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A COMPARISON OF THREE TECHNIQUES FOR THE PREDICTION OF ISOLATED PROPELLER PERFORMANCE [1]

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Abstract

Strip theory, blade element, and free-wake techniques have been used to analyse the performance of an isolated propeller. Their capabilities have been assessed for a range of flight conditions. The techniques have been compared on the basis of their agreement relative to each other and to published data, as well as ease of application, ability to model inplane forces and moments, ability to analyse flow upstream or downstream of the propeller disc, and computational efficiency. The blade element technique has been shown to give the best compromise, combining fast solution time with reasonable thrust and torque coefficient prediction. In addition, it has the ability to model forces and moments in the plane of the propeller disc, and the potential to empirically approximate the wake dependant force and moment coefficient increments on the wing and tail aerodynamic surfaces.

Nomenclature

C_P	Power coefficient
C_Q	Torque coefficient
C_T	Thrust coefficient
h	Altitude [m]
J	Advance ratio
n	Propeller speed [RPS]
N	Number of Blades
r_c	Incremental radius [m]
s	Solidity
$\frac{t}{c}$	Thickness to chord ratio
V_∞	Freestream velocity [ms^{-1}]
x	Radial station
X_P	Propeller based X axis, pointing forward from propeller hub
Y_P	Propeller based Y axis, point out along starboard wing
Z_P	Propeller based Z axis, Orthogonal to X_P and Y_P
ϵ_0	Zero lift angle [rad]
θ	Blade pitch angle [rad]
ϕ	Wake helix angle [rad]
ψ	Blade azimuth angle [rad]
ζ_1	Slipstream locus angle from propeller to wing [rad]
ζ_2	Slipstream locus angle from wing to tail [rad]

(I) Introduction

The prediction of the flow through a propeller has long been one of the more difficult aerodynamic problems to solve. The general nature of the flow has been well understood from an early stage, but accurate prediction of propeller performance has always been troublesome.

This paper compares three schemes for estimating the performance of propellers, and discusses the enhancements in performance information that have become available with improved propeller modelling. The first scheme is a classical analysis of the flow through a propeller, based largely on empirical data gathered from testing. It is typical of the type of scheme used before the advent of computers. The second scheme is based on a conventional blade element momentum theory solution, as commonly used to estimate the thrust, torque and power coefficients of a propeller. However, it has been modified such that variations in inflow factors and blade loading around the propeller disc can be evaluated. The third and final scheme is a modern three dimensional vortex lattice panel code, which models not only the propeller but also the propeller slipstream and its influence on the flow, via "free-wake" techniques.

All three schemes are applied to the analysis of a propeller of known geometry operating at a number of flight conditions, and the results of the three schemes are compared together with published performance data for the specified propeller geometry [1]. The propeller of the Pilatus PC-9A (Hartzell Propellers Inc. blade sec-

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tion HC-D4N-2A/D9512AK) has been chosen for comparison of the three schemes because of the availability of geometric and performance data.

The three schemes are evaluated on a number of bases. Firstly, a comparison of thrust and torque predictions is made with respect to each other and the published performance data for the specified flight conditions. Secondly, their ease of application to the problem at hand is compared. Ideally the techniques should be simple to apply with little setting up. Thirdly, their abilities to estimate forces and moments other than thrust and torque are evaluated. A propeller generates additional forces and moments when not aligned with the oncoming flow, and these can also be significant. Fourthly, on the basis of their ability to be used for analysis of the flow further upstream or downstream of the propeller disc. A propeller is usually located either upstream or downstream of other aerodynamic surfaces and the influence of the propeller on these surfaces can also be significant. Finally, on the basis of the computational power required to complete the solution. An improvement in thrust and torque predictions may be achieved by a more complex technique, but the increase in solution time may be prohibitive.

This work is being conducted as part of an ongoing investigation of the effects of propellers on the stability and handling characteristics of single engined high-powered monoplanes [8][1].

(II) Analysis Techniques

The three techniques selected for comparison in this paper are representative of pre-computer methods, well established computational methods, and latest technology computational methods respectively.

Strip Theory

Strip theory style techniques were first used to calculate propeller performance around the 1920's and continued to be the main technique used until the 1960's. The particular technique used in this paper was published in 1945 by Lock, Pankhurst, and Conn [2]. The propeller blade is divided into radial segments (strips). Empirical relations are then used to evaluate the incremental values of thrust and torque coefficients at each strip on the basis of N, V_∞, n, h, r_c, $\frac{t}{c}$, ϵ_0, s, and θ, and these are then integrated along the propeller blade. The empirical relations exist in tabular form as functions of r_c, N, J, and ϕ, and can be found in, for example, [2].

It is important to note that the tables of empirical relations used for strip theory calculations in this paper are only valid for radial stations between 0.3 and 1.0; and for advance ratios greater than or equal to 1.0.

Blade Element Theory

The majority of blade element propeller models assume inflow to be either constant across the entire propeller disc or to vary only along the propeller blade (not around the disc), and in this regard are largely a computerised version of the strip theory previously described. However, the blade element technique used in this paper has been modified to allow the variation of axial and rotational inflow around the entire propeller disc to be modelled, hence allowing the full variation in propeller blade loading to be captured [7, 8]. Not only does this allow the forces and moments along and about the Y_P and Z_P axes due to the propeller blade loading to be estimated, it also provides improved estimates of thrust and torque (about the X_P axis) for cases where the flow is not aligned with the propeller axis. Additionally and, perhaps, more significantly the inflow is evaluated via an iterative process combining blade element and momentum conservation principles (rather than the empirical relationships of the strip theory technique), which should produce a more accurate result.

Free-Wake Vortex Lattice Theory

Free-wake vortex lattice panel codes represent the cutting edge technology for modelling propellers and helicopter rotors, and are beginning to come into general use. Their main advantage over earlier techniques is the ability to model the radial flow along the propeller blades, as well as the ability to take into account the flow field upstream or downstream of the propeller. These two factors should allow free-wake techniques to provide better estimates of the forces and moments generated by the propeller, as well as allowing the interaction of the propeller and the aircraft structure to be modelled.

The particular free-wake model used in this investigation [3] considers the propeller blades as lifting surfaces. As such blade twist, pitch and chord are modelled but section thickness is not. At each time step a wake panel is shed off each trailing edge panel. The strength of the wake panel is set equal to the strength of the panel from which it was shed, at the time it was shed. The influences of these wake panels are included in the calculation of the surface panel strengths for the next time step. At each time step the wake panels are moved according to the induced velocity at their particular location, and the influences of all panels (surface and wake) are taken into account in this calculation.

(III) Test Cases

The analyses presented in this paper are based on the Hartzell propeller fitted to the Pilatus PC-9A. Figure 1 illustrates the example propeller together with the axes system used in the analysis. The principal specifications of the propeller are listed in Table 1.

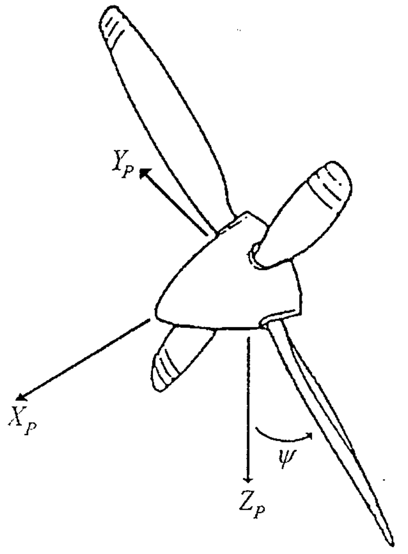

Figure 1: Propeller based axis system.

Propeller diameter	8 ft.
Number of blades	4
Propeller weight	162.3 lbs.
Spinner diameter	18.5 in
Propeller RPM	2000 (constant)
Blade section mean line	NACA 65 series

Table 1: Propeller specifications

This propeller is fitted to a constant speed unit that varies the propeller blade pitch angle as a function of advance ratio (J) and power coefficient (C_P), and all three computational models account for this. The blade pitch angle for each flight condition investigated is determined from data supplied by Hartzell Propellers Inc. [1] (Figure 2).

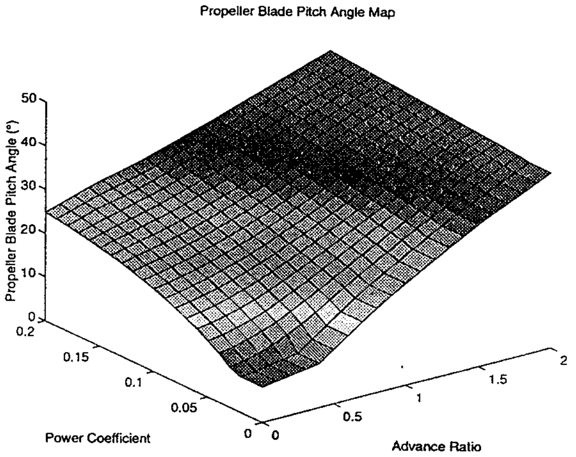

Figure 2: Variation of blade pitch due to constant speed unit.

Nine flight conditions have been investigated for this paper, and these have been selected to represent the full range of the flight envelope of aircraft to which this propeller is typically fitted (such as the PC-9A). The combinations of J and C_P corresponding to these flight conditions are given in Table 2.

Flight Condition	J	C_P
1	1.50	0.180
2	1.00	0.180
3	0.50	0.180
4	1.50	0.100
5	1.00	0.100
6	0.50	0.100
7	1.50	0.030
8	1.00	0.030
9	0.50	0.030

Table 2: Flight conditions

(IV) Computational Model

All three computational techniques utilise the same baseline model tailored as required for application to each scheme. The baseline model only considers the actual blades, and no attempt is made to model the presence or influence of the propeller spinner. As such, the propeller blades are modelled as extending from the edge of the spinner out (Figure 3), with the exception of the strip theory model which considers the portion of the blade outside $x = 0.3$ only.

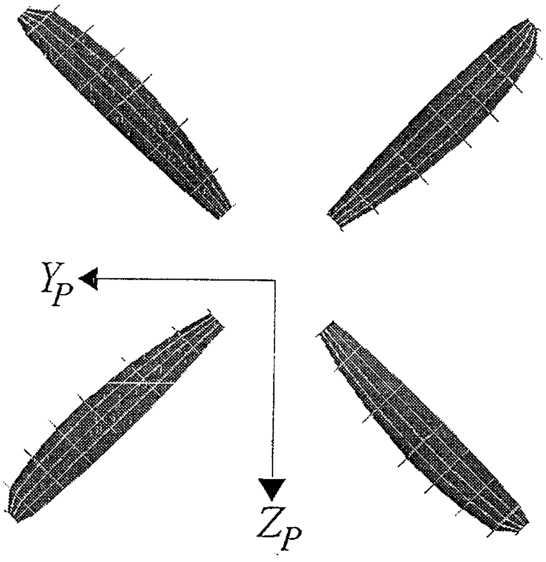

Figure 3: Computational model of propeller blades.

The baseline model represents each blade as eight radial strips/segments/panels. Table 3 gives the geometric data at the eight radial stations used to construct the model.

Station	x	c	θ_0	$\frac{t}{c}$
1	0.20	0.1164	56.74	0.466
2	0.30	0.1562	47.57	0.232
3	0.45	0.1854	37.26	0.126
4	0.60	0.1909	29.57	0.084
5	0.70	0.1809	25.42	0.070
6	0.80	0.1593	22.32	0.064
7	0.90	0.1260	20.24	0.054
8	0.95	0.0929	19.40	0.053

Table 3: Propeller blade station geometric data

(V) Results

This section is intended to provide an overall assessment of the suitability of the techniques to the modelling of an isolated propeller. In line with this, the thrust and torque coefficients predicted by the three techniques are compared to each other and the manufacturer's data. In addition, the ease of application, ability to model forces and moments in the plane of the propeller disc, ability to model the flow up or down stream of the propeller, and the computational efficiency/solution time are compared.

Comparison of Thrust and Torque Predicted

The published data presented in Tables 4 and 5 were supplied by the propeller manufacturer. The data are believed to have been sourced from a computational model, rather than wind tunnel testing, however, this could not be verified. As such while the published data have been used for comparison, they cannot be considered a "control" for the experiment. Independent wind tunnel or flight testing of the propeller would be required to confirm the manufacturer's data.

Flt. Cond.	C_T Pub. Data	Strip Theory	BE Theory	FW Theory
1	0.114	0.116	0.118	0.140
2	0.151	0.151	0.174	0.192
3	0.202	-	0.228	0.232
4	0.059	0.052	0.053	0.070
5	0.089	0.077	0.092	0.121
6	0.141	-	0.151	0.169
7	0.005	-0.008	-0.012	0.010
8	0.018	0.008	0.005	0.036
9	0.040	-	0.032	0.065

Table 4: Comparisons of computed thrust coefficients to published data

Flt. Cond.	C_Q Pub. Data	Strip Theory	BE Theory	FW Theory
1	0.030	0.021	0.031	0.036
2	0.029	0.028	0.032	0.037
3	0.029	-	0.029	0.036
4	0.016	0.009	0.014	0.019
5	0.016	0.014	0.016	0.021
6	0.016	-	0.017	0.020
7	0.004	-0.001	-0.002	0.007
8	0.005	0.002	0.002	0.008
9	0.005	-	0.004	0.007

Table 5: Comparisons of computed torque coefficients to published data

Examination of the tables shows that for high power coefficient conditions (flight conditions

1, 2, and 3) the strip theory provides good agreement, with variations from the published data and the blade element results of less than 2% for C_T, and 30% for C_Q. The agreement of these results suggests that the published data may be based on techniques very similar to those of the strip theory or blade element analyses used here-in. The free-wake results show significantly more variation from the published data with differences in C_T ranging from 15% to 27%. C_Q results show a similar level of agreement with differences ranging from 22% to 28%. The blade element results are in better agreement, with variations in C_T ranging from 3% to 15%, and variations in C_Q ranging from 1% to 12%. Both blade element and free-wake techniques show consistently greater C_T and C_Q results compared to the manufacturer's data, at these flight conditions.

The moderate power coefficient results (flight conditions 4, 5, and 6) generally show slightly higher levels of variation from the published data (than the high power coefficient conditions). The strip theory results show differences in C_T around 13%, and differences in C_Q between 12.5% and 44%, compared to the published data. The blade element results (when compared to the published data) show slightly reduced variation compared to the high power coefficient conditions. Variation in C_T ranges from 3% to 10%, and in C_Q from 0% to 12.5%. The free-wake results again show higher levels of variation in C_T, ranging from 18% to 36%, and C_Q, ranging from 19% to 31%. In general, the published data, strip theory, and blade element all give thrust and torque coefficients of similar magnitude, while the free-wake analysis gives significantly higher results.

All three techniques show significantly more disagreement for the low power coefficient flight conditions (7, 8, and 9). Differences in C_T as large as 0.017 occur for some flight conditions when compared to the published data. This is not so much indicative of the capabilities of the techniques employed, but a reflection of the sensitivity of the thrust and torque coefficient calculations in these conditions, due to their small magnitudes. In these conditions the propeller is producing virtually no thrust and in some cases (flight condition 7) some of the results indicate windmilling.

The consistent over estimation of thrust and torque coefficients by the free-wake technique, for all flight conditions, is most likely a result of consistent under estimation of the inflow at the propeller disc. This would result in over prediction of lift and induced drag on the propeller blade and, therefore, propeller thrust and torque. Detailed reasons of the mechanism of this are the subject of further research and are beyond the scope of this paper.

As a general conclusion the tables show that the blade element technique gives results in better agreement with the published data than the free-wake technique. Although under the conditions at which it has been applied, the strip theory gives better agreement still (particularly for high C_P flight conditions), this is probably a function of the similarity of the techniques used in the strip theory and the calculation of the published data. It should be noted that it could not be confirmed whether the published data applied to a propeller in isolation or installed on an aircraft. Installation effects could also explain some of the discrepancy.

Ease of Application

All analysis techniques used in this paper require detailed knowledge of the blade geometry in terms of pitch angle, twist, and chord as functions of radial station. In addition, the strip theory and blade element techniques require lift curve and drag polar data at each radial station; and the free-wake technique requires mean-line data for the propeller blade sections. With the necessary data available, the ease of use of each technique becomes a deciding factor in the choice of technique.

The "hand calculation" nature of strip theory makes it a slow and tedious process requiring large amounts of "look-up" and interpolation of data tables. This can be somewhat alleviated via use of a spread-sheet or digitisation of tables, but the process is still time consuming. While the free-wake technique only requires knowledge of the blade geometry, setting up the panel model requires extensive data input and is extremely time consuming if done by hand. Appropriate pre-processing of the geometry data can allow the panel setup to be read into the programme from a suitable data file. In addition to the geometric setup, the boundary conditions on the blades must be imposed, and the flight condition being analysed must be specified. The blade element technique is the easiest to apply. With the propeller geometric setup in data files, all that is required is to specify the flight condition being analysed.

Ability to Evaluate Forces and Moments other than Thrust and Torque

The development of forces and moments in the plane of the propeller disc due to the on-coming flow not being aligned with the propeller axis has long been known. Several early techniques determined the side-force developed by the propeller by calculating the force on an "equivalent" fin located at the propeller hub [4, 5, 6]. The dimensions of the fin were determined on the basis of the propeller geometry via empirical equivalence relations derived from wind tunnel testing. While these techniques would provide an estimate of the side-force developed, they do not compute the moments about the propeller hub. Because of this limitation and the fact that they are completely separate to the strip theory, they are not considered here.

The blade element technique evaluates the loading on the propeller blade at intervals around the propeller disc that are specified at the time of calculation. As such the variation of lift and drag (thrust and torque) on each blade element are evaluated and these are integrated over the entire propeller to give the forces and moments at the propeller hub about the Y_P and Z_P axes.

The free-wake technique evaluates the forces on each propeller blade directly at each time step and these are summed about the origin of the axes (which is located at the propeller hub in this case). As such, because the blades of the propeller are moved at each time step, not only are the in-plane force and moment components about the propeller hub evaluated, but also their variation in the time domain.

Figure 4 shows the variation in C_Z and C_n with angle of attack (at zero sideslip), as predicted by the blade element and free-wake techniques.

Inspection of the figure shows that both techniques predict force and moment coefficients of the same order of magnitude. The C_Z coefficients agree well. The C_n coefficient predicted by the blade element technique is approximately double that predicted by the free-wake technique. This discrepancy is most likely due to differences in radial blade loading. The blade element technique does not account for the tip loses of the blades, while the free-wake technique does. As such the blade element analysis will predict higher loading at the blade tips (compared to the free-wake analysis) that will, due to the moment arm, have a significant impact on C_n while only having a minimal impact on C_Z.

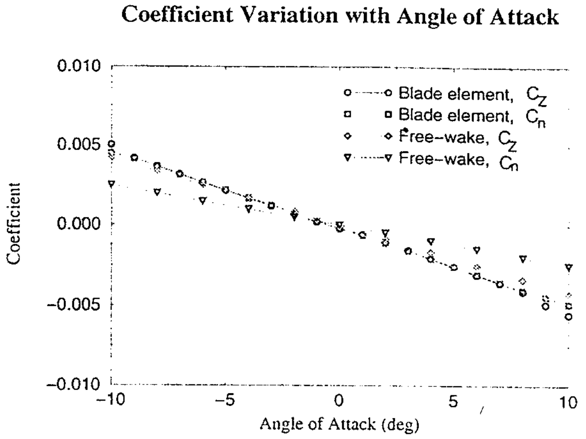

Figure 4: Coefficient variation with angle of attack for an isolated propeller

While the published data for the propeller does not provide values of the in-plane force and moment coefficients, the agreement of the blade element and free-wake techniques with the published data for thrust and torque, suggests a reasonable degree of accuracy for the prediction of the in-plane forces and moments.

Ability to be used for Analysis of the Flow Up or Down Stream of the Propeller

Strip theory was and will only ever be a technique suitable for giving a first approximation to the thrust and torque generated by a propeller. It is too cumbersome and does not provide the necessary information to allow modelling of the flow upstream or downstream of the propeller. However, blade element and free-wake techniques both allow modelling of the propeller slipstream/airframe interaction with some degree of accuracy.

The blade element technique can be coupled to a semi-empirical analysis of the location of the slipstream locus as it progresses downstream, taking into account the upwash, the downwash, and the side-wash of the propeller, wing, and tail, and the side-wash induced by the fuselage. The locus is assumed to move in a straight line from the propeller hub to the wing quarter chord, and from the wing quarter chord to the tail (Figure 5).

With the location of the slipstream relative to the wing and tail known, the induced velocities on these surfaces can then be calculated and the forces and moments integrated to yield total aerodynamic coefficient increments due to the slipstream/airframe interaction.

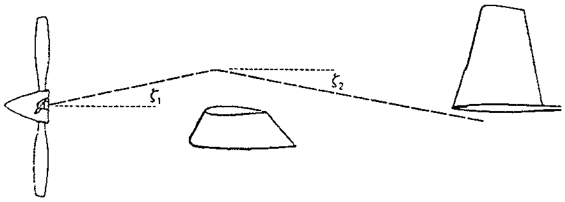

Figure 5: Blade element technique approximation to motion of slipstream locus.

The lack of empennage interference information suggests that the force and moment increments due to the wake interference with the empennage are difficult to estimate with any degree of accuracy, and remain an uncertainty in the method.

The free-wake technique provides the greatest possibility of truly accurate modelling of the slipstream/airframe interaction. In this technique the wake panels are shed off the trailing edge of the propeller blades and allowed to move with the induced velocities (due to all panels) at their location (refer to Figure 6 for an example of the shed wake shape). As such, the wake is free to move as it would in the "real world" and should develop the correct shape.

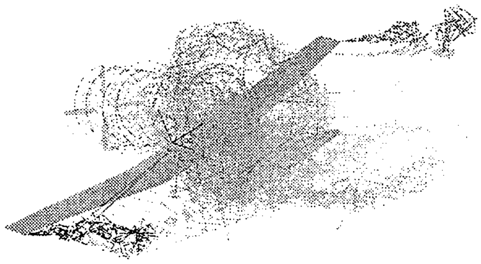

Figure 6: Free-wake interaction of propeller slipstream with wing and tail.

However, if the time step used in the solution is to large this can lead to a wake panel passing through a solid surface (a situation which is physically impossible), and caution must be used to ensure this does not happen.

The variation of thrust and torque predicted by the free-wake technique, compared to the other techniques, suggests that the inflow at the propeller disc is being consistently under estimated. This will have the follow-on effect of producing errors in the helix angle and velocities within the propeller wake. As such, the structure of the slipstream/airframe interaction predicted by the free-wake technique will be correct, but the magnitude of the forces and moments predicted may be in error.

Computational Efficiency

While the free-wake model of the propeller does hold better hope of accurately evaluating the forces and moments on an aircraft in power-on flight, due to the ability to correctly model the slipstream/airframe interaction, Table 6 shows that a simple model of the isolated propeller required a three hundred fold greater solution time compared to the blade element technique combined with a semi-empirical approximation of the slipstream/airframe interaction. The data given in Table 6 are for a typical analysis.

Technique	Solution Time [sec]
Strip theory	≈1500
Blade element	1
Free-wake	300

Table 6: Comparison of solution times

The greater solution time given will only be further exacerbated by increasing model complexity and the inclusion of other aerodynamic surfaces, from which additional wake panels will be shed. As such increasing model complexity or solution length will result in exponential increase in the solution time required. Combined with the evidence from Tables 4 and 5 that the blade element technique generally provides better results for the propeller forces and moments, this suggests that the free-wake technique has not matured sufficiently to be suitable for everyday usage. It is, however, evident from initial results that if solution times can be reduced, and thrust and torque prediction improved/verified, the free-wake technique holds great promise for spot checks of more empirical techniques, and eventually everyday usage.

The solution time given for the strip theory technique is an approximate value based on the hand calculations used in this paper. The hand calculation nature of the technique means establishing a more precise solution time is near impossible. It can, however, be seen that there is a substantial time penalty over the other techniques.

(VI) Conclusions

Strip theory will provide a good first approximation to thrust and torque coefficients. However, it is a time consuming processes and the empirical basis of the technique throws doubt over its general applicability.

Blade element analysis is an easy to use technique that provides estimates of thrust and torque, as well as forces and moments in the plane of the propeller disc, with reasonable accuracy. When coupled to a semi-empirical analysis of the motion of the slipstream locus it can also be used to estimate the forces and moments due to the slipstream/airframe interaction.

Free-wake analysis is the technique that holds the greatest promise of correctly modelling not only the forces and moments generated by the propeller, but also those due to the interaction of the slipstream and the airframe. However, the greater solution time over the blade element technique currently prohibits it from being used for solution throughout the entire flight envelope.

For general usage, blade element analysis offers a good level of accuracy, along with a solution time that makes it suitable for investigation of the complete flight envelope [8]. It also provides an initial estimate of the forces and moments due to the slipstream/airframe interaction. In this regard it is currently the most attractive and amenable method for large scale analysis.

Given improvements in thrust and torque prediction and solution time the free-wake analysis may become suitable for more general application in the future.

(VII) References

[1] Performance maps of the Hartzell HC-D4N-2A/D9512AK propeller as fitted to the Pilatus PC-9A aircraft, 1996.

[2] C. N. H. Lock, R. C. Pankhurst, and J. F. C. Conn. Strip theory method of calculation for airscrews on high-speed aeroplanes. Reports and Memoranda 2035, Aeronautical Research Committee, October, 1945.

[3] D. M. Newman. *Application of time domain modal aerodynamic analysis to aeroelastic and flight dynamic modelling.* PhD thesis, Department of Aeronautical Engineering, University of Sydney, 1997.

[4] H. S. Ribner. Formulas for propellers in yaw and charts of the side-force derivative. Technical Report 819, National Advisory Committee for Aeronautics, 1945.

[5] H. S. Ribner. Propellers in yaw. Technical Report 820, National Advisory Committee for Aeronautics, 1945.

[6] H. S. Ribner. Proposal for a propeller side-force factor. Wartime Report L-336, National Advisory Committee for Aeronautics, December, 1943. Originally published as NACA RB-3L02.

[7] N. van Bronswijk and P. W. Gibbens. Modelling of propeller forces and moments due to asymmetric loading across the propeller disc. In *Proceedings of the International Aerospace Congress 97*, pages 785–800, February, 1997.

[8] N. van Bronswijk, P. W. Gibbens, D. M. Newman, and K. C. Wong. Investigation of the effects of propeller power on the stability and control of a tractor-propeller powered single-engined low-wing monoplane. Aero Tech Note 9801, November, 1997.