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DEVELOPMENT OF AN ANALYTICAL EXPRESSION AND A FINITE ELEMENT PROCEDURE TO DETERMINE THE RESIDUAL STRESSES IN BONDED REPAIRS

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Abstract

In recent years bonded repairs using boron patches have been used as a means of life extension for cracked aircraft structures. Bonded repairs are typically cured at 80-100°C for a duration of several hours. However the difference in coefficients of thermal expansion of the components may result in significant residual stresses after cooling to operating temperatures. While an analytical analysis are available for residual stresses for repaired one dimensional strips, no analytical expression exists for the case of heat conduction in a repaired two dimensional plate. In this paper a derivation of a closed form solution will be given for the direct residual stress for a circular patch on a circular plate. These closed form solutions have direct application for the design of bonded repairs to aircraft wing skins. The validation has been carried out using F.E. analysis.

Introduction

In recent years the use of bonded repairs[1] has become a cost effective means of aircraft life extension. The bonding of a high strength patch to a plate involves heating of the patch and adhesive, and localised heating of the plate. This bonding process may be carried out at a temperature of between 80 - 120°C depending on the type of adhesive, and may involve a duration of several hours. When cured, the patch and plate are allowed to cool to room temperature. As a result of the cooling and the different coefficients of expansion between the patch and the plate, residual stresses will occur in the plate, patch and adhesive. Boron/epoxy has been used extensively for repair of aircraft structures and has a thermal coefficient of expansion which is very much lower than aluminium alloys. In this paper the bonding of a circular patch on a circular plate is considered, and an analytical expression is derived for the residual thermal stresses induced by curing.

Heating of a circular plate

Consider now a circular plate shown in Fig. 1. This plate is uniformly heated within a radius $r = R_I$ to a temperature T_I, while the temperature at the boundary is T_O. The temperature solution[2] which satisfies the Laplacian operator:

$$\nabla^2 T = 0 \qquad \text{...(1)}$$

is given by:

$$T = T_O + \frac{(T_I - T_O)\ln(r / R_O)}{\ln(R_I / R_O)} \quad \text{for} \quad R_I \le r \le R_O \text{ and}$$

$$T = T_I \text{ for } r \le R_I \qquad \text{...(2)}$$

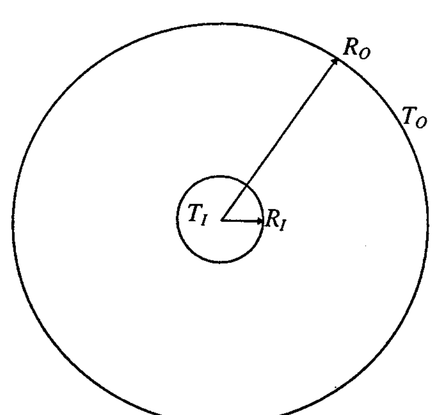

Figure 1.Heating of a circular plate.

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For this temperature distribution, the following differential equation(2) is applicable for the elastic displacement, assuming plane stress:

$$\frac{d}{dr}\left(\frac{1}{r}\right)\frac{d(ru)}{dr} = \alpha(1+\nu)\frac{dT}{dr} \qquad ...(3)$$

where:

r is the radius
u is the radial displacement
ν is Poisson's ratio
α is the coefficient of thermal expansion
T is the temperature.

Analysis of bonded repair

In a bonded repair both the patch and plate are heated to the cure temperature T. In the case of a plate in which the edges are restrained, the heating gives rise to an initial stress in the plate before curing of the adhesive and cooling takes place. In the case of the patch no restraint exists and the initial stress is negligible. To compute the initial stress in the plate we have the following expressions for displacement and radial stress(2) which are obtained from the integration of equation(3):

$$u = \frac{\alpha_1(1+\nu)}{r}\int_0^r Trdr + Cr \qquad ...(4)$$

$$\sigma = -\frac{\alpha_1(1+\nu)}{r^2}\int_0^r Trdr + \frac{E_1}{(1-\nu)}C \qquad ...(5)$$

From the boundary condition that $u = 0$ at $r = R_O$ we obtain the integration constant C. From this the general form for the radial displacement and stress is:

$$u = \frac{\alpha_1(1+\nu)}{R_O}\left\{\frac{R_O}{r}\int_0^r Trdr - \frac{r}{R_O}\int_0^{R_O} Trdr\right\} \qquad ...(6)$$

$$\sigma = -\alpha_1 E_1\left\{\frac{1}{r^2}\int_0^r Trdr + \frac{1+\nu}{(1-\nu)R_O^2}\int_0^{R_O} Trdr\right\} \qquad ...(7)$$

Specifically for $r = R_I$ the displacement is given by:

$$u = \frac{\alpha_1(1+\nu)}{2}R_I(T_I - T_O).\left\{1 + \frac{1}{2\ln(R_I/R_O)}\left(1 - \frac{R_I^2}{R_O^2}\right)\right\} \qquad ...(8)$$

Specifically for $r \le R_I$ we have a constant state of stress given by:

$$\sigma = -\frac{\alpha_1 E_1}{2}\left\{T_I + \frac{(1+\nu)}{(1-\nu)}\left[T_O + \frac{(T_I - T_O)}{2\ln(R_I/R_O)}\left(\frac{R_I^2}{R_O^2} - 1\right)\right]\right\} \qquad ...(9)$$

Equation (9) gives the required initial stress. The second part to the solution of residual stresses involves the analysis of the plate and patch. We require the state of stress corresponding to a cooling of the plate and patch. If we start at a temperature 0°C corresponding to zero initial stress, then cooling to a negative temperature equal in magnitude to the cure temperature ($-T_I$) will give the required stress components. A summation with the initial stresses will give the residual stress state.

In this analysis, the adhesive will not be considered, and the patch is assumed to be rigidly connected to the plate. It will be seen that this assumption is necessary to obtain sufficient equations necessary for the solution. Radial stresses and displacements will be obtained for the skin and patch. Consider a plate shown in figure 2, which has properties E_1, α_1, t_1 for $r \ge R_I$ and overall properties E_O, α_o, t_o for $r \le R_I$. Again the temperature boundary conditions are given by T_I being constant for $r \le R_I$. and $T = T_O$ at $r = R_O$. The displacements are given by:

for $r \ge R_I$

$$u_1 = \frac{\alpha_1(1+\nu)}{r}\int_{R_I}^r Trdr + C_2 r + \frac{C_3}{r} \qquad ...(10)$$

for $r \le R_I$

$$u_o = \frac{\alpha_o(1+\nu)}{r}\int_0^r Trdr + C_1 r \qquad ...(11)$$

The stresses are given by:

for $r \geq R_I$

$$\sigma_1 = -\frac{\alpha_1 E_1}{r^2}\int_{R_I}^{r} Trdr + \frac{E_1}{(1-\nu^2)}\left[C_2(1+\nu) - \frac{C_1(1-\nu)}{r^2}\right] \quad ...(12)$$

for $r \leq R_I$

$$\sigma_O = -\frac{\alpha_O E_O}{r^2}\int_0^r Trdr + \frac{E_O}{(1-\nu^2)}C_1(1+\nu) \quad ...(13)$$

The solution of these equations must satisfy the following conditions:

(a) The displacement u_1 and u_o is equal at $r = R_I$, hence from equations (10) and (11):

$$C_2 r + \frac{C_3}{r} = \frac{\alpha_o(1+\nu)}{R_I}\int_0^{R_I} Trdr + C_1 R_I \quad ...(14)$$

(b) Equilibrium must be maintained across the boundary at $r = R_I$, using equations (12) and (13):

$$\frac{E_1 t_1}{(1-\nu^2)}\left[C_2(1+\nu) - \frac{C_3(1-\nu)}{R_I^2}\right] = -\frac{\alpha_o E_o t_o}{R_I^2}\int_0^{R_I} Trdr + \frac{E_o t_o}{(1-\nu)}C_1 \quad ...(15)$$

(c) Also we have the further boundary condition that at $r = R_O$:

$$u_1 = 0 = \frac{\alpha_1(1+\nu)}{R_O}\int_{R_I}^{R_O} Trdr + C_2 R_O + \frac{C_3}{R_O} \quad ...(16)$$

At this stage we have enough information for the evaluation of the constants C_1, C_2 and C_3.

It is more convenient to have the equations in the form that represent a patch over the skin for $r \leq R_I$ as shown in Fig. 2.

As before, the skin has the properties E_1, t_1, α_1 while the patch has the properties E_2, t_2, α_2. It is necessary to derive an expression for α_o in terms of these quantities. From equilibrium considerations we have :

$$E_o t_o \alpha_o = (E_1 t_1 + E_2 t_2)\alpha_o = E_1 t_1 \alpha_1 + E_2 t_2 \alpha_2 \quad ..(17)$$

hence:

$$\alpha_o = \frac{(\alpha_1 + s\alpha_2)}{(1+s)} \quad ...(18)$$

where:

$$s = \frac{E_2 t_2}{E_1 t_1} \quad ...(19)$$

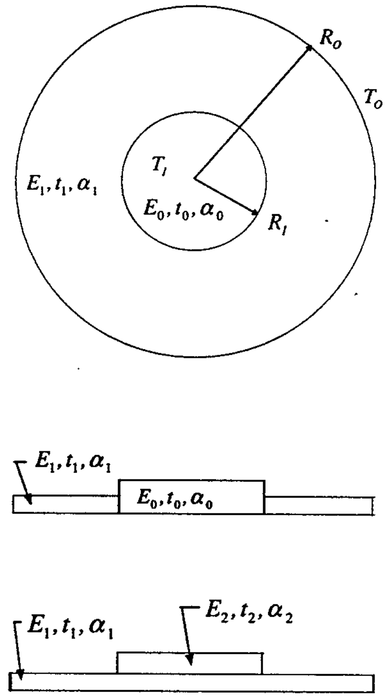

Figure 2.Idealized bonded repair.

The expression for the stress state in the plate just outside the patch is given by equation (12) for $r = R_I$:

$$\sigma = E_1\left[\frac{C_2}{(1-\nu)} - \frac{C_3}{(1+\nu)R_I^2}\right] \quad ...(20)$$

We will now derive the expressions for the stress state in the plate beneath the patch and in the patch. From equation (11) with $r \leq R_I$, under a uniform temperature, the displacement is given by:

$$u = \left[\frac{(1+\nu)\alpha T_I}{2} + C\right]r \quad ...(21)$$

Since the displacement is the same in both the patch and skin we have at $r = R_I$:

$$\frac{(1+\nu)\alpha_1 T_I}{2} + C_1 = \frac{u}{R_I} \qquad ...(22)$$

$$\frac{(1+\nu)\alpha_2 T_I}{2} + C_2 = \frac{u}{R_I} \qquad ...(23)$$

where the displacement u corresponds to the location $r = R_I$

The radial stresses for the plate and patch are given by:

$$\sigma_1 = -\frac{\alpha_1 E_1}{r^2}\int_0^r T_I r dr + \frac{E_1 C_1}{(1-\nu)} \qquad ...(24)$$

$$\sigma_2 = -\frac{\alpha_2 E_2}{r^2}\int_0^r T_I r dr + \frac{E_2 C_2}{(1-\nu)} \qquad ...(25)$$

Using equations (22), (23), (24) and (25) we have the expressions for the radial stresses in the plate beneath the patch and in the patch:

$$\sigma_1 = \frac{E_1}{(1-\nu)}\left[-\alpha_1 T_I + \frac{u}{R_I}\right] \qquad ...(26)$$

$$\sigma_2 = \frac{E_2}{(1-\nu)}\left[-\alpha_2 T_I + \frac{u}{R_I}\right] \qquad ...(27)$$

To obtain the residual stress in the plate beneath the patch it is necessary to sum equations (9) and (26), but with $T_I = -T_I$ in equation (26). Hence the final expression for the residual stress beneath the patch is:

$$\sigma = -\left(\frac{\alpha_1 E_1}{2}\right)\left\{T_I + \frac{(1+\nu)}{(1-\nu)}\left[T_O + \frac{(T_I - T_O)}{2\ln(R_I / R_O)}\left(\frac{R_I^2}{R_O^2} - 1\right)\right]\right\} + \frac{E_1}{(1-\nu)}\left(\alpha_1 T_I + \frac{u}{R_I}\right) \qquad ...(28)$$

Since the initial stress in the patch is zero, then the residual stress in the patch is given by equation (27), but again with $T_I = -T_I$ hence:

$$\sigma = \frac{E_2}{(1-\nu)}\left(\alpha_2 T_I + \frac{u}{R_I}\right) \qquad ...(29)$$

and the final expression for the residual stress just outside the patch is given by the summation of equations (9) and (20), hence:

$$\sigma = \frac{\alpha_1 E_1}{2}\left\{T_I + \frac{(1+\nu)}{(1-\nu)}\left[T_O + \frac{(T_I - T_O)}{2\ln(R_I / R_O)}\left(\frac{R_I^2}{R_O^2} - 1\right)\right]\right\} + E_1\left[\frac{C_2}{(1-\nu)} - \frac{C_3}{(1+\nu)R_I^2}\right] \qquad ...(30)$$

The displacement u at $r = R_I$ for equations (13, 14) and integration constants C_2, C_3 are given in the appendix.

These equations now give the residual stress in terms of the cure temperature T. For operating temperatures different from room temperature, equations (20), (26) and (27) can be used to calculate the stresses. In this case $T_I = T_O$ =uniform temperature change from room to operating temperature. The final stresses are obtained by superimposing these stresses on the residual stresses.

Validation

The solution of these equations has been carried out and the following quantities have been evaluated for the comparison with F.E. results, the mesh is shown in figure 3:

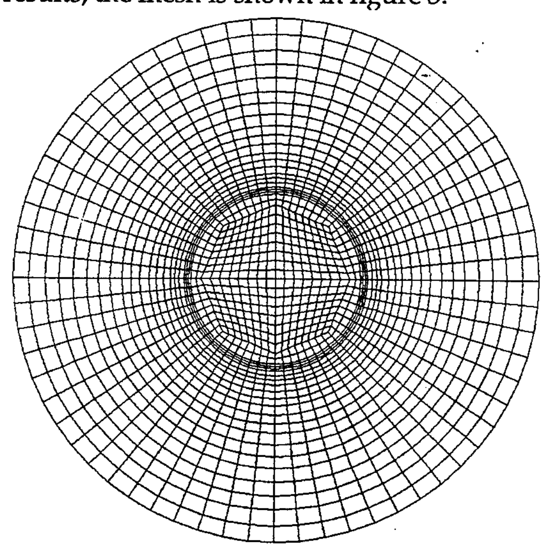

Figure 3.Finite element mesh.

1. residual stress just outside the patch at $r=R_I$

2. residual stress in the skin beneath the patch (σ_1)

1. residual stress in the patch (σ_2)

restrained in the radial direction if the repair is bounded by significant structural elements such as spars.

The effect of relative plate and patch sizes is shown in figure 6. For large plate diameters in comparison to the patch , the results are not significantly affected by patch diameter.

Table 1 Mechanical properties.(Boron properties(4))

Component	Young's Modulus (MPa)	Poisson's ratio	Thermal Coeff. Exp. /^oC	Thickness (mm)
Plate	71016.	0.3	$23x10^{-6}$	1.0
Patch	156000.	0.3	$4.1x10^{-6}$	0.5 .

As an example a circular patch and plate are considered whose mechanical properties are shown in Table 1. These properties are representative of a quasi-isotropic boron patch reinforcement of aluminium plate (although the value of α used here for boron corresponds to uni-directional boron and should have been $6.2x10^{-6}$.for the laminate). While this not a perfect representation of an actual repair it is acceptable for estimating residual stresses. Also this assumes that bending is restrained which is applicable when the repair is bounded by structural elements such as spars and ribs. Consider the case in which the plate edge is restrained in the radial direction. The analytical and F.E. results are shown in figure 4 where the curves are from equations (28), (29) and (30), and the points on the curves are F.E. results. In all cases very good agreement between analytical and F.E. results are obtained, (to four significant figures). The results for the case of a plate in which the edge is not restrained is shown in figure 5. Again very good agreement between analytical and F.E. results is obtained, (to four significant figures). In the latter case the stresses are significantly higher than those obtained in the restrained case. This is simply due to the lack of initial stresses which arise as a result of the restrained edges of the plate when heated up to the cure temperature. The assumption of edge restraints is important. Typically a repair to an aircraft wing skin can be considered as fully

However significant variations occur when the plate diameter approaches the patch size. Limiting values exist for large plate diameters, i.e. $R_I/R_O \to 0$. These limiting values are conservative for stresses in the plate beneath the patch and in the patch, but are un-conservative for stresses just outside the patch.

So far, the adhesive has not been considered in the analysis. However F.E. results have been obtained in which the patch and plate have been coupled using 3D adhesive elements. To make a useful comparison with the previous analytical work the bending of the plate has had to be restrained. The introduction of the adhesive has resulted in an error of only 2% in direct stresses, showing that the use of a closed form solution is sufficiently accurate for patch design.

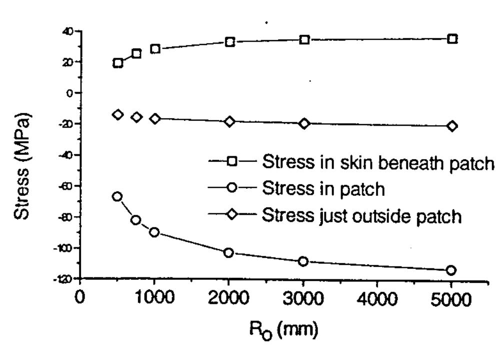

Figure 4.Comparison between analytical and F.E. results for case in which the edge is restrained.

$R_I = 162mm$

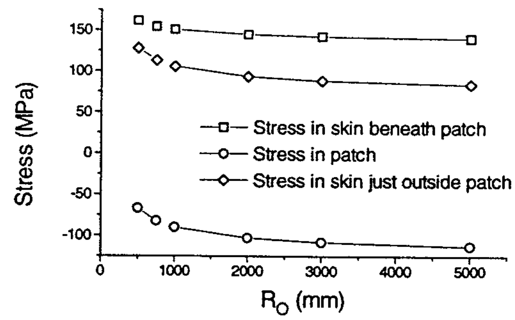

Figure 5.Comparison between analytical and F.E. results, for case in which no edge restraint exists on plate. $R_I = 162mm$

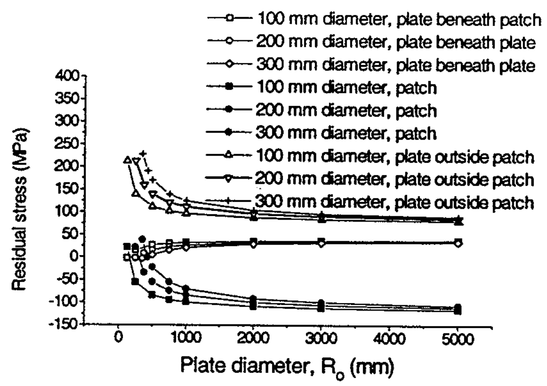

Figure 6 Variation of stresses in repair as a function of relative patch size.(therm/icas98t.org, plot 3) $R_I = 162mm$ (edges restrained)

The main concern in this paper is the equations for direct stresses in the repair. It is important to know the direct stress in the plate beneath the patch in order to predict crack growth rate or simply the residual strength of the repaired structure. While this paper has not considered the adhesive stresses, a closed form solution has been recently derived(5). The adhesive itself has no effect on the maximum value of the direct stresses.

Finite Element Procedure

Some F.E. programs have the capability in which the material properties can be temperature/time dependent. In this case a simulation of the bonding process can be carried out. The adhesive properties change during the curing process. At the end of the curing process the adhesive has developed a shear stiffness and as the repair is cooled to room temperature residual stresses develop. If the simulation capability is available, then residual stresses are directly obtained from the analysis.

If this capability is not available then a superposition procedure can be used. The analysis is carried out in two steps. The first analysis is equivalent to heating up of the plate to the curing temperature (without the patch, since the adhesive has no stiffness at this stage). Secondly, another analysis is carried out with the patch included, subject to a cooling temperature equal to the cure temperature.($T_I = - T_I$). In the work presented here this two stage procedure has been shown to be very accurate. The superposition of these two analyses gives the residual stresses in the repair. Since the adhesive shear modulus is temperature dependent, an arithmetic average value of the shear modulus should be used during the cooling process.

Conclusions

1. A closed form solution has been derived and validated for the direct residual stresses in the patch and in the plate beneath the patch and in the plate just outside the patch.
2. A 3D F.E. analysis has shown that effect of an adhesive has no influence on the direct stresses in the patch.
3. The closed form solutions indicate that the residual stresses are significantly influenced by the plate edge support conditions.

Future work

It is known that during the heating of structure that heat is not only conducted but is also lost to the atmosphere by convection and radiation. Temperature measurements taken during a simulation of a bonded repair (at 90°C) on a F-111 wing have shown that within a distance of *1000mm* room temperature is attained. As a result the closed form solutions are likely to be conservative in that they over-predict the stresses in bonded repairs. An analytical method to account for heat losses is currently being investigated.

Acknowledgment

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Appendix

The displacement at $r = R_I$ is given by:

$$u = \frac{(1+\nu)}{2} R_I \frac{2(((\alpha_O T_I - \alpha_1 T_O) + \alpha_1 R_I^2 (T_I - T_O)/(R_O^2 - R_I^2)) + \alpha_O T_I s) + \alpha_1 (T_I - T_O)/(\ln R_I / R_O)}{2R_O^2/(R_O^2 - R_I^2) + s(1+\nu)}$$

The integration constants are given by:

$$C_2 = -\frac{(1+\nu)}{4} \frac{D_1 + D_2}{2R_O^2 + s(R_O^2 - R_I^2)(1+\nu)}$$

where

$$D_1 = 2(T_O s(R_O^2 - R_I^2)\alpha_1(1+\nu) + 2\alpha_O T_I R_I^2(1+s)) - (R_O^2 - R_I^2)s\alpha_1(T_I - T_O)(1+\nu)/(\ln R_I / R_O)$$

and

$$D_2 = (2s\alpha_1 R_I^2(1+\nu)(T_O - T_I) + 4\alpha_1(T_O(R_O^2 - R_I^2) - R_I^2(T_I - T_O))) - 2\alpha_1(R_O^2 - R_I^2)(T_I - T_O)(1+\nu)/(\ln R_I / R_O)$$

$$C_3 = \frac{(1+\nu)}{4} R_I^2 \frac{F_1 + F_2}{2R_O^2 + s(1+\nu)(R_O^2 - R_I^2)}$$

where

$$F_1 = 2(s\alpha_1(1+\nu)(T_O R_O^2 - T_I R_I^2) + 2\alpha_O T_I R_O^2(1+s))$$

and

$$F_2 = -(R_O^2 - R_I^2)s\alpha_1(T_I - T_O)(1+\nu)/(\ln R_I / R_O)$$