the design process is hampered due to the local minimum issue associated with nonlinear design spaces. The present study shows that the efficiency of the design process is significantly affected by the choice of shape functions.

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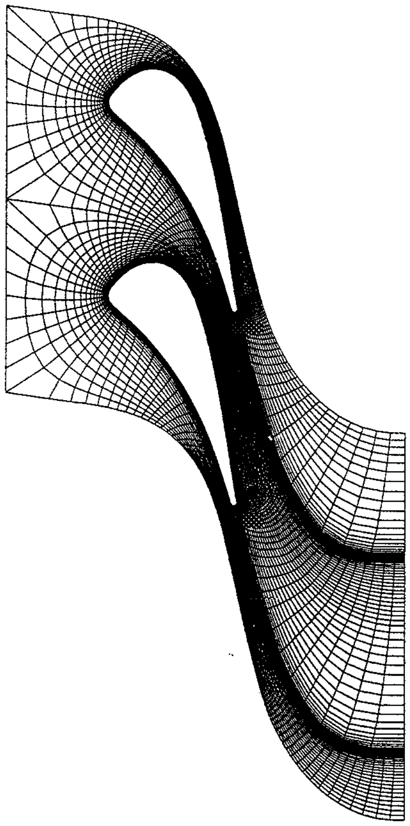
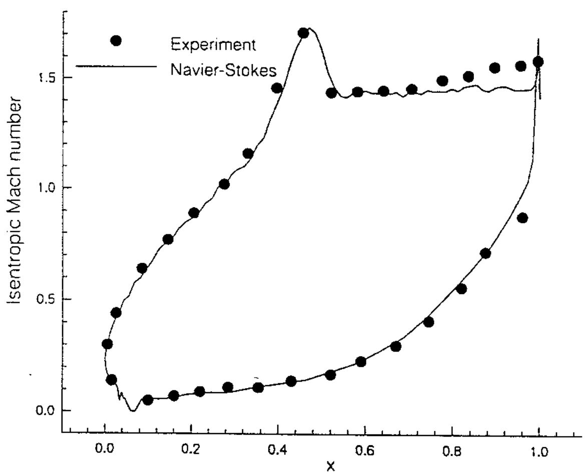
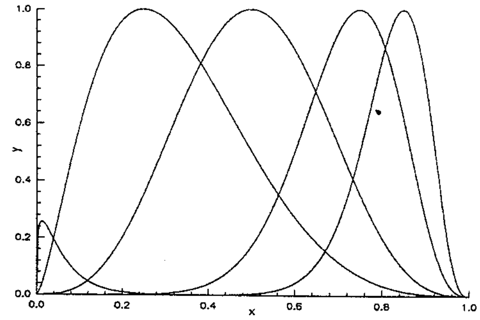


a) Computational grid (257x33)

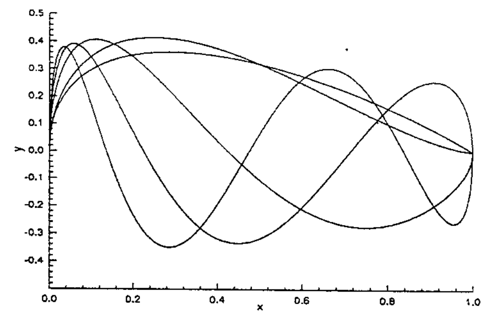


b) Isentropic Mach Number

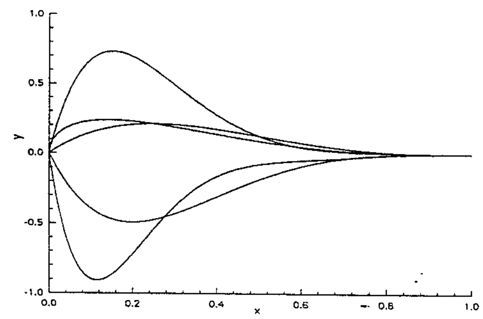
Figure 1. Analysis validation with VKI-LS82 turbine blade at M_{2is}= 1.43, Re=10^6.



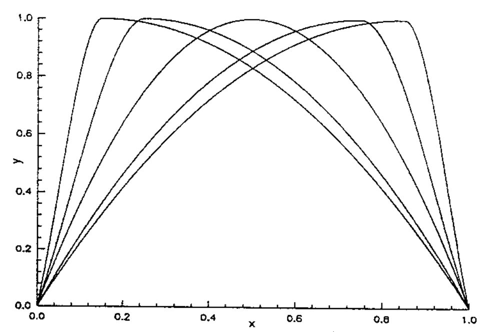
a) Hicks-Henne functions



b) Wagner functions



c) Legendre polynomials



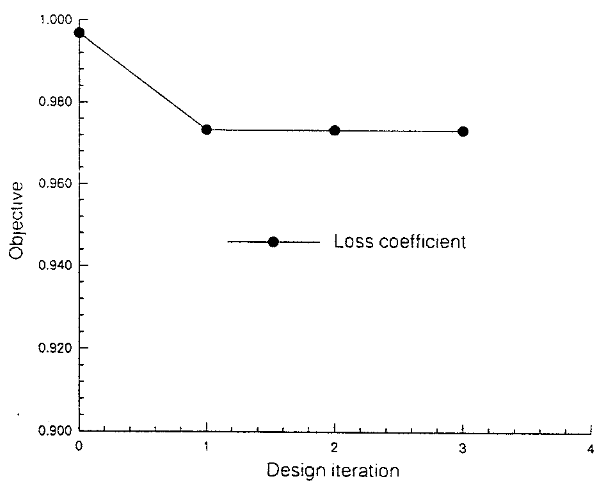
d) Patched polynomials

Figure 2. Shape functions used to perturb the geometry.

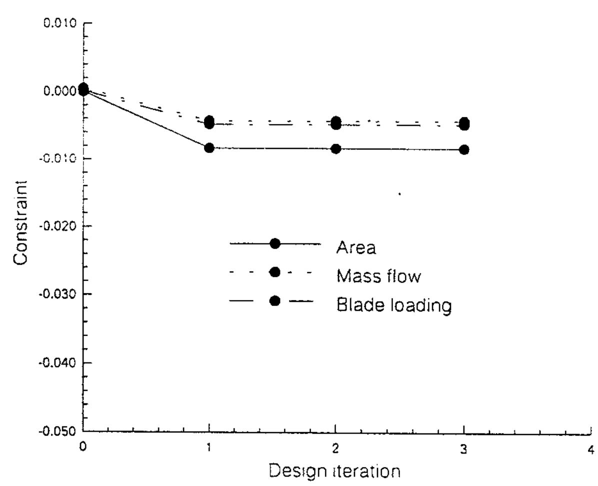
Table 1. Design optimization of the VKI-LS82 turbine blade with Hicks-Henne functions.

	Initial	Design	Change (%)
C_{loss}	0.80340E-01	0.78629E-01	-2.130
Area	0.56510E+00	0.56983E+00	0.837
Mass Flow	0.16114E+00	0.16189E+00	0.465
Blade loading	0.21993E+00	0.22100E+00	0.487

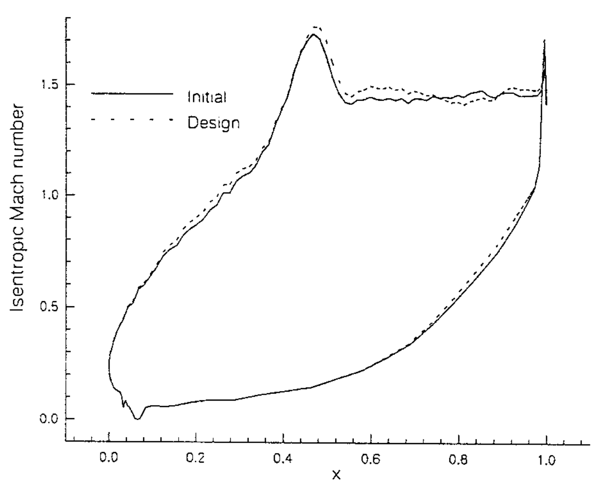
Number of function calls = 37, CPU = 5 hours 40 minutes



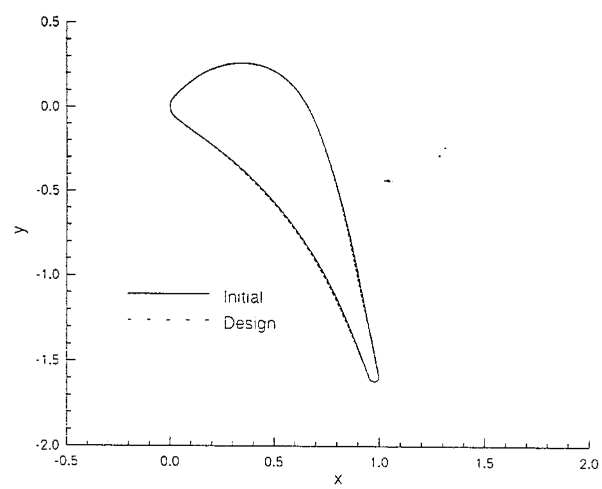
a) Loss coefficient



b) Constraints



c) Isentropic Mach number



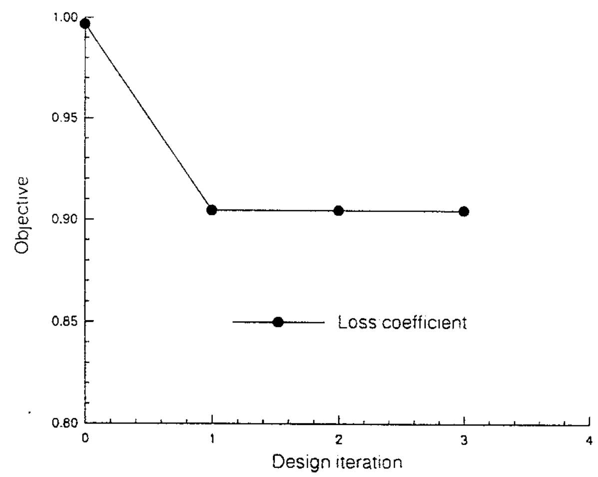
d) Blade geometry

Figure 3. Design optimization of the VKI-LS82 turbine blade with Hicks-Henne functions.

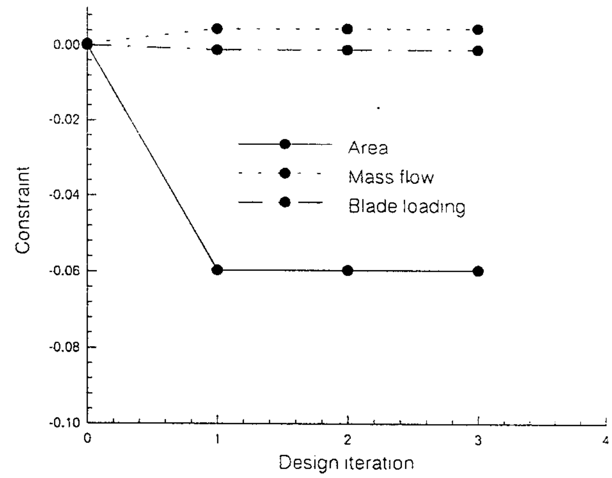
Table 2. Design optimization of the VKI-LS82 turbine blade with Wagner functions.

	Initial	Design	Change (%)
C_{loss}	0.80340E-01	0.73486E-01	-8.531
Area	0.56510E+00	0.59881E+00	5.965
Mass Flow	0.16114E+00	0.16050E+00	-0.397
Blade loading	0.21993E+00	0.22021E+00	0.127

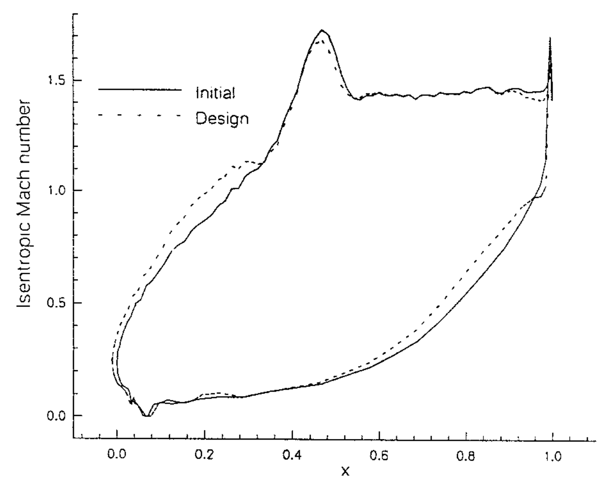
Number of function calls = 39, CPU = 11 hours 45 minutes



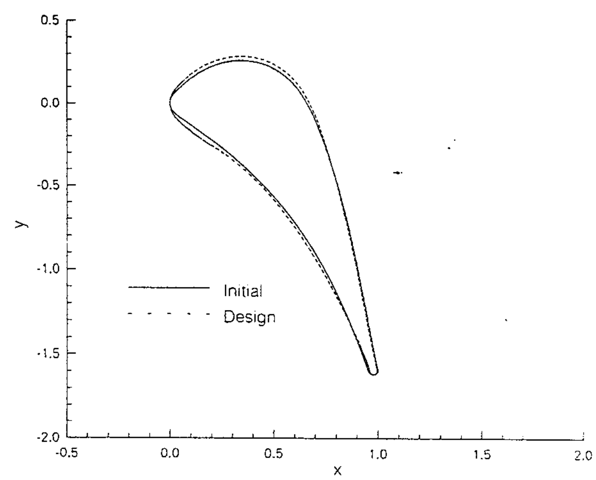
a) Loss coefficient



b) Constraints



c) Isentropic Mach number



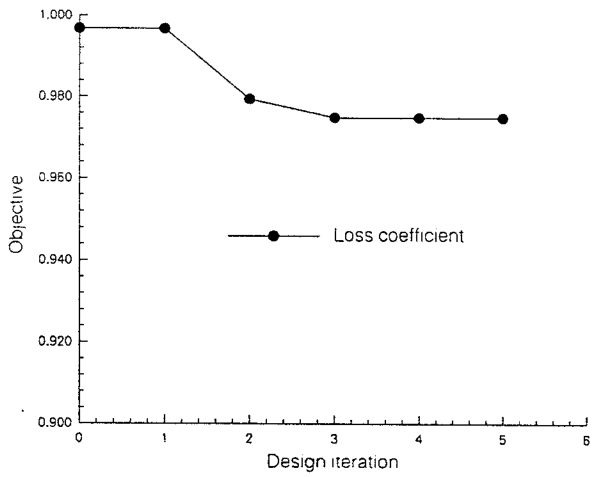
d) Blade geometry

Figure 4. Design optimization of the VKI-LS82 turbine blade with Wagner functions.

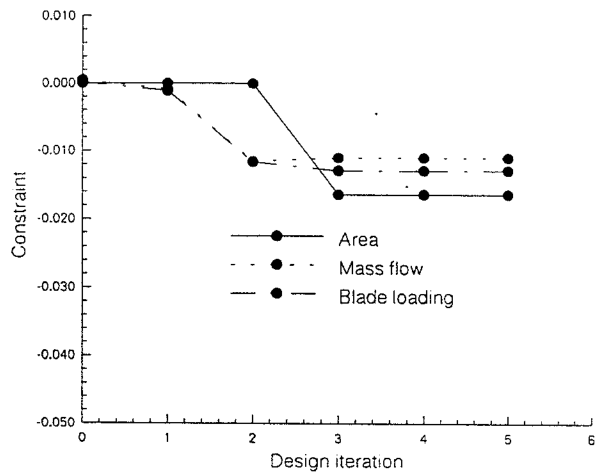
Table 3. Design optimization of the VKI-LS82 turbine blade with Legendre polynomials.

	Initial	Design	Change (%)
C_{loss}	0.80340E-01	0.78565E-01	-2.209
Area	0.56510E+00	0.57438E+00	1.642
Mass Flow	0.16114E+00	0.16299E+00	1.148
Blade loading	0.21993E+00	0.22280E+00	1.305

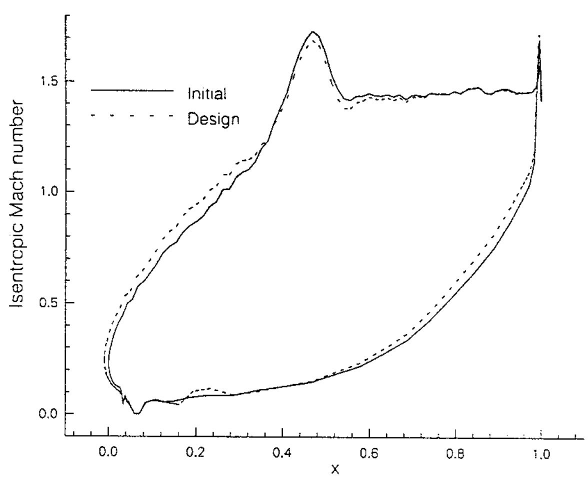
Number of function calls = 108, CPU = 13 hours 3 minutes



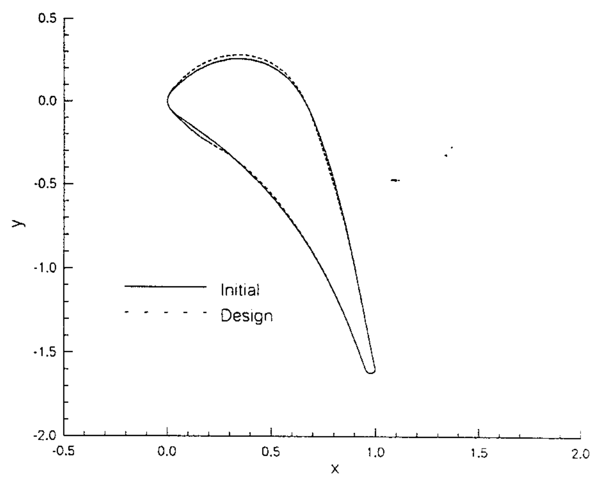
a) Loss coefficient



b) Constraints



c) Isentropic Mach number



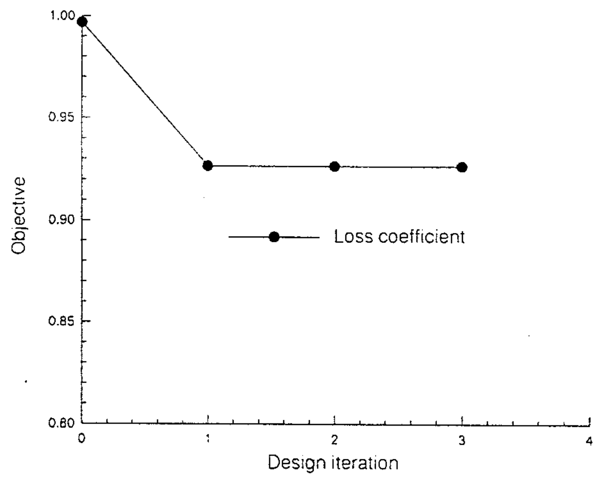
d) Blade geometry

Figure 5. Design optimization of the VKI-LS82 turbine blade with Legendre polynomials.

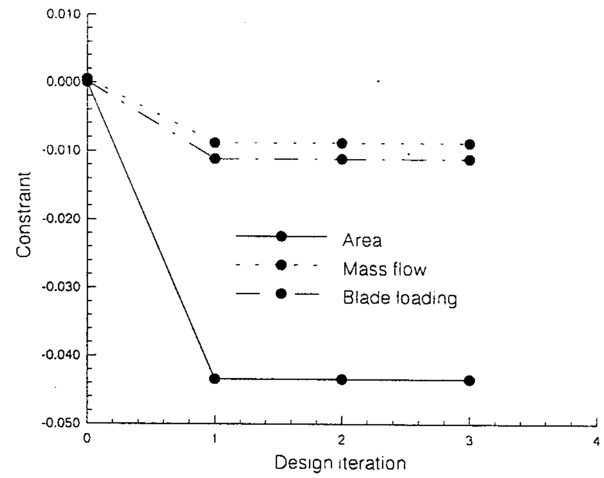
Table 4. Design optimization of the VKI-LS82 turbine blade with Patched polynomials.

	Initial	Design	Change (%)
C_{loss}	0.80340E-01	0.74685E-01	-7.039
Area	0.56510E+00	0.58964E+00	4.343
Mass Flow	0.16114E+00	0.16258E+00	0.894
Blade loading	0.21993E+00	0.22237E+00	1.109

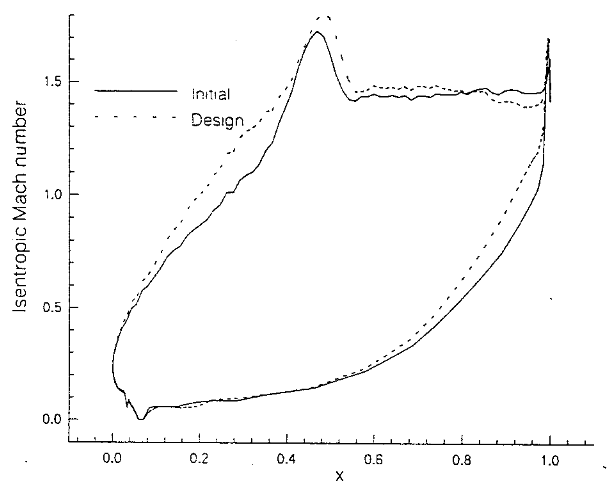
Number of function calls = 31, CPU = 6 hours 25 minutes



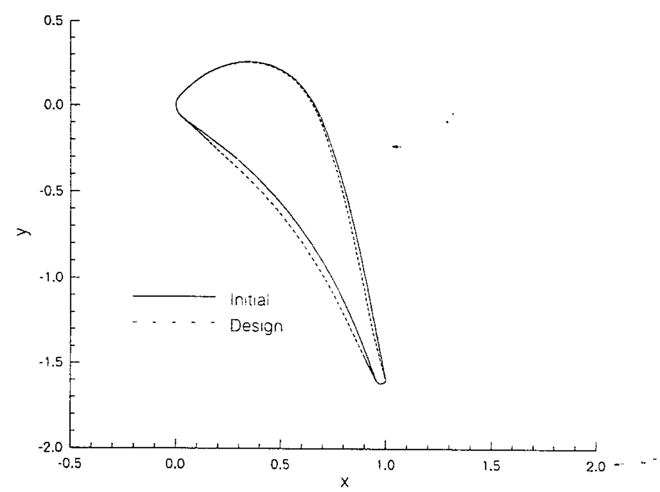
a) Loss coefficient



b) Constraints

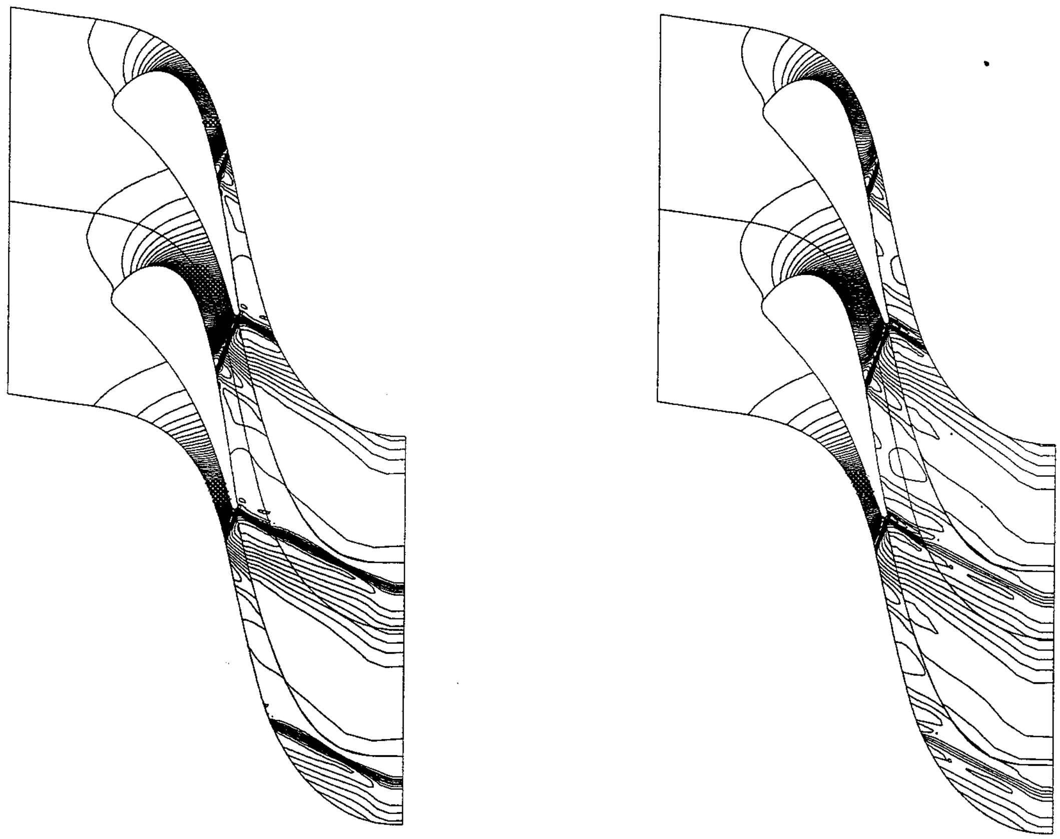


c) Isentropic Mach number



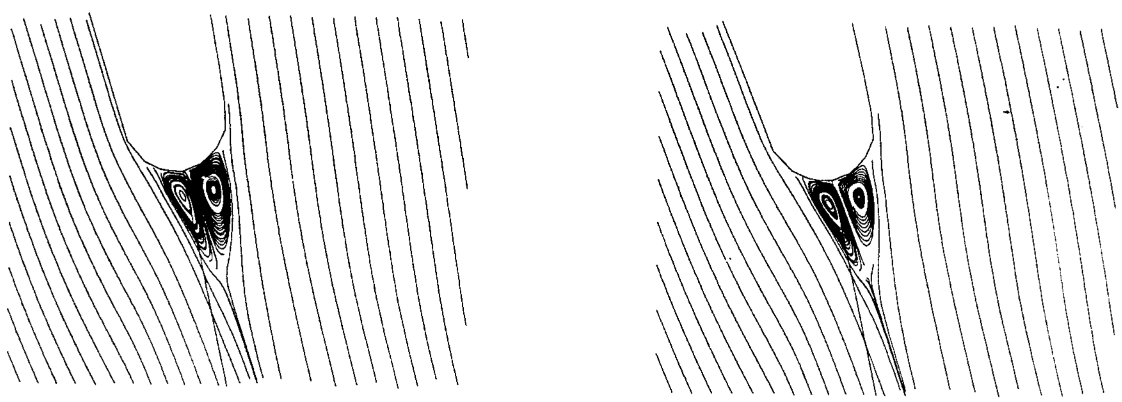
d) Blade geometry

Figure 6. Design optimization of the VKI-LS82 turbine blade with Patched polynomials.



a) Pressure contours of VKI-LS82 turbine blade

b) Pressure contours of designed blade



c) Particle traces of VKI-LS82 turbine blade

d) Particle traces of designed blade

Figure 7. Design optimization of VKI-LS82 turbine blade with Patched polynomials.