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## THEORETICAL STUDY OF TRANSONIC FLUTTER/BUZZ IN THE FREQUENCY AND TIME DOMAIN

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**Abstract.** The iterative method for transonic flutter calculation is developed. The problem of unsteady flow near elastic deforming wings at transonic Mach numbers is solved using Godunov finite-difference method of Euler equations integration. As initial approximation of frequency and deformation shape the result of flutter linear problem or experimental data are applied. The flow near the wing vibrating with specified frequency and mode is analyzed by nonlinear transonic theory. Calculated pressure difference coefficients are used to obtain aerodynamic matrices through harmonic linearization procedure. Linear flutter equations with these matrices are solved in frequency domain. The process is then repeated up to flutter frequency convergence.

Proposed method is illustrated by some flutter characteristic calculations for several aeroelastic structures: standard AGARD wing 445.6, fin with rudder, heavy transport aircraft. Various parameters influence on the transonic flutter is investigated.

The controllable equations of motion of elastic aircraft are obtained taking into account nonlinear unsteady aerodynamic loads in the transonic flow. The method is proposed and developed to solve time domain equations. The comparative results of structure aeroelastic behavior are presented for linear, harmonically linearized and nonlinear transonic cases.

### 1. Introduction.

In the last decade there have been extensive developments in computational methods in response to the need for computer codes which is available to study fundamental aeroelastic problems in the critical transonic regime. The computational methods provide a new tool which can be used in the combination with test facilities to reduce the cost and time of the aircraft designing. Many difficulties arise in the flutter analysis of modern wings. Numerous experiments have shown that dips occur at

transonic Mach number in the flutter boundaries. This phenomenon is caused by the motion of shock waves on the wings. To describe of the physics of such moving shock wave it is require to use the computational methods solving nonlinear partial differential equations for regions of mixed subsonic and supersonic flow. The solution of the fluid dynamics problem is usually accompanied by the solution of the aeroelastic problem to obtain the structural deflections, stability limits and flutter speeds.

Some modern aircrafts encounter a phenomenon known as a limit cycle oscillations (LCO), which is a type of flutter where the aerodynamic forces are essentially nonlinear. Semi-empirical methods are currently being developed to predict LCO but they require a lot of wing tunnel experimental data<sup>(1)</sup>.

Here results will be presented for iterative method of calculation which combine linear classical flutter analysis with Godunov finite-difference method of Euler equation integration. For the computational flutter investigations a software package KC-2 is applied. KC-2 uses doublet-lattice method<sup>(2)</sup> to compute airloads in subsonic flow and panel method<sup>(3)</sup> in supersonic flow. The nonlinear aerodynamic theory allowing to account for the thickness of an airfoil, the amplitude of vibrations and shock movement are used in proposed iterative method (TRAN computer code<sup>(4)</sup>).

In the case of transonic flow where airloads are essentially nonlinear the method of fluid/structure direct coupling is proposed (TRTDR computer code). For both frequency and time domain calculations the flow is modeled by using the Euler equations of the ideal gas.

The purpose of the work is the description of the transonic flutter investigation method. Proposed method is illustrated by some flutter characteristic calculations for several aeroelastic structures in frequency and time domain.

## 2. Aeroelastic equations of motion

Aeroelastic equations of motion are derived on the base of methods which are developed in KC-2 computer code. The Ritz method is used in KC-2 when the deformations  $W$  are represented as polynomial functions of the spatial coordinates. The whole structure is represented by a set of thin, originally flat elastic surfaces which can be arbitrarily located in the space. For each elastic surface the distribution of mass and stiffness is specified. All elastic surfaces are joint in unified structure by the elastic connections which allow to simulate various fastening conditions in the attachment points between them with required accuracy.

The displacement  $W(x, z, t)$  of the aircraft structure is represented by a set of  $N$  polynomial terms:

$$W(x, z, t) = \sum_{k=1}^N y_k(t) f_k(x, z),$$

where  $f_k(x, z) = x^{m_k} z^{n_k}$ ,  $m, n = 0, 1, \dots$

For each elastic surface the polynomial can be chosen separately. The factors  $y_k(t)$  are used as generalized coordinates of the polynomial method. The directions of the axes of the coordinate system are shown in fig. 1.

The aerodynamic loads which may be found by some aerodynamic theory or taken from experimental data are acted on the elastic surfaces in the flow.

For the calculation of the aerodynamic forces acting on the aeroelastic surface the displacements and the local angles of attack are determined at the aerodynamic points. Sometimes it is necessary to use the deformation shapes of elastic surface represented as the displacements in  $L$  points which are not the aerodynamic points (for example, when the displacements are

determined from the calculations using the finite element method or from the experiment). Then the known interpolation formula is used<sup>(5)</sup>:

$$W(x, z) = \sum_{i=1}^L a_i R_i^2 \ln R_i^2 + b_1 + b_2 x + b_3 z$$

$$R_i^2 = (x - x_i)^2 + (z - z_i)^2$$

Unknown coefficients  $a_i$  and  $b_i$  are determined from the known displacements in  $L$  points  $x_i, z_i$  and from the additional conditions

$$\sum_{i=1}^L a_i = 0 \quad \sum_{i=1}^L a_i x_i = 0 \quad \sum_{i=1}^L a_i z_i = 0$$

Thus we can obtain the matrices of the equations of motion for the united vector of the polynomial method for the whole elastic system. For solving the dynamic aeroelasticity problems (flutter, dynamic response, aeroservoelasticity) the equations are reduced to the generalized coordinates corresponding to the natural modes without flow. They can be written in matrix form as:

$$C \ddot{q} + D_0 \dot{q} + Gq = Q^a + R\delta_r + F \quad (1)$$

$$y = H_0 q + H_1 \dot{q} + H_2 \ddot{q}$$

where  $q = \text{col}(q_r, q_e, \delta)$  - generalized coordinate vector including the motion of the aircraft as the rigid body  $q_r$ , its elastic deformations  $q_e$ , control deflections  $\delta$ ;

$C, D_0, G$  - matrices of inertia, damping and stiffness of structure;

$Q^a$  - vector of generalized aerodynamic forces;

$\delta_r$  - vector of actuator rod deflections;

$R$  - matrix of actuator influence efficiency;

$F$  - vector of the external concentrated forces;

$y$  - output parameters vector including displacements, angles, accelerations and angular rates at sensor locations and also loads in chosen sections of structure;

$H_0, H_1, H_2$  - matrices of transformation from generalized coordinates, their velocities and accelerations to physical ones.

In the case of the linear aerodynamics the generalized aerodynamic forces in the equations

of motion are often determined under the harmonic assumption. In this case:

$$Q^a = -\rho V D \dot{q} - \rho V^2 B q + \rho V D^w w$$

where  $D$ ,  $B$  - matrices of aerodynamic damping and stiffness computed for specified reduced frequency (Struhal number);

$\rho$  - air density;

$V$  - true air speed;

$D^w$  - vector of the gust efficiency;

$w$  - gust intensity.

Solution of homogeneous and nonhomogeneous equations with linear aerodynamics in frequency and time domains is considered in References 6, 7.

The computation of the transonic flow about the wing is a nonlinear problem and computational results depend on both the surface deformation and on its amplitude. It is necessary to note that in linear case the flutter mode is determined up to any factor, while in transonic case the vibrations with flutter frequency should have specified amplitude because unsteady pressure difference  $\Delta p(x, z, t)$  depends on it. The airloads calculations are based on the forced vibration method and the principle of harmonic linearization.

As to determine unsteady pressure difference  $\Delta p(x, z, t)$  it is necessary to specify previously the wing deformations and vibration frequency under flutter conditions. The results of linear flutter problem are usually used as initial approximation. After that the flow near the wing vibrating with specified frequency and mode is analyzed using nonlinear transonic theory. Then the sine and cosine components of main frequency are extracted from the found dynamic pressure distribution  $\Delta p(x, z, t)$  and new aerodynamic matrices are computed. The equations of vibration in the flow are solved anew. These equations have the same form as in the linear case. Thus new frequencies and flutter mode appear. The process is repeated up to flutter frequency convergence. Usually linear flutter analysis is performed for the structure in the beginning. But there may be some regions of parameters (Mach number,

reduced frequency, air density) in which linear flutter is absent. For this reason one of the vibration eigen mode and frequency (calculated or experimental) may be taken as first approximation. More detailed calculation algorithm is presented in Reference 4.

A lot of additional useful information about dynamic response can be obtained from time domain analysis in the case of essentially nonlinear system. When investigating the transonic aeroelastic phenomena the direct coupling numerical integration of the equations (1) and Euler equations of transonic flow is executed in time domain. On each step of integration the whole field of the velocities, pressure and density are determined by the Godunov method [4]. Boundary conditions in the points of the aerodynamic grid of moving elastic surface  $\dot{W}$ ,  $\frac{\partial W}{\partial x}$ ,  $\frac{\partial W}{\partial z}$  are calculated through the vectors of generalized coordinates and velocities

$$\begin{aligned} \dot{W} &= XU\dot{q} - w(x, z, t); \\ \frac{\partial W}{\partial x} &= X^x U q; \quad \frac{\partial W}{\partial z} = X^z U q \end{aligned}$$

where  $X$ ,  $X^x$ ,  $X^z$  - corresponding polynomial transformation matrices;

$U$  - modal matrix;

$w(x, z, t)$  - distribution of the gust velocities.

Obtained dynamic pressure difference  $\Delta p(x, z, t)$  is summed through the vibration modes for the determination of the generalized aerodynamic forces:

$$Q^a = X^T U^T S \Delta p$$

where  $S$  - diagonal matrix of the aerodynamic element areas.

Thus to determine the right hand of the equation (1) at the given time moment all field of the flow parameters depending on the preceding process, vibration amplitude and the motion of shock waves is used. Integration of the equations is executed by Euler method of the 1-st order. For choosing the optimal integration step the system with linear aerodynamic is investigated beforehand. Usually the step size is limited by the stability

of Godunov finite-difference procedure rather than elastic oscillations.

The examples illustrating the possibilities of the proposed methods of transonic flutter computation in frequency and time domains are considered below.

### 3. AGARD wing 445.6

AGARD wing 445.6 is frequently used for comparisons of the computational methods. The comparative results are given in fig. 2 for the variant "2.5 foot weakened model 3" (8). Four first vibration modes were taken for calculations. Their frequencies are listed in the following table:

No	f, Hz NASA	f, Hz TsAGI	Mode
1	9.6	9.56	1-st bending
2	38.1	38.09	1-st torsion
3	50.7	48.15	2-nd bending
4	98.5	92.04	2-nd torsion

The non-dimensional flutter speed coefficient  $\bar{V}_f$  for each Mach number was calculated using linear unsteady subsonic and supersonic aerodynamic theory, and iterative transonic method.

As stated above in transonic flow the flutter speed depends on amplitude of oscillations. For this reason as initial approximations the results of linear flutter problem are used when the mode shapes are multiplied on a certain coefficient. Thus the dimensional amplitude can be introduced in the procedure of solving the equation of the wing vibrations in the flow.

On fig. 2 the results of TRAN-method are represented for two values of the amplitudes of flutter mode shape, corresponding to torsion angle on the wing tip: 1 and 6 degrees. As it is shown on fig. 2 the change of the amplitude of oscillation can affect the transonic flutter speed. This can be explained by the fact that in transonic dip the characteristics of flutter

motion are changed and the nature of instability becomes essentially nonlinear. In iterative method unsteady aerodynamic coefficients are computed with allowance for the shock movement and its intensity changes caused not only by the displacement variations but also by the velocity of the displacement. The discrepancy between experimental data and nonlinear theory is due to the presence of the viscosity in the flow. (TRAN solves Euler equations, the viscosity is not taken into account). At large subsonic Mach numbers nonlinear theory underestimates flutter speed coefficient  $\bar{V}_f$  in comparison with experimental data. On the other hand linear aerodynamic theory overestimates flutter speed coefficient  $\bar{V}_f$ . Out of the transonic dip the discrepancies between linear, nonlinear theory and experiment are very small; the characteristics of flutter become independent on vibrations amplitudes.

Calculated results in time domain for flow parameters, corresponding experimental flutter point, are presented on fig. 3. At Mach number 0.96 two different types of the excitations are used to start time process. In case a) force 2.5 kg is applied on the tip of the wing, in case b) initial condition is specified as deflection upon the first eigen mode. Wing tip displacement indicates flutter. For both cases calculated displacements for linear and nonlinear theories are presented. As it is shown on fig. 3 the linear process is damped, nonlinear process is antidamped and flutter frequency is 14.5 Hz. The comparison of results in time domain and in frequency domain gives satisfactory agreement.

### 4. One DOF Flutter.

The control surfaces transonic autooscillations (buzz) were investigated numerically. This phenomenon is characterized by the negative aerodynamic damping in transonic flow (9).

The plan view of the fin having 10% of relative thickness with rudder is shown on fig. 4. The fin bending and torsion stiffness was specified rather high, to have possibility to consider

structure as a system with one degree of freedom (DOF):

The amplitude influence on the aerodynamic hinge moment ( $\delta=\delta(t)$  angle of rudder deflection) was investigated. Figure 4 gives result of hinge moment coefficient  $m_h^\delta$  calculation versus the amplitude  $\delta$  at Mach 0.95 and vibration frequency  $\omega=30$  Hz. The limit cycle is realized at the amplitude  $\bar{\delta}$  when aerodynamic damping changes the sign. The peculiarity of this type of autooscillations is that the occurrence of buzz is shock related. It was found from numerical investigations that the shock oscillations were synchronized, but not in phase, with the motion of the rudder. This phase lag between the shock wave motion and the motion of the rudder plays a significant role in the mechanisms producing control surface buzz.

Representative results from nonlinear transonic method calculation in time domain for control displacement are shown on fig. 5. If initial amplitude  $\delta_0$  is smaller than limit cycle amplitude, it increases with time up to limit value. When time response process is started from greater than limit amplitude, it is damped at the same Mach number and frequency of oscillation. It is need to note that an estimations of limit amplitudes, receiving in frequency and time domain procedures, have not coincided exactly. Based on harmonical linearization approaches proposed iterative method gives only qualitative agreement with direct coupling method.

Two process calculated on nonlinear transonic theory gives qualitatively the same results. In the first of them generalized aerodynamic forces are computed on every time step by coupling method. The second is based by the principle of harmonically linearization. Linear aerodynamic theory (Doublet-Lattice method) gives damped time response process for any initial conditions.

### 5. Heavy transport aircraft.

The flight test results showed a self-excited sinusoidal vibrations with limit amplitudes of the major aircraft components at Mach number 0.86 and single frequency near 3 Hz. Linear

classical flutter analysis can not find any instabilities in the mentioned above flight parameters. The relative thickness of the wing is 14.5% at the root chord and 9% at the tip. A mixed subsonic/supersonic flow with shock waves realizes on the surface of such wing. For this reason nonlinear transonic theory is applied for airloads calculations.

Mathematical model of aircraft elastic structure includes 13 symmetrical vibration modes. The fourth eigenmode is selected as initial approximation for the iterative method. This eigenmode has frequency  $f=2.40$  Hz; mode shape is shown on fig. 6.

The critical mode with frequency 2.84 Hz appears in the next iterative steps. Fig. 7 gives results of flutter calculations at few values of the structure damping coefficient (decrement) at Mach number 0.86. The critical flutter dynamic pressure  $Q$  depends on the amplitude  $\alpha$  of flutter oscillations (torsion angle at the wing tip).

Limit value amplitude can be found for fixed Mach number and damping coefficient  $\eta$ . If the amplitude of vibrations is smaller than limit value, then critical flutter mode exist. Flutter absent if the amplitude is larger than limit value.

This phenomenon, probably, is LCO of the investigated critical flutter mode. In order to estimate the limit cycle amplitude it is need to explore the damping curve for the critical mode. It has a zero crossing and zero incidence in the point of limit cycle parameters. Flutter mode shapes in two phases at  $\varphi=0^\circ$  and  $\varphi=90^\circ$  (corresponding to  $1/4$  of the period of flutter oscillations) at  $\eta=0.05$ ,  $\alpha=3.5$  degrees are presented on fig. 8.

Time process of the transonic flutter oscillations is shown on fig. 9. Load factor and torsion angle on the tip of the wing were calculated at the same flight condition ( $M=0.86$ ) and the same initial condition for nonlinear transonic case and linear aerodynamics (Doublet-Lattice method). It can be seen from the figure that in transonic case unstable oscillations appear; the amplitude of

the oscillation is limited through the time. In linear case process is stable.

## 6. Conclusions.

One of the versions of mathematical model of elastic structure in transonic flow is developed for frequency and time domains. The model is based on polynomial Ritz method and Godunov's finite-difference method for solution unsteady Euler equations.

Appropriate computer code is generated to compute aeroelastic behavior of structure.

Computations on PC Pentium 200 Pro show that it takes reasonable computing time to obtain flutter boundaries and time domain dynamic responses taking into account specific transonic phenomena for practical structure.

Mathematical model and computer code allow also to analyze aeroservoelasticity problems in transonic flow, but a power of accessible computers limits serious numerical investigations in this field for us.

We hope that more power computers give us opportunity to perform numerical simulations of flight test in transonic flow for the elastic aircraft with control system, including active flutter suppression investigation.

## 7. References

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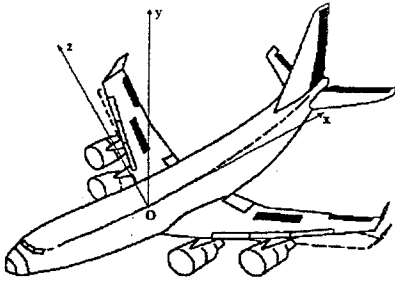


Fig. 1. Reference coordinate system

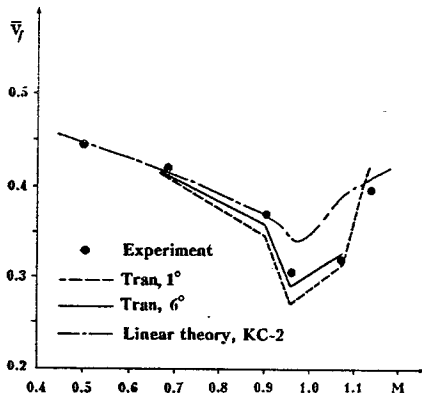


Fig. 2. Comparison of flutter speed coefficients of 445.6 weakened wing

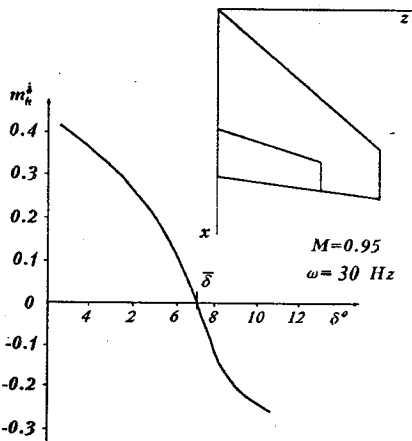
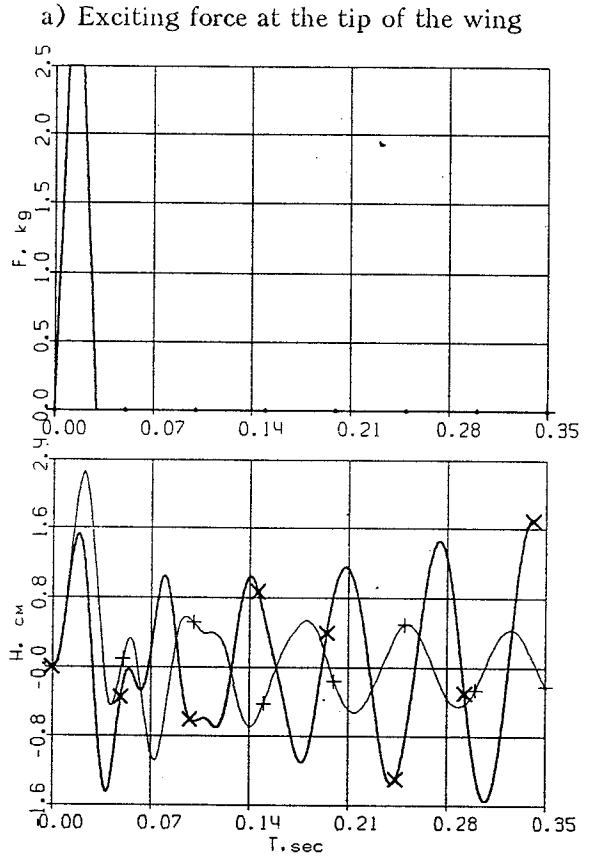


Fig. 4. Amplitude influence on the aerodynamic hinge moment



b) Initial condition  $q_1(0)=0.1$

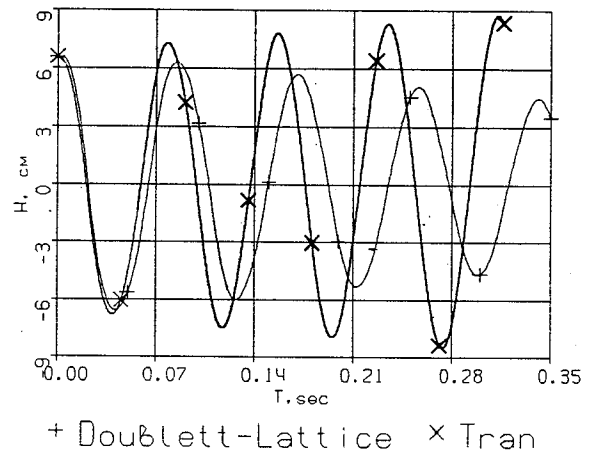


Fig. 3. Wing AGARD 445.6 Time Domain Responses

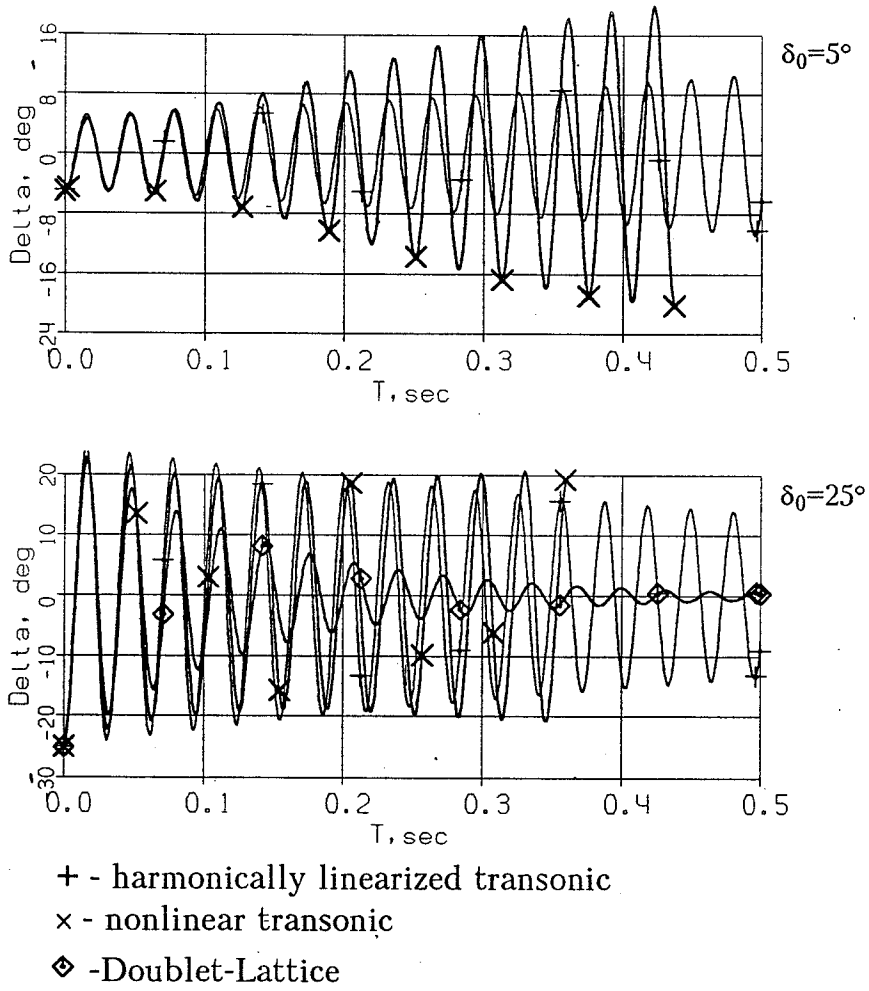


Fig. 5. 1 DOF oscillations in transonic flow

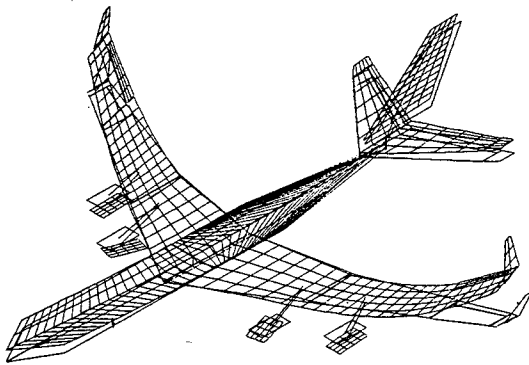


Fig. 6. Symmetrical mode 4:  $f=2.40$  Hz

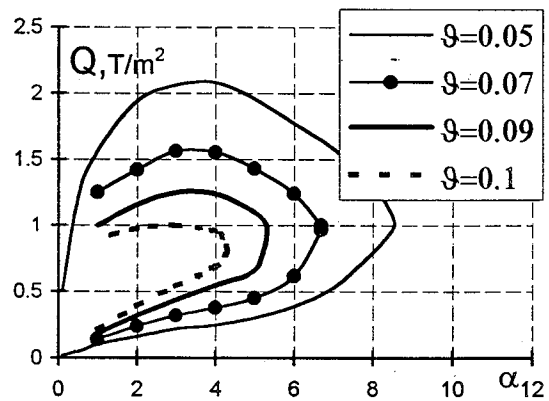


Fig. 7. Amplitude influence on the flutter dynamic pressure ( $M=0.86$ ,  $f=2.84$ )



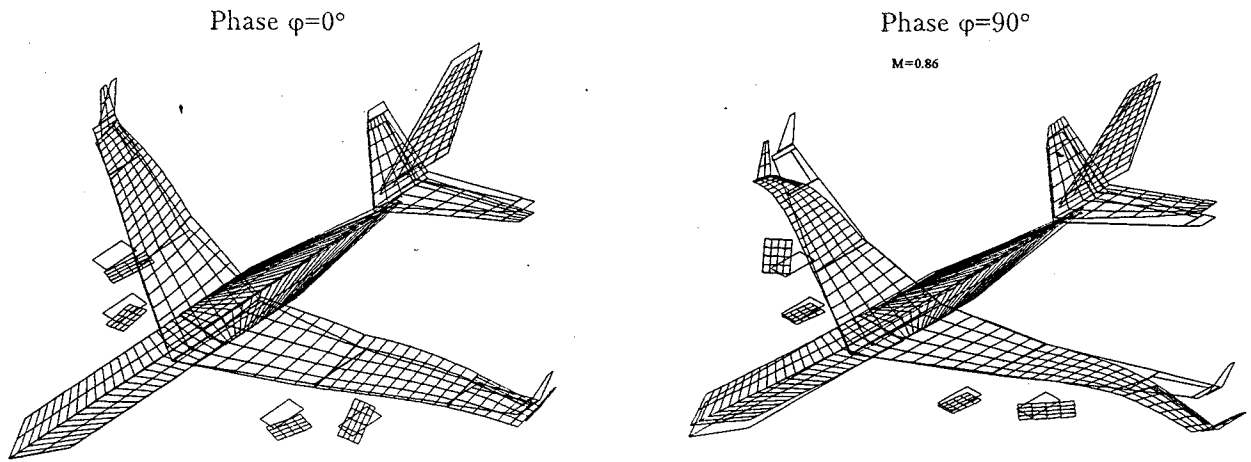


Fig. 8. Flutter mode shapes in two phases ( $f=2.84$  Hz,  $\eta=0.05$ ,  $\alpha=3.5^\circ$ )

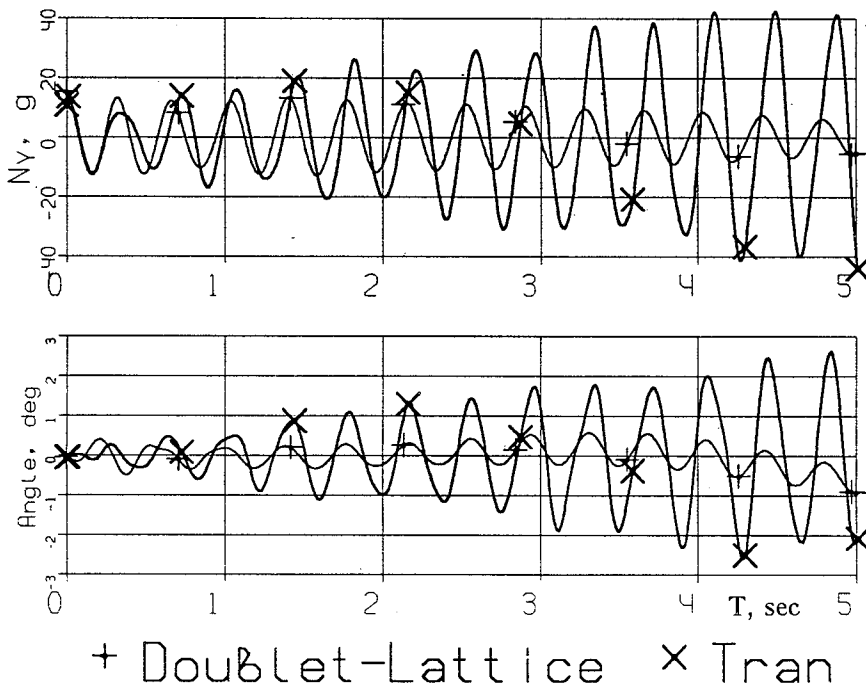


Fig. 9. Heavy transport aircraft: Load factor and torsion angle at the tip of the wing