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A NUMERICAL STUDY OF LIFTING SURFACE AEROELASTIC INSTABILITY USING TRANSONIC UNSTEADY AERODYNAMIC CODE - NTRANS

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ABSTRACT

A conceptually novel and computationally efficient technique for computing aeroelastic instability boundary of lifting surfaces in transonic flow is presented. The advance capabilities in structural modeling of NASTRAN are integrated with a newly developed unsteady transonic aerodynamic computer code called NTRANS (Nusantara TRANSonic code). This integration is carried out through the replacement of each natural mode shapes required for aeroelastic calculation in NTRANS with mode shapes calculated from NASTRAN. An interface module is developed, based on the DMAP module of NASTRAN; in order to make this integration efficient for routine calculations. NTRANS code is developed based upon the nonlinear transonic small disturbance flow equations and employed an Approximate Factorization finite difference solution technique. Features that distinguished this solution procedure from the other solution techniques are the use of: a). Cyclic acceleration scheme which could increase the convergence rate of the solution without degrading the accuracy, and b). Unit impulse transfer function method in conjunction with Pade approximation function for the calculation of the elements of GAF matrix as function of oscillation reduced frequency. In this method, elements of the GAF matrix for each combination of free stream Mach number and structural mode shape for a wide range values of the oscillation reduced frequency are obtained by a single flow solution. It has been shown that this method was much faster compared to the conventional method. Numerical results show that this procedure is accurate and efficient for routine analysis and design use.

INTRODUCTION

Aerodynamics and structures interaction plays a critical rule in airframe design. It becomes even more significant when viewed in the context of emerging multidisciplinary design concept, because the accuracy of both the aerodynamic and structural models improves the reliability of the optimal solutions. This static and dynamic aeroelastic problem is govern by the mutual interaction of elastic and inertial forces of the structure with the unsteady aerodynamic loading induced by the oscillation of the part of the aircraft structure itself[1].

Conventional design practice required that this flutter boundary of an aircraft structure be outside the flight envelope by a margin of at least 15 % in equivalent airspeed. Therefore, a correct understanding of flutter behavior of an aircraft structure is important for safety reasons as well as for overall performance of the aircraft.

At present time, there is a continuous effort to improve the performance of subsonic transport aircraft. One attempt is to improve the fuel efficiency by extending the flight regime to high sub-transonic Mach numbers to increase lift-to-drag ratios and flight speed. But, an increase in Mach number into transonic regime will bring other important problems of high induced drag and nonlinear aeroelastic response phenomenon, termed as transonic dip, where the aircraft experience an undesirable reduction in the flutter speed (as much as 50% of its value at subsonic speed). To accurately predict the nonlinear flutter characteristic at transonic speed, it is necessary to model the flows with an appropriate flow

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equation or system of equations. Navier - Stokes equations are capable of presenting mathematically the physical phenomena encountered in most of fluid dynamic problems such as transonic flows, including shock waves and boundary layers. This flow equations consists of system of nonlinear, second order partial differential equation in space and time. Its numerical solution requires the implementation of the tangency boundary condition on the body surface, for which a time dependent, body conforming grid system have to be used. This requirement adds the overall complexity and computational effort and resources of the problem. Consequently a simpler form of equation, but still can describe a typical transonic flow structure, is often utilized. At present time, transonic small disturbance (TSD) equation is widely used in the prediction of unsteady aerodynamic loads for aeroelastic analysis, besides several older linearized aerodynamic theory that had been developed 30 years ago, such as: Doublet - Lattice and Vortex - Lattice theory, quasi steady Mach Box theory and unsteady Piston theory. Most of these linearized aerodynamic theory, however, can not directly taking into account several important parameters such as: lifting surface thickness and camber, angle of incidence and oscillation amplitude and frequency[2] Some empirical corrections procedure to these theory have been developed and used for routine aircraft design purposes.

NASTRAN, a well known and widely use computer code today for aeroelastic design and analysis, was developed based upon uncoupled aeroelastic solution procedure. Structural equation of motions is solved using the finite element discretization method. Mean while, the unsteady aerodynamic loads working on the structure are calculated using a linearized aerodynamic theory such as Doublet - Lattice theory and Mach Box theory. The linearized aerodynamic theories being used could accurately predict the unsteady aerodynamic load only for flow in low subsonic and high supersonic regimes. Outside these flow regimes, where the flow nonlinearities increases, significant error in the prediction of the aerodynamic load may occurs. Similar limitation is also applied for ELFINI [3], an aeroelastic code developed by Dassault based on empirical correction procedure and THINAIR [4], developed by Boeing Company, which apply linear aerodynamic Panel theory.

The most advanced procedure for nonlinear transonic aeroelastic analysis commonly used at present time are based on the TSD theory, such as ATRAN3S and CAP-TSD (both developed at NASA Langley and is limited for US company use only). The ATRAN3S code, NASA Ames version of XTRAN3S, is a three-dimensional code based upon a time-accurate, finite difference methods using alternating direction implicit (ADI) algorithm. Several terms of the ADI algorithm used in this code treated explicitly, which leads to time steps restriction based upon numerical stability consideration. Therefore, it is becomes very expensive for three-dimensional applications not just because of the small time-step needed to obtain convergence results, but also because not all sweep in the algorithm can be written in vectorized form [5]. Meanwhile, an approximate factorization (AF) algorithm [6] that is applied in CAP-TSD was proven to be more efficient for three-dimensional calculations. This AF algorithm consists of a time-linearization procedure coupled with a Newton iteration technique. In this algorithm, the Newton iteration process occupied most of the computing time needed. Even though both ATRAN3S and CAP-TSD computer code are much faster compared to ENSAERO, their use for design and analysis is still considered to be expensive and is limited only for analysis during the final design stage of an aircraft.

The main objective of this work is to developed a prediction method for unsteady transonic aerodynamic load called NTRANS (Nusantara TRANSonic) based upon the solution of TSD flow equations using modified AF algorithm which has higher efficiency and accuracy compared to the original scheme. From previous study, it was found that Newton iteration step employed in the original AF algorithm is the major source of the slow convergence. Also, the use of harmonic oscillation technique in the calculation of generalized aerodynamic forces (GAF) during the aeroelastic analysis of the system required calculations of aerodynamic response of each combination of free stream Mach number, oscillation frequency and structural mode shapes which is expensive and inefficient. Features that distinguished this new solution procedure from the other solution techniques are the implementation of

1. Cyclic acceleration technique[7] for improvement in convergence rate. In this technique an acceleration coefficient, α, with cyclic values is added during the sweeping process in the chordwise direction. The addition of this coefficient will gives stable and accurate results with less iteration number per time step.
2. Unit pulse transfer function method[8] in conjunction with Pade approximation function for the calculation of the elements of the aerodynamic load matrix as function of oscillation frequency. In this method, elements of the aerodynamic load for each combination of free stream Mach number and structural mode shape for a wide range values of the oscillation frequency are obtained indirectly, in a single flow solution, from the aerodynamic response due to a smoothly varying exponentially shaped pulse.

The structural natural mode shapes and natural frequencies needed for calculations of the aerodynamic load in this procedure are obtained from NASTRAN structural dynamic solution. Calculations of the aeroelastic instability boundary and responses are carried out either in NASTRAN (frequency-domain solution) or in NTRANS (time-response solution). NTRANS module is used to replace the linear aerodynamic module available in NASTRAN, so that the integrated NTRANS - NASTRAN module can be applied for aeroelastic analysis and design in transonic speed. This integration is carried out through the DMAP module of NASTRAN. Numerical results for a wing and wing - body transonic aircraft configuration shows that this algorithm is accurate and efficient for routine design use.

GOVERNING EQUATIONS

NTRANS computer code is developed based upon the linearized parabolic transonic flow equations, which is the modified transonic small disturbance equation. The transonic small disturbance equation is obtained by combining the continuity and Bernoulli equation for a perfect gas with the isentropic flow relation and written in conservation form as

$$\frac{\partial f_o}{\partial t} + \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = 0 \qquad [1]$$

where t is the nondimensional time $= k\,\bar{t}$ with k represent the oscillation reduced frequency. The f_o, f_1, f_2 and f_3 coefficients are defined, in term of the disturbance velocity potential, as follows :

$$f_o = -B\phi_x + A\phi_t \;,\; f_1 = E\phi_x + F\phi_x^2 + G\phi_y^2$$
$$f_2 = \phi_y + H\phi_x\phi_y \;,\; f_3 = \phi_x$$

where A , B , F , G and H coefficients are function of free stream Mach number, motion reduced frequency and specific heat coefficients. The pressure coefficient on the lifting surface, expressed in terms of perturbation potential velocity, are calculated from the relation

$$C_p = -2\phi_x - 2\phi_t - (1 - M^2)\,\phi_x^2 - \phi_y^2 \qquad [2]$$

in which the cubical and higher powers of the perturbation velocity are neglected.
Numerical computations are carried out in computational domain, within a rectangular region conform to the body, which is obtained by a coordinate transformation of the physical domain. The physical grid system in the (x, y, z) - plane is transformed into some (ξ, η, ζ) - plane, so that the mesh spacing in all directions can be kept uniform in the computational domain, using trigonometric transformation function. The flow boundary conditions that are imposed on the far-field (outer) boundary are similar to the nonreflecting boundary conditions introduced by Kwak [9] and the flow tangency conditions on the surface are applied on the mean plane of the oscillating surface, which is located along the axis parallel to the streamwise direction, z = 0, equidistantly between two horizontal gridlines. For unsteady flow calculations based upon the TSD equation, the surface tangency boundary conditions need not to be applied on the actual surface. Instead, it is

applied on the mean plane of the body. Therefore, a body-fitted grid system is not required.

APPROXIMATE FACTORIZATION ALGORITHM

A modified Approximate Factorization (AF) algorithm is used for the solution of the flow equation, Eq. [1] . This scheme consists of a time linearization, to determine an estimate value of the perturbation potential, coupled with a Newton iteration technique to provide time accuracy in the solution. In this algorithm, flow equation is represented as triple product of differential operator, which is

$$L_\xi \, L_\eta \, L_\zeta \, (\Delta\phi) = R\left(\phi^*, \phi^n, \phi^{n-1}, \phi^{n-2}\right) \qquad [3]$$

where L_ξ, L_η, L_ζ represent differential operators in the ξ, η, ζ direction , respectively , $\Delta\phi$ is the error in the perturbation potential velocity , R represent residual of the equation , ϕ^* is the estimate value of the perturbation potential velocity , and $\phi^n, \phi^{n-1}, \phi^{n-2}$ is the perturbation velocity potential at time level n , (n-1) and (n-2) , respectively. The definition of L_ξ, L_η, L_ζ operators and R could be found in ref. 10.

Equation [3] is solved through three - sweeps in the computational domain by sequentially applying the differential operators L_ξ, L_η, L_ζ as follows

$$\begin{aligned} \xi - \text{sweep} &: \quad L_\xi\,(\Delta\tilde{\phi}) = -R \\ \eta - \text{sweep} &: \quad L_\eta\,(\Delta\bar{\phi}) = \Delta\tilde{\phi} \\ \zeta - \text{sweep} &: \quad L_\zeta\,(\Delta\phi) = \Delta\bar{\phi} \end{aligned} \qquad [4]$$

Once these entire three sweeps completed, the updated values of ϕ at each grid points are computed by applying the last values of $\Delta\phi$ into the previous perturbation potential velocity:

$$\phi^{n+1} = \phi^* + \Delta\phi \equiv \phi^*_{new} \qquad [5]$$

The computation is started with an estimate value of ϕ^* and is carried out until a convergence solution of ϕ^{n+1} is obtained (until the perturbation error $\Delta\phi$ reaches the values of 10^{-6}). In most of the computation that had been performed [10], a maximum of 3 Newton iteration is needed for convergence solution at each time step.

Using the new ϕ^{n+1} values, the time linearization step is carried through to obtained the new estimate values of ϕ^* for the iteration of the next time step. In this step, the body surfaces are put at their new position and updated surface boundary conditions are applied. The unsteady solution are initiated using the steady solution as the first estimate values. Since the solution at each sweep depends entirely on the values that have been computed at the previous sweep, all sweeps can be coded in vectorized form.

CYCLIC ACCELERATION TECHNIQUE

Since all terms in this scheme are treated implicitly, this scheme does not have a time step restriction. In steady flow calculation and during the Newton iteration step in unsteady flow calculation, however, it is possible to accelerate the convergence rate of the procedure. This can be achieved by adding an a cyclic acceleration coefficient, α, into the right hand side of Eq. [4] during the ξ - sweep, so that this equation become

$$\xi - \text{sweep} : \quad L_\xi\,(\Delta\tilde{\phi}) = -\alpha R$$

The value of α is given a variation according to geometric sequence defined by

$$\alpha_k = \alpha_{max}\left[\frac{\alpha_{min}}{\alpha_{max}}\right]^{\frac{(k-1)}{(k_n-1)}} \qquad [6]$$

where $k = 1,2,3, ..., k_n$ with k_n represent the number of α_k values to be defined, between 4 to 8. The α_{max} and α_{min} parameters represent the maximum and minimum values of selected α, respectively, which are defined as

$$\alpha_{max} = 1 \quad \text{and} \quad \alpha_{min} = \frac{4}{(\Delta x)^2}$$

where Δx is the grid spacing in the chordwise - direction. It was found that the stability and convergence rate of solutions is strongly depends on the number of cyclic values of α_k being used. For each different case, an investigation has to be made for the definition of an appropriate value of this parameter.

UNIT PULSE TRANSFER FUNCTION METHOD

For aeroelastic stability solution, the generalized aerodynamic forces (GAF) matrix elements have to be compatible with the associated aeroelastic stability equation given in the Laplace variable, which is

$$\left[[M]s^2 + [C]s + \left(qc^2\right)[A] + [K] \right]\{h_o\} = 0 \qquad [7]$$

where [A] represent the Laplace transform of the generalized aerodynamic force matrix, [A1], and defined by the following relation

$$A1_{i,j} = -qc^2 \int_s \frac{\Delta p_j(x,y,t)}{q} h_i(x,y) \frac{ds}{c^2} \qquad [8]$$

The coefficient $A1_{i,j}$ may consider as the generalized force coefficients from the pressure induced by mode -j acting through the displacement of mode -i. $\Delta p_j(x,y,t)$ is the lifting pressure at discrete point (x,y) due to wing displacement in the j-th mode, whereas h_i (x,y) represent the I-th mode shape of the structure.

Elements of the GAF matrix, [A1], are computed at finite number of values of the oscillation reduced frequency for each combination of free stream Mach number and structural mode shape. For three-dimensional problems, calculation of these elements using the method of harmonic oscillation is very expensive because for each of this combination (free stream Mach number, mode shape and reduced frequency) a complete flow solution is required. To make this calculation procedure more efficient, a unit pulse transfer function method is developed. In this method, the aerodynamic loads for each combination of structural mode shape and Mach number are obtained in a single flow solution for a wide range value of reduced frequencies.

In this method, the aerodynamic loads are computed indirectly from the aerodynamic response due to a smoothly varying exponentially shaped pulse, as shown in Figure 1.

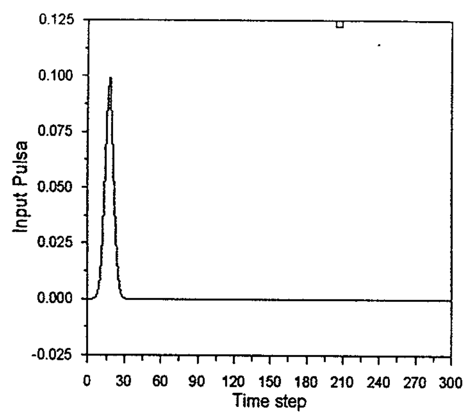


Fig. 1. Unit Pulse input

The advantages of using pulse input (rather than step function input) is to avoid the non-physical transient that was found when using step function input. The pulse is expressed as

$$r(t) = r_o\, e^{-a(t-t_o)^2} \qquad [9]$$

where r_o is the pulse amplitude, a and t_o are constants related to the width and center of the pulse. In this work a and t_o are selected equal to 0.25 and 17.5 Δt, respectively, where Δt is the nondimensional time step. Deformation z at any point (x,y) given to the structure surface at any time t is determined by the product of the pulse and the structural mode shapes, h (x,y), as

$$z(x,y,t) = r_o\, e^{-a(t-to)^2} h(x,y) \qquad [10]$$

This deformation is applied to the structure and the aerodynamic transients response, which is expressed in term of the total lifting pressure at point (x,y) on the surface $\Delta p(x,y)$, are computed. The aerodynamic loads, in k-domain, are defined as the ratio between the Fast Fourier Transform (FFT) of the transient response divided by the FFT of the deformation. Once these quantities are computed, the GAF matrix elements, $A1_{i,j}$, are obtained by substituting these values into Eq. [8]. In a three-dimensional problems, for the purpose of the compatibility between the structure and aerodynamic terms in the derivation of the aeroelastic equation of motion, h (x,y) are selected as the structure natural mode shapes. It is important to note that the transient response has to be calculated for a time interval that is long enough such that the final value of the response become steady and equal to its initial value. Otherwise, the Fourier transform of the response will not be accurate.

NUMERICAL VALIDATION

The accuracy and efficiency of the present calculation procedure employ in NTRANS code is evaluated by selecting several basic test cases recommended by AGARD and comparing the numerical results with experimental data and numerical results obtained using other algorithm. Validations are carried out in three-phase, which are: a). Unsteady flow with harmonic motion, b). Unit pulse motion, and c). Aeroelastic calculation. Numerical calculations are performed on rectangular wings configuration with aspect ratio (AR) equal to 20 and have a NACA 0012 airfoil without control surfaces.

Unsteady Flow Solutions

For unsteady calculations, wing structure or its control surface is given a sinusoidal harmonic pitching oscillation in the form

$$\alpha(t) = \alpha_0 + \alpha_1 \sin(k\bar{t})$$

where α_0 , the initial angle of attack, is selected equal to 0.02 degree and α_1 , the pitching amplitude , equal to 2.51 degree.

Numerical calculations that will be shown here are obtained for wing with NACA 0012 airfoil at free stream Mach number M = 0.755 and oscillation reduced frequency k = 0.05 , where the flow field has a strong shock on the upper surface of the wing. Each oscillation cycle is divided into 360 time steps and the aerodynamic responses are taken after 3 complete cycles.

Structure oscillation and the aerodynamic time response (given in the form of lift and moment coefficients, C_l and C_m) of the first wing is given in Figure 2.

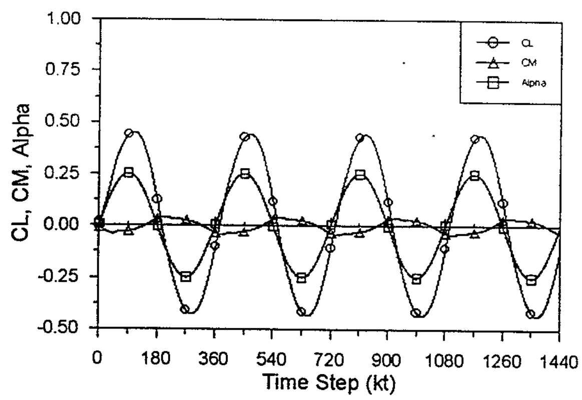
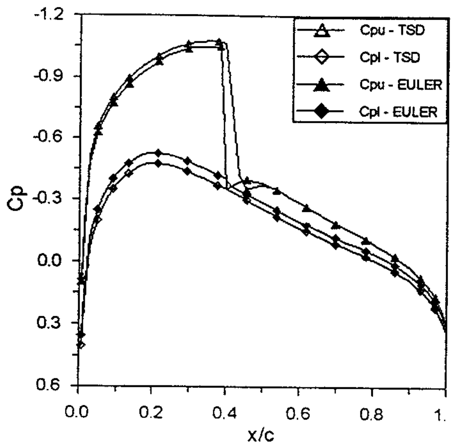
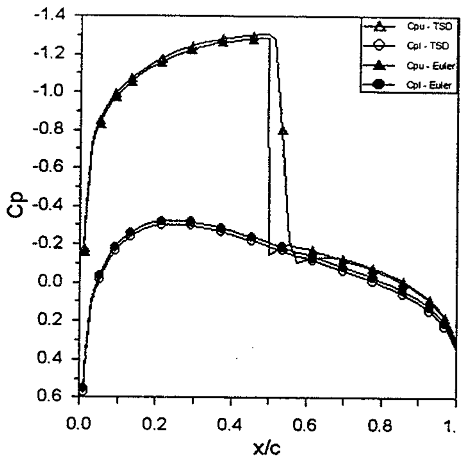


Fig. 2. Time histories of α, CL and CM of NACA 0012 M = 0.755, k = 0.05, xc = 0.25c, $\alpha_0 = 0.02°, \alpha_1 = 2.51°$

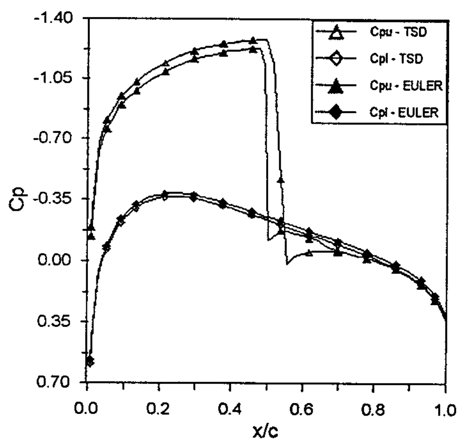
Unlike linear aerodynamic response in subsonic flows, responses in these figure shows that they generate a phase - lag with the structure oscillation and between themselves. The aerodynamic responses are late by a phase φ from the oscillation due to the discontinuity that occurs across the shock wave, which change the propagation behavior of the disturbance. As the oscillation continue, the phase - lag becomes higher and higher. This phase - lag phenomenon in turn will increase or decrease the aerodynamic damping of the flow depends on wheter it is in-phase or out of-phase with the oscillation and becomes the main factor in the generation of transonic dip. It was found that phase - lag in aerodynamic lifting forces is more significant compared to the one of the aerodynamic moment. Surface pressure distribution at time step $k\bar{t}$ = 36, 72, 144 and 180 at wing section 25% span from the root are shown in Figure 3.



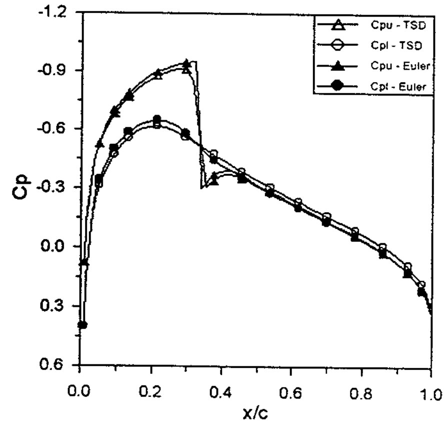
a) kt = 36



b) .kt = 72



c) kt = 144



d) kt = 180

Fig. 3. Unsteady pressure distribution of NACA 0012 M = 0.755, k = 0.05, xc = 0.25c, $\alpha_0 = 0.02°, \alpha_1 = 2.51°$,

Comparisons with the Euler solutions at all four-time steps are in very good agreement except for the shock position and strength. As was found in the steady cases, shock positions predicted in this study is further forward and stronger compared to its Euler position. This is mainly due to the fact that the present procedure has not yet employ boundary layer displacement correction on the surface. Shock movement during the oscillation is given in Figure 4.

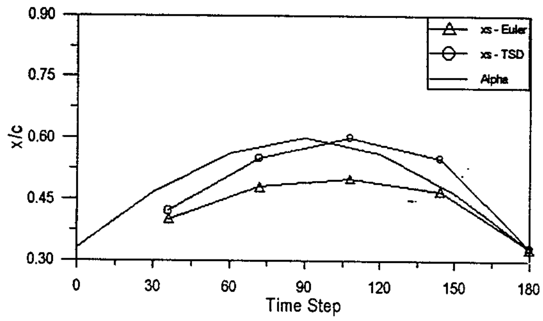


Fig. 4. Shock location during oscillation of NACA 0012, M = 0.755, k = 0.05, xc = 0.25c, $\alpha_0 = 0.02°, \alpha_1 = 2.51°$

This figure shows that there is phase - lag, also, between pitching movement of the wing with the shock movement on the upper surface of the wing. Comparison with Euler results shows that the inviscid shock movement amplitude is larger than that for viscous flow which means that shock - boundary layer interaction in general will decrease the strength of the shock and also its movement amplitude. The different in shock position at higher oscillation frequency (k = 0.162) is more significant, as shown in Figure 5.

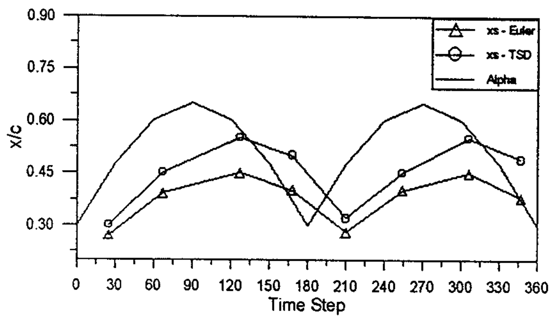


Fig. 4. Shock location during oscillation of NACA 0012, M = 0.755, k = 0.162, xc = 0.25c, $\alpha_0 = 0.02°, \alpha_1 = 2.51°$

Unit Pulse Displacement

The unit pulse transfer function method is used to calculate the elements of GAF matrix for flutter calculation of the wing. Figure 6 shows the total lift and moment response, C_l and C_m, at the grid point located at the wing tip, quarter chord length from the leading edge, induced by the displacement in the second mode, at Mach number M = 0.8.

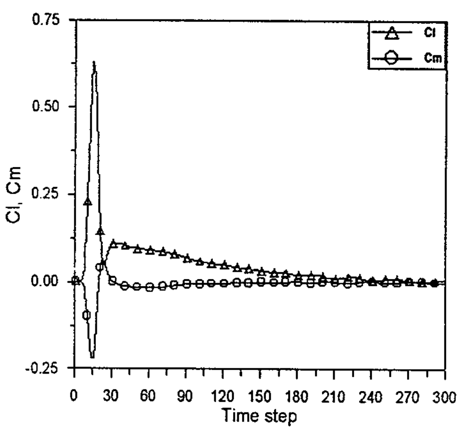
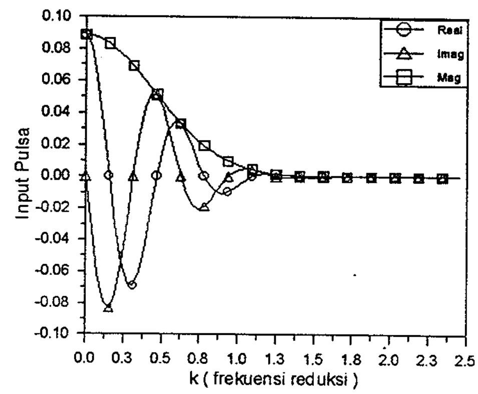
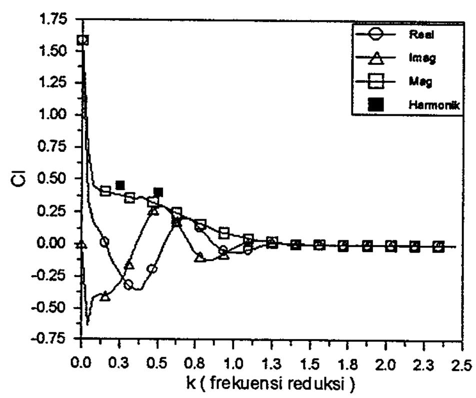


Fig. 6. C_l and C_m response

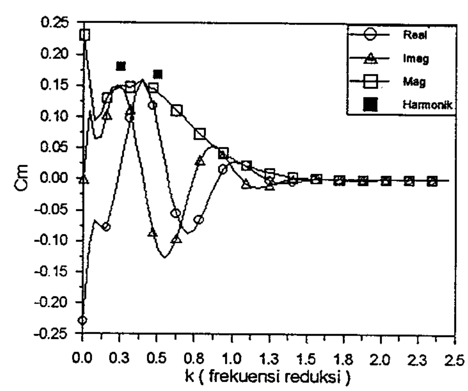
The wing is given a displacement according to Eq. [10] with the pulse amplitude, r_o, is taken equal to 0.1 , a = 4.05 , t_o = 17.5 Δt and Δt = 0.01. The pulse amplitude have to be selected in such a way that it is not so large that the response become divergence, but so small that the response becomes inaccurate. The response becomes steady, returning to its initial value, quickly in 300 time steps after the wing returning to its steady position.



a). FFT of pulse input



b). FFT of C_l response



c). FFT of C_m response

Fig. 7. FFT of pulse input and aerodynamic output

Since at this pitching amplitude, the flow has weak shocks so that the flow is not highly nonlinear, a quick converged response can be expected. Fourier transform of this unit pulse and response is shown in Figure 7.

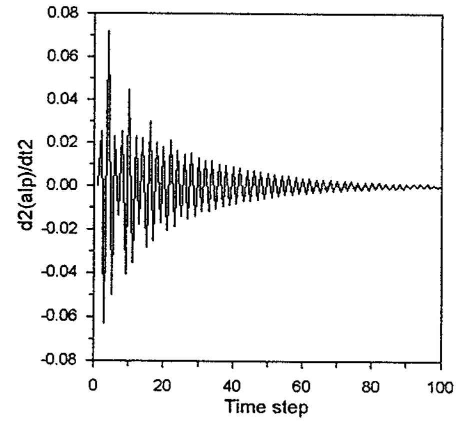
The dots in this figure represents the discrete values of the response after FFT, meanwhile continuous line represent their Pade polynomial approximation. Because the response is smooth enough to give a well distributed FFT values; Pade approximation could represents them with a smooth and continuous function also. This is very important for the use in flutter analysis of the wing. Using these Pade function, the elements of GAF matrix at certain value of Mach number and wing mode shape as function of motion reduced frequency, k, are calculated from Eq. [8]. Flutter analysis of the wing required the calculations of the GAF matrix for every value of Mach number, M, and mode shapes used in the analysis. Using this unit pulse method, the number of flow field calculation needed to compute all of these elements, at a certain value of M, is equal to the number of mode shapes being used in the analysis. In the harmonic oscillation method, this number will be equal to the number of the mode shapes multiplied by the number of reduced frequencies of interest.

Aeroelastic Calculation

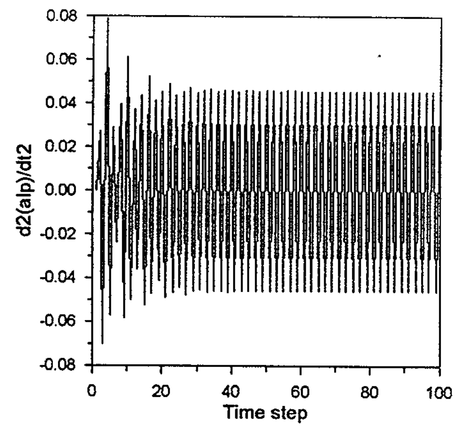
Flutter calculations in frequency - domain are carried out using mode shape and natural frequency obtained from NASTRAN. Calculation are made by solving the aeroelastic stability equation, Eq [7], for free stream Mach number M = 0.8 at cruising altitude of 35.000 ft above sea level

Figure 8 shows the wing response at various values of dynamic pressure, q.

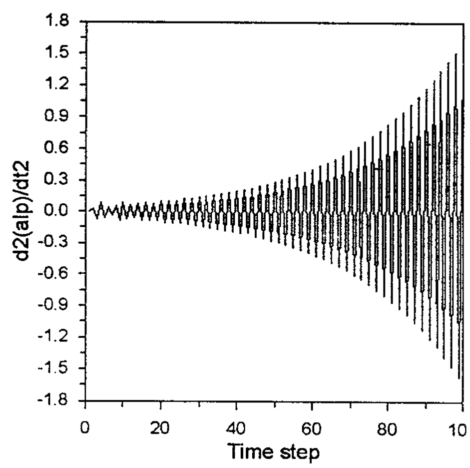
Flutter velocity is defined when the aerodynamic damping equal to zero or when the two frequencies coalesce. It was found that wing response start to diverge when q become larger than 1.375 or $q_{critical}$ = 1.375. Variation of critical dynamic pressure (or flutter index) with Mach number is given in Figure 9. As the free stream Mach number increasing, flutter index of the wing decreasing until the minimum value of 0.855 at Mach number M = 0.85 before it increasing back.



a). q = 1.25



b). q = 1.375



c) q = 1.5

Fig. 8. Acceleration response of torsion modes AR = 20, NACA 0012, M = 0.8,

This dip in flutter index is so significant because of the shock that appears in the flow field is so strong with high phase - lag.

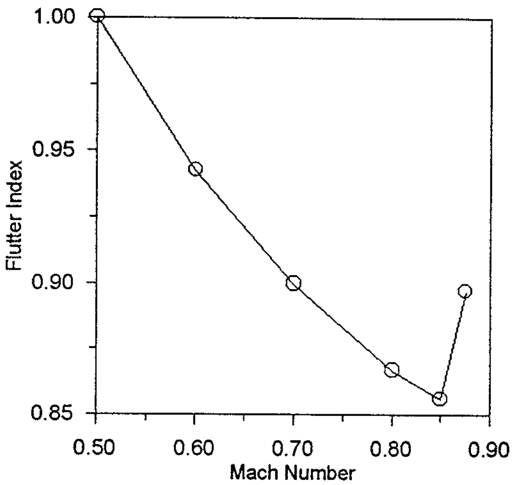


Fig. 9. Flutter Index versus Mach number

CONCLUSIONS

A three - dimensional flow solution procedure, called NTRANS, was developed based on the solution of nonlinear transonic small disturbance equations. The solution procedure applied in this module is the finite difference AF algorithm with two significant improvements, which are : a). The use of cyclic acceleration technique to increase the convergence rate of the calculations and b). Application of unit pulse transfer function method in the calculation of the GAF matrix. An integration module was a device to integrate this module with NASTRAN code (which calculate the structure natural frequencies and mode shapes) for aeroelastic calculations.

The procedure was applied to determine the unsteady aerodynamic loads on aircraft structure in subsonic and sub-transonic flow regimes. Numerical results show that this procedure is accurate and efficient for routine use in aircraft structural design and analysis.

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