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# INVESTIGATION OF SHAPE OPTIMISATION TECHNIQUES FOR THE DESIGN OF PLATES WITH CUT-OUTS

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## ABSTRACT

The optimisation of holes in both isotropic and laminated plates under a variety of loading conditions has been investigated using the design sensitivity method of MSC/NASTRAN. A least squares objective function was shown to successfully produce a constant stress around the hole boundary in isotropic cases for uniform and non-uniform biaxial load cases. For non-uniform biaxial loading, it was found that a range of harmonic holes exists which depend on the constraints imposed. The optimisation of holes in laminated composite plates showed that the optimum hole shape depends on the degree of orthotropy. Quasi-isotropic laminates produced holes of similar shape to the isotropic case while laminates without fibres in the primary loading direction produced unexpected shapes.

## INTRODUCTION

Much of the previous work performed in the field of structural optimisation has involved the development of solution algorithms and their computational implementation into finite element (FE) codes. However, recent focus has been in the area of optimisation techniques or methods as applied in the structural design environment. Optimisation techniques are effectively the design strategy applied to a given design problem to achieve the intended optimised solution. Given the large number of variables generally involved with structural design, the quality of the optimisation technique applied can greatly increase the effectiveness of the design.

Generally the technique for structural optimisation revolves around three distinct groups: objectives; constraints; and design variables. The choice of optimisation objectives, constraints and design variables effectively controls the type of optimum design that will result. This poses the following questions for the optimisation of a structure: "What is to be optimised (objective)?"; "What is to be constrained?"; and "What are the design variables?". The answers to these questions lead to the development of structural optimisation techniques which make use of existing optimisation algorithms to solve given design problems.

Shape optimisation methods refer to the process of generating improved local geometries through the use of an optimisation numerical algorithm. There exists a wide range of shape optimisation techniques which are well documented throughout the literature. The addition of structural optimisation into commercial FE codes has provided the power to apply these techniques to a variety of structural problems.

The optimisation of the shape of holes or cut-outs is an important detail design problem and is also becoming more prominent in the area of fatigue life extension of existing structures. Such design problems often have constraints imposed, such as material can be removed but not added or the hole cannot be larger than a certain limit. These constrained optimisation problems are often more difficult to solve and little work can be found in the literature. This work concentrates on the application of shape optimisation methods in MSC/NASTRAN for the optimisation of holes in both isotropic and laminated composite plates containing holes. A variety of load cases was considered, as well as several constrained optimisation cases.

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## GENERAL SHAPE OPTIMISATION APPROACH

There exist many methods suitable for shape optimisation of holes in plates, with each possessing unique advantages. Optimisation analyses require the definition of the design variables, the objective function and also constraints.

### Sensitivity Analysis Method

Sensitivity analysis methods require calculation of stress gradient information so that the sensitivity of the design objective function with respect to the design variables can be determined. With respect to shape optimisation, these design variables are the boundary movements at the stress concentrator. Most industry-standard FE analysis codes which have an optimisation capability use this approach. MSC/NASTRAN uses the sensitivity analysis method for design optimisation.(1)

While sensitivity analysis is a powerful general method for optimisation, it can be relatively demanding on computer time. Also significant is the fact that typically a great deal of effort is required (compared to standard FE analyses) to generate suitable initial model specifications to obtain successful results. Mesh distortion and appropriate specification of the objective function are typical problems.

In the open literature, a number of papers deal with the development of this method applied to relatively simple geometries. However, few references can be found to the application of this method to more complex constrained optimisation problems.

### Other Optimisation Methods

Other optimisation techniques exist which are suitable for shape optimisation problems. Many of these are gradientless which reduces the complexity of the computations required. The biological growth analogy method proposed by Mattheck and Burkhardt(2) is a gradientless shape optimisation method which aims to achieve a constant Von Mises stress field along a given free boundary. The Nueber notch stress reduction method, developed by Schnack,(3) is also a gradientless approach. This aims to reduce notch stresses to a minimum by producing a constant tangential stress boundary. Another gradientless approach, termed the moving boundary method, has been proposed by Kaye and Heller.(4) This method also aims to produce a constant boundary stress field by appropriate boundary nodal movements.

Another method known as homogenisation optimisation was developed by Rozvany et al.(5) This method aims to achieve identical strain energy in each element. A similar method, termed evolutionary optimisation, has been developed by Xie and Steven.(6) The principle behind this method is the removal of elements which have low Von Mises stress. However, it is not ideally suited to stress concentrator problems as the removal of elements leaves an irregular boundary.

### Objective Functions

The objective functions in MSC/NASTRAN can be selected from a set of standard objectives (eg. weight, stress, etc.) or in the form of user defined mathematical equations using FE analysis output. While there exists a multitude of possible objectives, two functions suitable for stress concentration problems were chosen. The first was termed the least squares objective function. This objective aims to produce a constant boundary stress through the application of an averaging equation which relates the stress at a point on the boundary to the average stress around the entire boundary. The objective takes the following form:

$$\text{Minimise } \frac{\Sigma(\sigma_i - \sigma_{av})^2}{k^2} \qquad (1)$$

where: $\sigma_i$ = Hole nodal boundary stress
$\sigma_{av}$ = Average boundary nodal stress
$k$ = Number of boundary nodes

In this form, the objective function effectively attempts to increase the lower valued stresses and decrease the higher valued stresses, ideally producing a constant stress boundary in many cases.

The second objective function is simpler in nature and less flexible than the least squares objective. This objective, termed the maximum stress objective function, aims to reduce the maximum stress around the hole boundary and can be written as follows:

$$\text{Minimise } \max(\sigma_1, \ldots, \sigma_n) \qquad (2)$$

where: $\sigma_1$ = First hole nodal boundary stress
$\sigma_n$ = Last hole nodal boundary stress

This function searches for the node with the highest stress and the boundary is moved to reduce this value.

Convergence of the objective function in all cases was considered to be achieved when two consecutive iterations were within 1%.

### Design Variables

The general term, design variable, refers to items which are varied throughout the optimisation process to achieve the required objective. In the case of shape optimisation of holes, the design variables were the location of the nodes along the hole boundary. To reshape the hole, the nodes are allowed to move finite amounts dependent upon the constraints applied.

### Shape Basis Vectors and Auxiliary Models

In MSC/NASTRAN, the design variables in shape optimisation are related to allowable shape variations using shape basis vectors (SBVs). SBVs are the possible changes in location for a group of nodes. The optimisation process determines the best linear combination of the SBVs for a given objective and constraints.

The SBVs are generated through another FE model termed an auxiliary model. This model shares the same geometry, element connectivity and often material properties but has different boundary conditions. A typical auxiliary model used is shown in Figure 1, which features individual radial point loads applied to the nodes on the boundary of the hole.

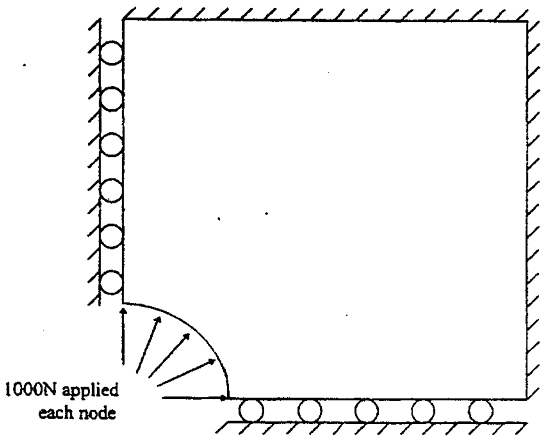


FIGURE 1 - Typical auxiliary model used for the shape optimisation of a hole on a plate

### Panel Configuration

The panel configuration used in this work was that of a 250 x 250 mm plate containing a centrally located 50 mm diameter hole. The sign conventions are shown in Figure 2. For the isotropic material problems, the properties of aluminium were used ($E = 69$ GPa, $\nu = 0.3$) and the plate thickness was 10 mm.

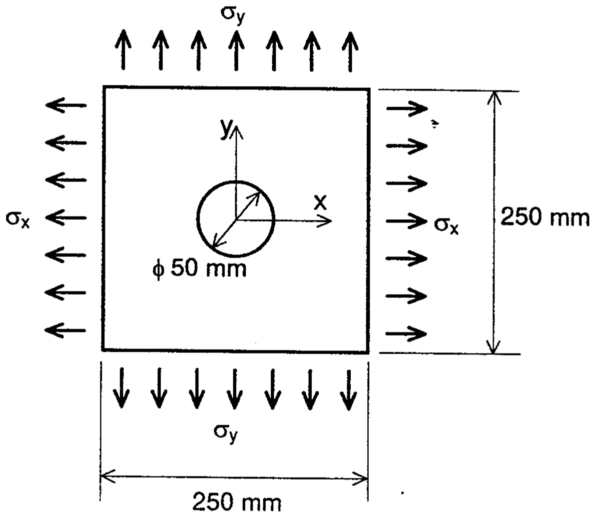


FIGURE 2 - Panel dimensions and sign conventions

## UNIFORM BIAXIAL STRESS FIELDS

The optimisation of a circular hole in a plate under a uniform biaxial stress field is a common shape optimisation benchmark problem. Both 2:1 and 4:1 biaxial loading was investigated, the latter representing a more complex situation due to the presence of compressive stresses around the hole boundary. The loading applied is presented in Table 1. Due to symmetry, only one quarter of the plate was modelled using the mesh shown in Figure 3. Both the least squares and maximum stress objective functions were utilised.

TABLE 1 - Loading used for uniform biaxial stress field

| Load Case | $\sigma_x$ | $\sigma_y$ |
|---|---|---|
| 2:1 Biaxial | 240 MPa | 120 MPa |
| 4:1 Biaxial | 480 MPa | 120 MPa |

### Least Squares Objective Function

Using the least squares objective function (Equation 1), the optimised hole shape for 2:1 loading, achieved in six iterations, was a 2:1 ellipse as shown in Figure 4. Figure 5 shows the boundary hoop stress distribution around the circular and optimised hole. This demonstrates that in the optimised case, the boundary stress is constant at 394 MPa, reduced from a maximum of 660 MPa for the circular hole.

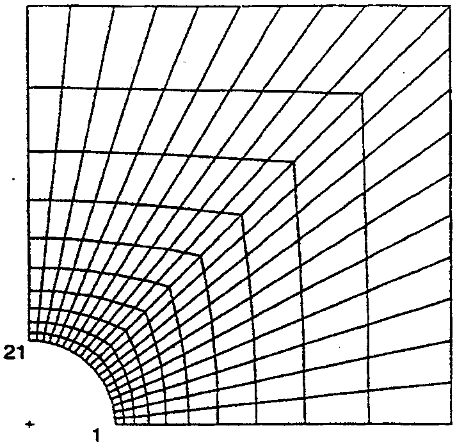


FIGURE 3 - Finite element model used for biaxial loading case

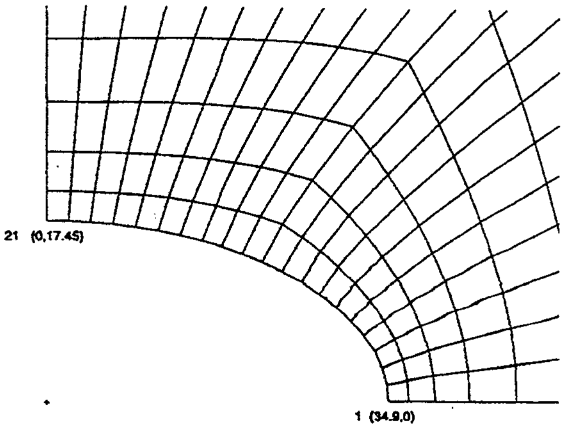


FIGURE 4 - Optimum hole shape for isotropic material under 2:1 uniform biaxial loading using the least squares objective

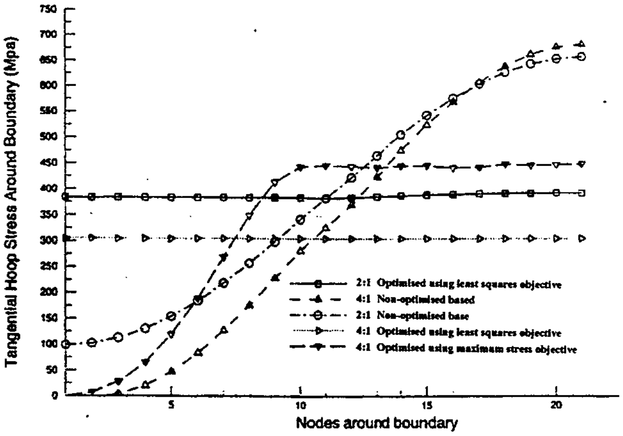


FIGURE 5 - Boundary stress distribution for isotropic material under uniform biaxial loading

For 4:1 biaxial loading using the least squares objective function, the optimised shape was a 4:1 ellipse shown in Figure 6. As shown in Figure 5, the boundary hoop stress was constant at 304 MPa, reduced from a maximum of 680 MPa for the circular hole.

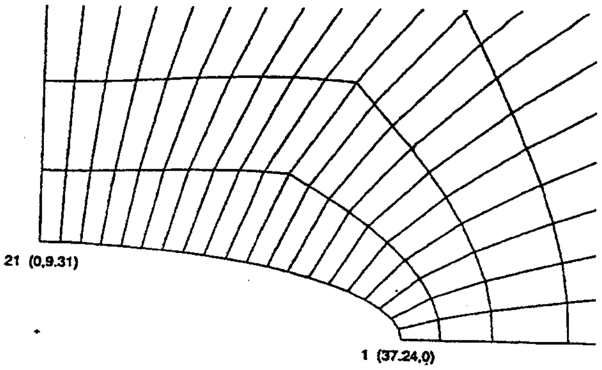


FIGURE 6 - Optimum hole shape for isotropic material under 4:1 uniform biaxial loading using the least squares objective

### Maximum Stress Objective

The plate under 4:1 biaxial loading was also optimised using the maximum stress objective function. The resultant optimised shape, which converged after five iterations, is shown in Figure 7 and was not an ellipse. The boundary stress distribution shown in Figure 5 is not uniform and is reduced to only 440 MPa. It appears that the maximum stress objective function becomes trapped in a local minimum preventing the true optimum solution from being reached. The least squares objective function proved to be more robust for the shape optimisation of holes in plates under biaxial loading.

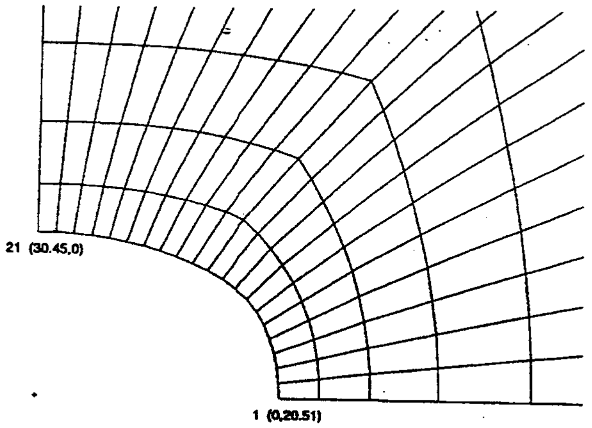


FIGURE 7 - Optimum hole shape for isotropic material under 4:1 uniform biaxial loading using the maximum stress objective

## NON-UNIFORM BIAXIAL FIELDS

Non-uniform biaxial stress fields provide a more difficult optimisation task in comparison with uniform biaxial fields. Non-uniform biaxial fields are a more realistic problem for the loads on a typical aircraft structure. Bjorkmann and Richards$^{(7,8)}$ have studied this problem and proposed an optimised solution known as a harmonic hole. It was further postulated that a harmonic hole was optimum as it reduced the magnitude of the boundary hoop stress to a minimum. A hole shape is termed harmonic when it satisfies the requirement that the first invariant of the original stress field remains everywhere unchanged after the hole has been included. The first invariant of stress is termed the bulk stress where the following equation must be satisfied:

$$\sigma_x + \sigma_y = CONSTANT \qquad (3)$$

### Loading and Constraints

For the non-uniform biaxial case, the loading considered was as follows (where $y$ is in millimeters):

$$\sigma_x = -10y + 250 \text{ MPa}$$
$$\sigma_y = 250 \text{ MPa}$$

Due to symmetry, a half model was constructed using the mesh shown in Figure 8. The optimisation utilised the least squares objective function defined in Equation 1 and was carried out with varying constraints. Although not stated explicitly, the problem investigated by Bjorkmann and Richards$^{(7,8)}$ appears to have had the top and bottom nodes of the holes completely constrained. To study the nature of harmonic holes further, the load cases considered in the present work were as follows:

Case A: Top and bottom nodes completely constrained.
Case B: All nodes free (no constraints).
Case C: Top and bottom nodes cannot move inwards.
Case D: No nodes can move inward (hole rework).

### Optimisation Results

The optimised hole shape for Case A is shown in Figure 9, which displays some mild mesh distortion. The resulting boundary stress distribution is presented in Figure 10 and was very similar to that predicted by Bjorkmann and Richards.$^{(7,8)}$ The bulk stresses of the plate containing the optimised hole were also compared with a plate without a hole under identical loading, as shown in Figure 11. This shows that the bulk stress is almost identical, except at the node on the hole boundary. This discrepancy is due to the numerical solution; the solution of Bjorkmann and Richards$^{(7,8)}$ was theoretical.

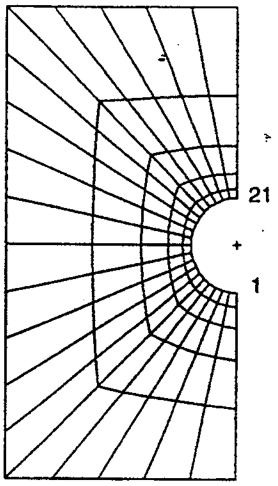


FIGURE 8 - Model used for optimisation under non-uniform biaxial loading.

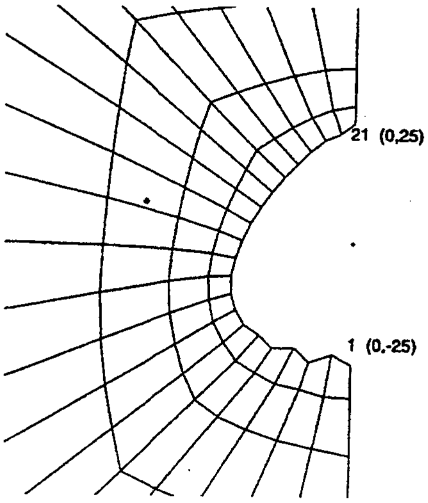


FIGURE 9 - Optimum hole shape under non-uniform biaxial loading for constraint Case A

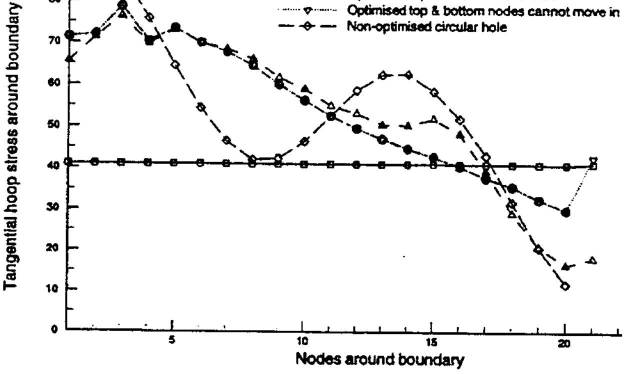
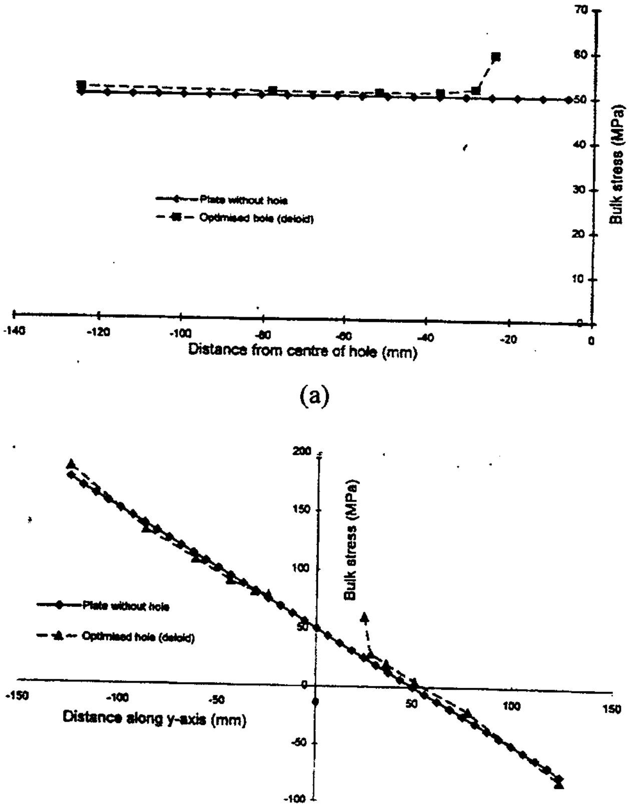


FIGURE 10 - Hole boundary stress distribution for plates under non-uniform biaxial loading

The optimised hole shape for the unconstrained problem (Case B) is presented in Figure 12. This hole is significantly smaller than the original circular hole and is positioned off-centre. The boundary stress distribution, shown in Figure 10, is constant and much lower than the value obtained for constraint Case A. A comparison of the bulk stress distribution (Equation 3) showed that this hole shape was also harmonic in nature.



(a)

(b)

FIGURE 11 - Bulk stress distribution along (a) the x-axis and (b) the y-axis of a plate under non-uniform biaxial loading for constraint Case A

Investigation of constraint Case C in which the top and bottom nodes could not move inwards produced a result which was identical to Case A. This is expected as the natural movement of the nodes, shown by Case B, would be inwards.

The final constraint Case D was a simulation of a hole rework problem in which material could only be removed, not added. This was a realistic problem for the fatigue life extension of existing structures through re-shaping of holes. The optimised hole shape is shown in Figure 13 and the resulting boundary stress distribution in Figure 10. This hole shape produced a stress distribution that was generally not as good as that achieved for the other constraint cases.

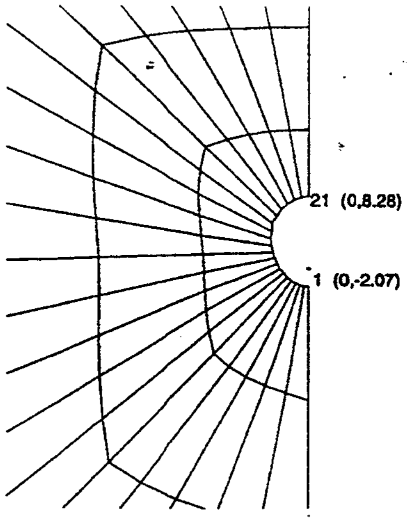


FIGURE 12 - Optimum hole shape under non-uniform biaxial loading for constraint Case B

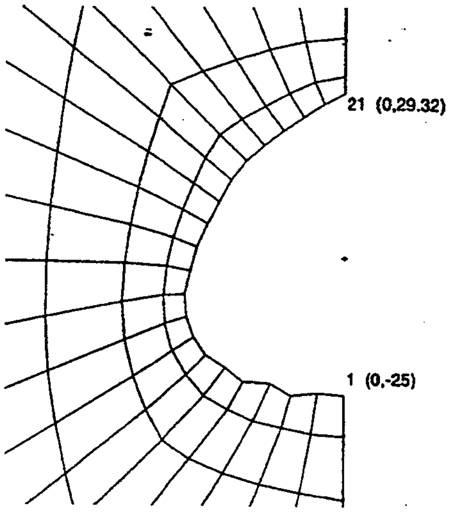


FIGURE 13 - Optimum hole shape under non-uniform biaxial loading for constraint Case D

These results indicate that a range of harmonic holes exist for given geometry and loading conditions which depend on the constraints imposed. The most optimum hole shape was achieved for the unconstrained problem (Case B) when a uniform boundary stress was achieved.

## OPTIMISATION OF LAMINATED PLATES

The optimisation of plates manufactured from laminated fibre composite materials offers more design variables than standard isotropic materials. For design purposes, the individual lamina orientation and thickness can be design variables. In this study, however, the optimisation problem is limited to the hole shape in an existing laminate.

Optimisation of holes in composite laminates has been investigated by a small number of researchers. Backlund and Isby(9) investigated optimum hole shapes in shear panels with the emphasis placed on minimising the weight rather than obtaining the hole shape which reduces the strains in the panel the most. Further, the shape of the holes was restricted by the use of curves fitted through the boundary nodes. Vellaichamy et al.(10) investigated the optimum orientation and aspect ratio of an elliptical hole for various laminates and loading conditions. In this case, the optimum hole shape was assumed to be elliptical. A more general study was conducted by Falzon et al.(11) who used evolutionary optimisation techniques to determine optimum hole shapes for quasi-isotropic laminates under several loading conditions. This method resulted in irregular edges due to the optimisation technique implemented.

For the present study, the loading conditions investigated were the 2:1 and the more demanding 4:1 uniform biaxial cases. The optimisation was conducted on a number of laminates with varying degrees of orthotropy.

### Objective Function

The least squares objective function applied to the isotropic problems (Equation 1) needs to be modified for the case of laminated composites in MSC/NASTRAN. When using laminated composites and shell elements, only element centroidal stresses, strains or failure indices can be output. Thus for the optimisation of laminated composite plates, the least squares objective function takes the following form:

$$\text{Minimise } \frac{\Sigma(FI_i - FI_{av})^2}{k^2} \qquad (4)$$

where: $FI_i$ = Element failure index
$FI_{av}$ = Average boundary failure index
$k$ = Number of boundary elements

In this case, the element failure index is the maximum (most critical) failure index of all plies in the element. Also, the average boundary failure index is determined by averaging the maximum element failure indices.

The maximum strain failure theory was used to calculate the failure indices, and is defined as follows:

$$FI = \max\left[\left(\frac{\varepsilon_1}{X}\right),\left(\frac{\varepsilon_2}{Y}\right),\left(\frac{|\gamma_{12}|}{S}\right)\right] \qquad (5)$$

where: $\varepsilon_1$ = Ply longitudinal strain
$\varepsilon_2$ = Ply transverse strain
$\gamma_{12}$ = Ply in-plane shear strain
$X$ = Allowable ply longitudinal strain in tension if $\varepsilon_1$ is positive, or compression if it is negative
$Y$ = Allowable ply transverse strain in tension if $\varepsilon_2$ is positive, or compression if it is negative
$S$ = Allowable ply in-plane shear strain

### Laminates

The geometry of the problem was the same as for the isotropic cases investigated (Figure 2). The laminates used for the orthotropic cases as detailed in Table 2, where 0° corresponds with the x-axis of Figure 2. The material used was T300/914C unidirectional carbon fibre epoxy tape and the properties are presented in Table 3.

TABLE 2 - Laminates used for the optimisation of flat plates containing holes

| Laminate Number | Lay-up |
|---|---|
| 1 | $[45/-45/90/0]_S$ |
| 2 | $[45/-45/90/0_2]_S$ |
| 3 | $[45/-45/0_2]_S$ |
| 4 | $[45/-45/90_2]_S$ |

TABLE 3 - Properties of T300/914C uni-directional pre-impregnated carbon fibre epoxy tape

| Property | Value |
|---|---|
| Longitudinal Modulus, $E_1$ | 130.0 GPa |
| Transverse Modulus, $E_2$ | 4.65 GPa |
| In-Plane Shear Modulus, $G_{12}$ | 4.65 GPa |
| Poisson's Ratio, $\nu_{12}$ | 0.3 |
| Ply Thickness | 0.127 mm |
| Long. Tension Ultimate Strain, $X_T$ | 9230 με |
| Long. Compression Ultimate Strain, $X_C$ | 7692 με |
| Trans. Tension Ultimate Strain, $Y_T$ | 10750 με |
| Trans. Compression Ultimate Strain, $Y_C$ | 25810 με |
| In-plane Shear Ultimate Strain, $S$ | 13980 μγ |

### Optimisation Results

The optimum hole shapes for the laminated plates under 2:1 and 4:1 biaxial loading are shown in Figures 14 and 15 respectively. The optimum hole shape is equivalent to having constant failure index in the elements around the boundary of the hole. As presented in Figures 16 and 17 for Laminate 1 under 2:1 and 4:1 biaxial loading respectively, the least squares objective function reduced the failure index in regions where it exceeded the average boundary stress and increased it in regions where it was below the average. The failure index in these graphs has been normalised so that the optimised shape gives a boundary failure index equal to one.

The ply orientations of the laminate do affect the optimum hole shape. The aspect ratios of the optimum hole for the various configurations under biaxial loading are presented in Table 4. It is evident that the optimised hole shapes of laminates which contain plies orientated in the four principal directions (0°, 90°, 45°, -45°) are very similar to the isotropic case. However, these holes are not truly ellipses as they do not satisfy the equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (6)$$

where: $a$ = Ellipse semi-length in $x$-direction
$b$ = Ellipse semi-length in $y$-direction

Laminate 3, which does not contain any 90° plies, results in an hole shape with a lower aspect ratio. The optimum hole shapes of Laminate 4, which does not contain any 0° plies, experienced some mesh distortion and are not presented in Figures 14 and 15. The shape was not close to an ellipse despite a constant failure index being achieved around the hole boundary. This case does not represent a realistic design due to the absence of fibres oriented in the primary load direction, meaning that much of the load would be carried in shear by the ±45° plies. This change in load affects the load paths and failure mode which has a great effect on the optimum shape. Further work is being conducted into this area.

TABLE 4 - Aspect ratio of the optimum hole shape for the various configurations under biaxial loading

| Laminate | Hole Aspect Ratio | |
|---|---|---|
| | 2:1 Biaxial | 4:1 Biaxial |
| Laminate 1 | 2.08 | 5.16 |
| Laminate 2 | 1.97 | 4.06 |
| Laminate 3 | 1.63 | 2.75 |
| Laminate 4 | 3.43* | 3.24* |

* Optimised shape was not close to an ellipse

The reduction in peak failure index around the hole for the optimised shape in the various configurations is presented in Table 5. The lowest recorded reduction was a significant 34% for Laminate 3 under 2:1 biaxial loading. These figures demonstrate the value of optimising hole shapes under biaxial loading conditions.

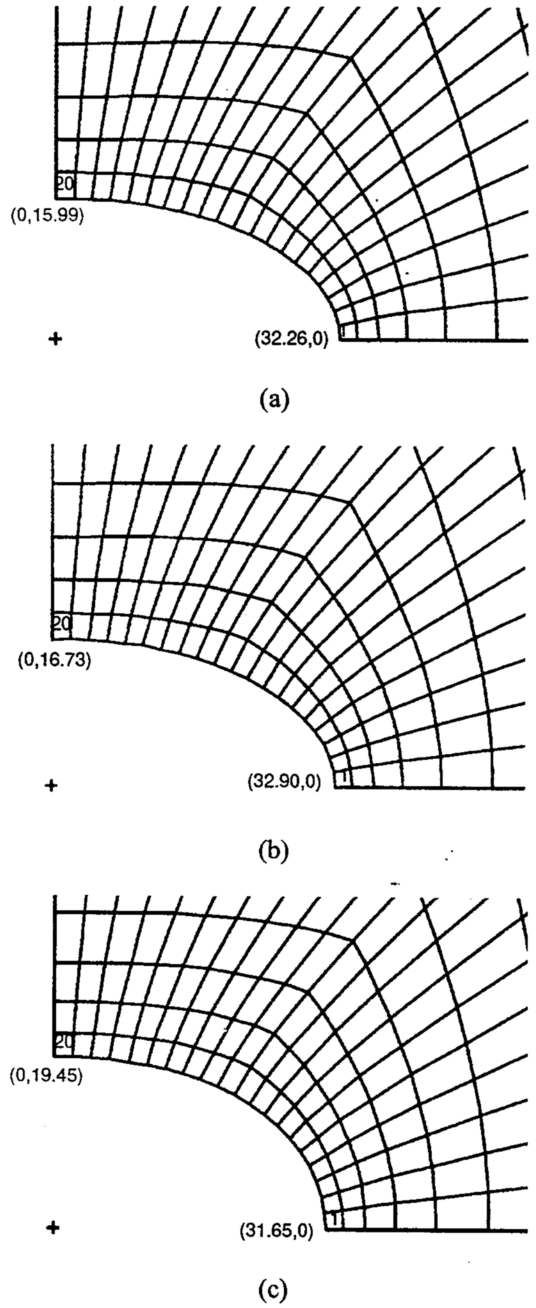


FIGURE 14 - Optimum hole shapes under 2:1 biaxial loading for (a) Laminate 1, (b) Laminate 2 and (c) Laminate 3

TABLE 5 - Percentage reduction in peak failure index for optimised hole shape for the various configurations under biaxial loading

| Laminate | % Reduction in Failure Index | |
|---|---|---|
| | 2:1 Biaxial | 4:1 Biaxial |
| Laminate 1 | 42 | 59 |
| Laminate 2 | 39 | 54 |
| Laminate 3 | 34 | 50 |
| Laminate 4 | 75 | 65 |

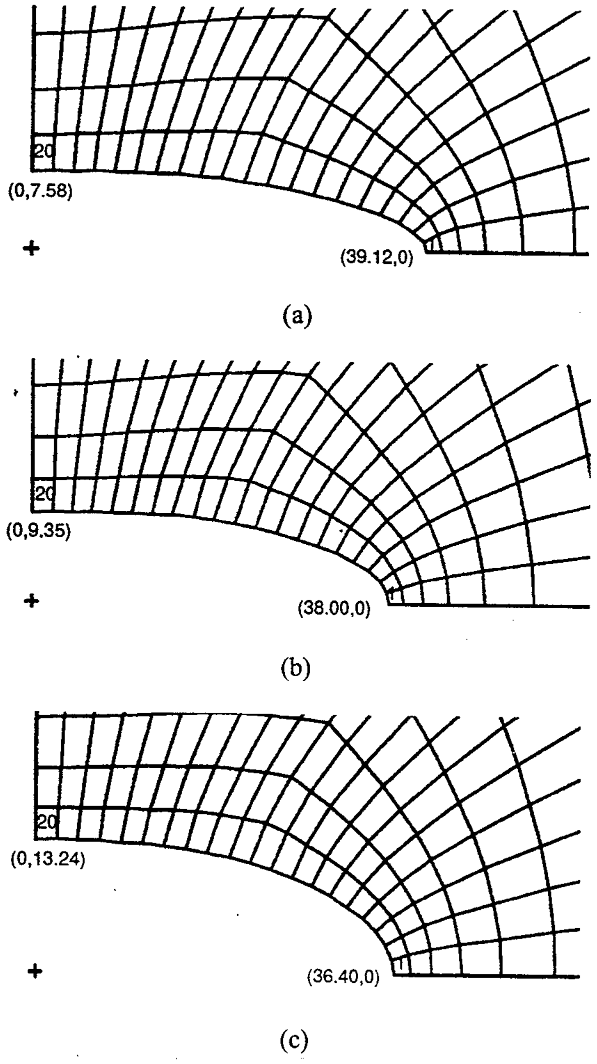


FIGURE 15 - Optimum hole shapes under 4:1 biaxial loading for (a) Laminate 1, (b) Laminate 2 and (c) Laminate 3

## CONCLUSIONS

The work presented demonstrates the suitability of the design sensitivity method of MSC/NASTRAN for the optimisation of holes in both isotropic and laminated composite plates. A least squares objective function was shown to successfully produce a constant stress around the hole boundary in isotropic cases for uniform and non-uniform biaxial load cases. For non-uniform biaxial loading, it was found that a range of harmonic holes exist which depend on the constraints imposed.

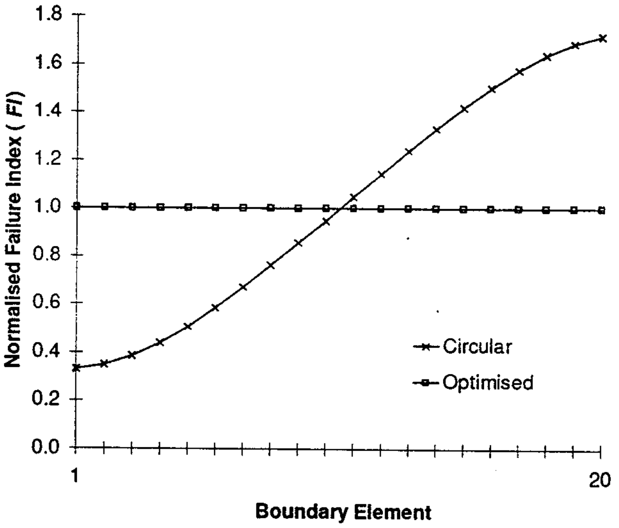


FIGURE 16 - Comparison of boundary failure indices between circular and optimised holes under 2:1 biaxial loading for Laminate 1

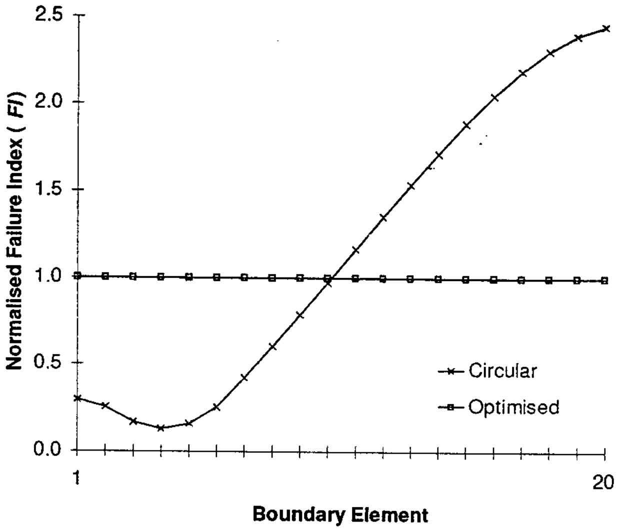


FIGURE 17 - Comparison of boundary failure indices between circular and optimised holes under 4:1 biaxial loading for Laminate 1

The optimisation of holes in laminated composite plates was conducted using a least squares objective function based on the failure index in the hole boundary elements.

The results showed that the optimum hole shape does depend on the degree of orthotropy of the laminate. Quasi-isotropic laminates produced holes of similar shape to the isotropic case while laminates without fibres in the primary loading direction produced unexpected shapes which are being further investigated.

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