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Structural Damage Detection Using Best Achievable 'Modal' Eigenvectors

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Abstract

The paper reports a modal formulation of the original method presented by Lim and Kashangaky, based on the use of the Best Achievable Eigenvectors in damage detection problems. The method requires the measurement of both frequencies and mode shapes. The structural damage is located by computing the Euclidean distances between the measured mode shapes and the best achievable 'modal' eigenvectors. The method is able to detect loss of both stiffness and mass properties, even though in this paper only the loss of stiffness will be analysed. Two numerical examples are reported to investigate the applicability of the modal formulation. Finally, an experimental validation is also included, using a ten-bay truss laboratory structure.

Introduction

During the last decade, the structural damage detection techniques based on modal data have been extensively investigated, aiming at the definition of an efficient and reliable method for health monitoring of complex systems (1-4). Such techniques are often based on periodically monitoring the modal behaviour in real-life conditions, in order to identify and localise possible damage. The damage detection is often formulated as an inverse updating problem, where the structural uncertainties are located on the real structure and the analytical model is considered as accurate. Unfortunately, the examples reported as successful generally concern laboratory test structures only. The main difficulty in the application of these methods is due to the intrinsically low sensitivity of the modal data to local changes of structural mass and stiffness. The minimum detectable structural damage is directly related to the minimum measurable variation on the modal data, i.e. frequencies and mode shapes. Another difficulty is related to the fact that these methods require both a refined analytical model of the analysed structure and a significant amount of measured modal data, making them very expensive and usually impractical for structures in real-life conditions.

It is convenient to split the structural damage detection problem into two distinct subproblems: the first concerning the localisation of the damage over the whole structure, the second relating to the evaluation of the damage magnitude. While some of the proposed methods try to solve these two subproblems simultaneously, others take advantage of this separation, trying to solve them step by step separately.

Different structural damage detection techniques have been proposed in literature. They are based on the classical optimal matrix modification methods (5-6), or on the sensitivity-based methods, such as the ones applied in the field of structural optimisation (7), or on strategies typical of identification and control problems (8). Recently, some applications of non-deterministic techniques, i.e. genetic algorithms and neural networks, have also been investigated (9).

This paper presents a different implementation of the method proposed in Ref. (10), based on the use of the Best Achievable Eigenvectors (*BAE*) for the localisation and identification of structural damage in space truss structures. In the original method, starting from a well-correlated finite element model of the analysed structure, the localisation algorithm is formulated in the physical space, represented by the degrees of freedom (*DOFs*) of the finite element model. This requires the use of global and local structural matrices expressed in the *DOFs* set. In the approach proposed in this paper, the original method is formulated in the modal space, represented by a limited number of low frequency measured modes. The availability of the local structural matrices in the *DOFs* set, very large even though sparse, and not easily obtainable with standard finite element codes, is thus no longer required.

Both numerical and experimental results are reported. In particular, the proposed method has been successfully applied to detect stiffness damage in a typical space truss structure, a 3.5 *m* long laboratory truss structure.

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Damage Localisation

As reported in the introduction, some damage detection methods split the original problem into two phases, respectively related to the localisation and to the magnitude identification of the damage. The method here proposed adopts this approach. It represents a modification in a modal form of the original Lim and Kashangaki (10) method where the search for structural damage is performed by computing the distance between two series of vectors, the measured eigenvectors on the damaged structure and the Best Achievable Eigenvectors (*BAE*).

Before introducing the modal formulation, it is convenient to recall the method in its original form, as reported in the original paper . For further details see Ref. (10).

Best Achievable Eigenvectors

The equation of motion of a generic *n DOFs* system may be expressed as :

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = f(\mathrm{t}) \qquad (2)$$

where **M** and **K** are mass and stiffness matrices of the system (n x n) and **x** and f (t) are the displacements and external forces vectors respectively (n x 1). The associated eigenvalues problem is :

$$\mathbf{K}\,\boldsymbol{\Phi} = \mathbf{M}\,\boldsymbol{\Phi}\,\boldsymbol{\Lambda} \qquad (3)$$

where $\boldsymbol{\Phi}$ is the eigenvectors matrix (n x r) and Λ is a diagonal matrix containing the eigenvalues (r x r).

When a finite element scheme of the original structure is adopted, the mass and stiffness matrices can be expressed as a sum of the element matrices, i.e.

$$\mathbf{K} = \sum_{i=1}^{p} \mathbf{K}_i \quad \text{and} \quad \mathbf{M} = \sum_{i=1}^{p} \mathbf{M}_i \qquad (4)$$

where $\mathbf{K}_i$ and $\mathbf{M}_i$ are mass and stiffness element matrices and p is the number of elements. When the structure shows a damage, the global mass and stiffness matrices can be represented as a sum of the original element ones multiplied by a reduction factor. While the method is able to consider damage concerning both the mass and the stiffness matrices, in the following only structural damage concerning the stiffness matrix will be taken into account.

The stiffness matrix of the damaged structure $\mathbf{K}_d$ is thus :

$$\mathbf{K}_d = \mathbf{K}_s + \sum_{i=1}^{p} a_i\,\mathbf{K}_i \qquad (5)$$

where $\mathbf{K}_s$ is the stiffness matrix of the original structure and a_i are the stiffness reduction factors for each element . For example, considering the coefficient a_k, if the corresponding element is not damaged, a_k is equal to zero, while if the corresponding element is completely removed, a_k is equal to -1 , so generally a_k ranges from -1 to 0.

If we measure r modes on the damaged structure, replacing Eq. (5) in Eq. (2) we obtain :

$$\sum_{i=1}^{p} a_i\,\mathbf{K}_i\,\boldsymbol{\Phi}_t = \mathbf{M}_s\,\boldsymbol{\Phi}_t\,\Lambda_t - \mathbf{K}_s\,\boldsymbol{\Phi}_t \qquad (6)$$

where the subscript t refers to the measured data. In the case of mode shapes, as they are measured only on some nodes of the structure, a completion operation extended to all the *DOFs* is required.

For the j-th measured mode, Eq. (6) can be rewritten as :

$$\sum_{i=1}^{p} a_i\,\mathbf{E}_j^{-1}\,\mathbf{K}_i\,\phi_{tj} = \phi_{tj} \qquad (7)$$

being

$$\mathbf{E}_j = \left(\omega_{tj}^2\,\mathbf{M}_s - \mathbf{K}_s\right) \qquad (8)$$

where ω_{tj} and ϕ_{tj} are the eigenvalue and eigenvector measured on the damaged structure. It is very important to underline that the matrix $\mathbf{E}_j$ is invertible since, due to the damage, $\omega_{tj} \neq \omega_j$.

Nevertheless, when the frequencies of the damaged structure are almost identical to the original ones, the matrix $\mathbf{E}_j$ can appear ill-conditioned. From this point of view, as reported by the authors, the method establishes a-priori a lower limit of the damage to be identified, one for which $\mathbf{E}_j$ becomes ill-conditioned.

Introducing the following matrix :

$$\mathbf{A}_{ij} = \mathbf{E}_j^{-1}\,\mathbf{K}_i \qquad (9)$$

the Eq. (7) can be expressed as :

$$\sum_{i=1}^{p} a_i\,\mathbf{A}_{ij}\,\phi_{tj} = \phi_{tj} \qquad (10)$$

If the reduction factors are grouped into a vector s, in order to identify the damaged element it is necessary to check the influence of each element on the considered mode shapes. Thus, for the k-th reduction factor, the basic equation of the damage detection algorithm is :

$$\mathbf{A}_{kj}\,\gamma_{kj} = \phi_{tj} \qquad (11)$$

where

$$\gamma_{kj} = s_k\,\phi_{tj} \quad \text{for } \mathrm{k} = 1, 2, \ldots, p \qquad (12)$$

Eq. (11) is very important because it can be satisfied only if the measured mode ϕ_{tj} is a linear combination of the columns of $\mathbf{A}_{kj}$, i.e. ϕ_{tj} must lie in the subspace defined by the columns of $\mathbf{A}_{kj}$. The main consequence is that if damage has been caused by a loss of stiffness in the k-th element, and this damage is reflected in mode j, then the vector ϕ_{tj} will lie exactly in the subspace defined by the columns of $\mathbf{A}_{kj}$.

To evaluate whether or not the j-th measured mode lies in the subspace defined by the columns of $\mathbf{A}_{kj}$ we have to use the definition of the *BAE*. The Best Achievable Eigenvector for the j-th mode and the k-th element can be computed by :

$$\phi_{tj}^{a} = \mathbf{A}_{kj}\,\hat{\mathbf{A}}_{kj}\,\phi_{tj} \tag{13}$$

where the superscript ^ indicates the pseudoinverse of the matrix. If the measured mode ϕ_{tj} lies in the considered subspace, then ϕ_{tj} and ϕ_{tj}^{a} are identical. If the damage is caused by another element or if it does not influence the j-th mode, the two vectors are different. The identification of the location of the damage requires the computation of the vector ϕ_{tj}^{a} for all the potential damaged elements and the calculation of the distance between these vectors and the measured one. A distance equal to zero indicates the damaged element. The distance between the two vectors can be computed using the Frobenious norm :

$$d_{kj} = \left\| \phi_{tj} - \phi_{tj}^{a} \right\|_F \tag{14}$$

For an assigned structure, having e potentially damaged elements and using r measured modes, the method requires, for the identification of the damage location, the evaluation of the $e \times r$ **D** matrix containing the Frobenious norm of the Euclidean distances. If the damage is located on the k-th element and mainly influences the j-th mode, then the d_{kj} element of the matrix will be close to zero and the others will be significantly higher. The localisation of the damage is simply performed by computing the minimum of the Euclidean distance matrices.

Best Achievable *Modal* Eigenvectors

In the original method previously reported, starting from a well-correlated finite element model of the structure analysed, the localisation algorithm is formulated in the physical space, represented by the degrees of freedom of the finite element model. This requires the use of global and element structural matrices expressed in the *DOFs* set. This can be difficult in many cases, when standard commercial codes are applied for the finite element analysis, since the element structural matrices in the *DOFs* set are not easily obtainable and they are very large even though sparse. The basic idea is thus to formulate the original method in the modal space, represented by a limited number of low frequency measured modes. In this way, the availability of the local structural matrices in the *DOFs* set is no longer required. In Eq. (6) and following, the global and element mass and stiffness matrices are simply replaced by the corresponding modal ones. These are obtained by pre- and post-multiplying the original matrices by the modal basis, composed of the low frequency measured modes. Since the operation represents a non-singular transformation, the main architecture of the damage location algorithm is not modified, and the localisation of the structural damage is once again performed by computing the minimum of the Euclidean distances matrix **D**.

Damage Identification

Generally, the damage identification phase is simpler than the previous one, especially in cases where only one damage at time is considered, as in the cases reported in the following. The method here adopted is based on a standard structural optimisation approach, as implemented in MSC/NASTRAN Sol200. The damage identification problem is formulated as an updating problem (11), where the design variables are the geometric or mechanical properties of the elements identified in the localisation phase as the most probably damaged. The constraints functions used during the optimisation are expressed in terms of eigenvalues and the objective function to be minimised represents the mean error between the analytical and measured frequencies.

Numerical Example #1: *Tenbar*

The first numerical example concerns a very simple structure, often used in literature as a test case for structural optimisation programs, and known as *Tenbar*. It is composed of 10 steel rod elements, connected to form the two-bay reticular truss shown in Figure 1.

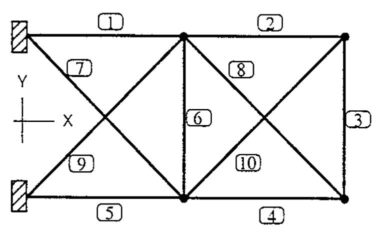

Figure 1 - The *Tenbar* structure

This test case, due to its simplicity, has been used to investigate the applicability of the modal formulation here presented. The structure is modelled by 10 elements and 4 nodes, for a total number of 8 *DOFs*. Many cases concerning different structural damages have been analysed, but here only the results related to a 10 % stiffness loss of element N.2 are reported. At first, the Best Achievable *Modal* Eigenvector method has been applied using all 8 modes of the structure. In this case, the modal approach is equivalent to the spatial one, since the number of modes is equal to the number of *DOFs* (8 in both cases). Figure 2 shows a 3D bar diagram representing the final Euclidean distances **D** matrix, where the highest values have been truncated in order to simplify the representation. It appears evident that the localisation method correctly identifies the element N.2 as the damaged one. In fact, in correspondence of this element, the Euclidean Distance is close to zero for all the considered modes.

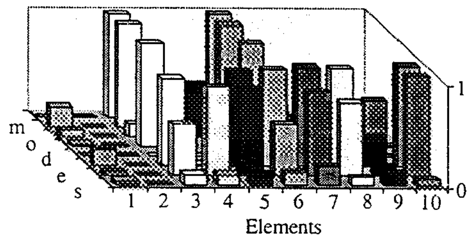

Figure 2 - The **D** matrix using 8 mode shapes.

In the second case only the first three modes have been used to localise the damage. Figure 3 shows the resulting **D** matrix.

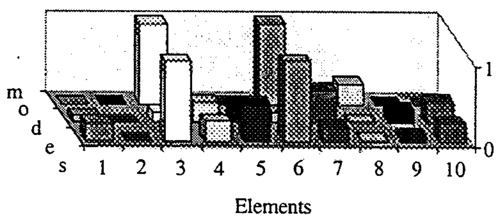

Figure 3 - The **D** matrix using 3 mode shapes.

As in the previous case, the damage is correctly associated to element N.2, even though now other elements, such as N.8 for example, show a distance close to zero, but not for all the modes considered.
In both cases, the magnitude of the damage has been successively computed by means of a very simple structural optimisation.

Numerical Example #2: *TESS*

The first numerical example, due to its simplicity, is not sufficient to demonstrate the applicability of the method here presented. A second more complicated numerical test case has thus been checked. The new numerical test case is represented by the *TESS* structure (Truss Experiment for Space Structures), a 19 *m*, 81 *Kg* slender truss structure composed of 54 cubic modules made of plastic tubes, developed at the Department of Aerospace Engineering of Politecnico di Milano. Figures 4 and 5 show the basic cubic module and the structure suspended from the ceiling by 3 pairs of metal springs during some dynamic tests. In the following, only the numerical model has been considered. It is composed of the first 24 natural modes, in the 0.3-24 *Hz* range, computed by MSC/NASTRAN using the updated mesh obtained after the modal tests.

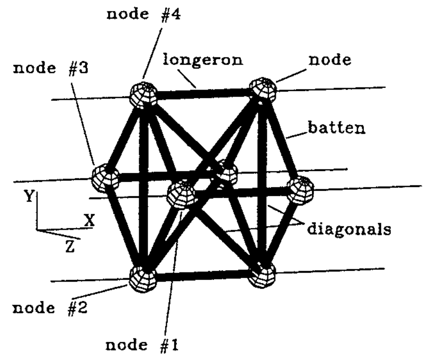

Figure 4 - The basic cubic module.

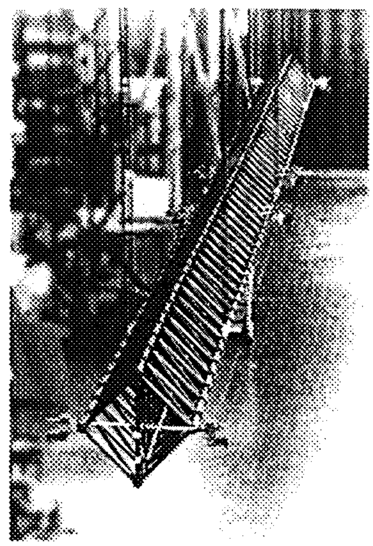

Figure 5 - The *Tess* structure.

Due to the configuration of the basic cubic modules, the truss shows the modes in the horizontal plane (horizontal bending and axial modes) completely separated from those in the vertical plane (vertical bending and torsion modes). As a consequence, the sensitivity of the modal parameters (frequencies) with respect to the structural elements also appears as separable, i.e. the horizontal longerons mainly influence the horizontal bending modes, while the vertical longerons and the diagonals mainly influence the vertical bending and torsion modes. Many tests based on different magnitude and location of structural damage have been investigated. In the following, only the ones concerning a loss of stiffness located in the horizontal longeron elements are reported.
The horizontal longerons are composed of 54 truss elements each . In order to maintain the symmetry of the structure, the same damage in the two corresponding elements of both the longerons has been supposed. Four structural damage cases have been considered, each with the same loss of stiffness (30 %) but with a different length along the truss, equal to 6, 3, 2 and 1 bays

respectively. To do so, four different variable linking operations have been used, grouping the elements of the horizontal longerons as reported in Tab. 1.

Case No.	Total number of Design Variables	No. of bays for each Design Variable
1	6	9
2	18	3
3	27	2
4	54	1

Tab.1 - The different variable linking test cases.

In all the cases analysed, the structural damage is associated to the design variable N.7. The modal basis used during the computation is composed of the first 8 modes in the horizontal plane, i.e. the first seven bending modes and one axial mode. The global and elements modal stiffness and mass matrices are exported from MSC/NASTRAN by altering the SOL103 eigenvalues analysis. Only the modal formulation of the *BAE* method has been used to localise the structural damage in the four cases considered. The results obtained have demonstrated the capability of the method to correctly localise the damage for the first three cases, while in the fourth case some numerical problems have been encountered, due to the ill conditioning of the matrix **L**. Nevertheless, it must be remembered that in this case, where the 30 % loss of stiffness is concentrated in the element N.7 with a damage length of only one bay, the maximum change in frequency is about 0.2 % for the sixth horizontal bending mode. Figure 6 shows the final **D** matrix for the smallest identifiable structural damage, i.e. damage length equal to two bays (test case N.3).

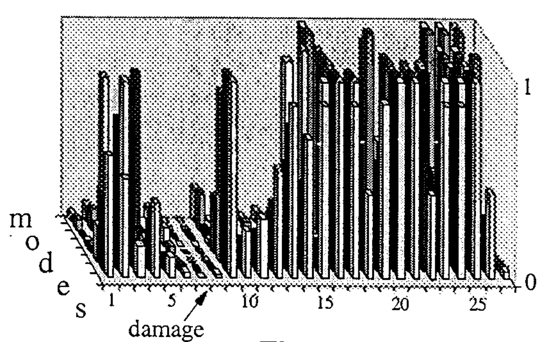

Figure 6 - The **D** matrix for the test case No. 3 : length of structural damage equal to two bays.

The localisation method correctly converges to the design variable N.7, the only one for which the Euclidean distances are close to zero for all the considered modes. The main effect of the modal formulation versus the spatial one is a sort of diffusion of the damage : the final **D** matrix shows many small terms, even though the column related to the damaged element is the only one composed of terms two orders smaller than all the others.

Experimental validation: the Mini*TESS* structure

To experimentally validate the BAE method in the modal formulation here presented, it has been applied to localise structural damage in a typical laboratory space truss structure, a 3.5 *m* long modular truss, composed of 10 basic tetrahedral modules, as shown in Figures 7-8.

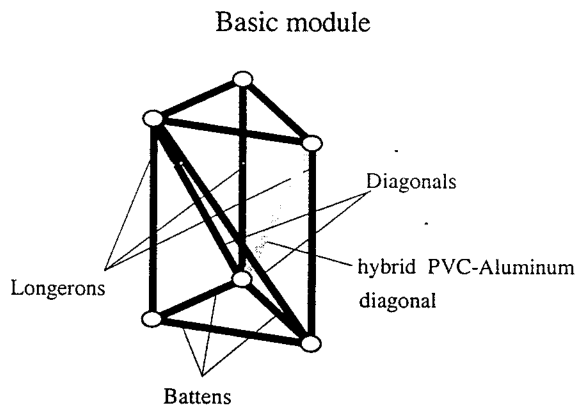

Figure 7 - The basic module.

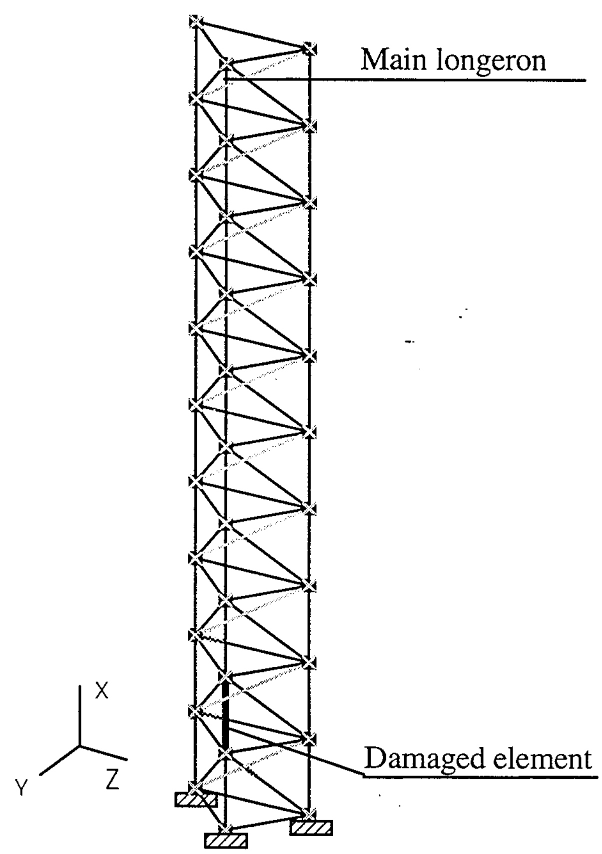

Figure 8 - The Mini*TESS* structure.

Each module is composed of 6 battens, 3 longerons and 2 diagonals, made of plastic tubes, and 1 diagonal made of a hybrid PVC-aluminum element.

First of all, an accurate modal analysis of the structure has been performed . A single-point excitation using a 40 Hz bandlimited noise and 30 piezoelectric accelerometers in the *Y* and *Z* directions have been used to identify the first 7 mode shapes, 3 bending modes in the *XY* plane, 2 bending modes in the *YZ* plane and 2 torsion modes. A very important step in the application of any damage detection method is the availability of a well-correlated finite element model of the structure. An MSC/NASTRAN finite element model of the Mini*TESS* structure has been adopted : Table 2 shows a comparison between the numerical and the measured frequencies.

Measured Freq. [Hz]	Numerical Freq. [Hz]	Err. [%]
2.97	3.14	-5.93
4.39	5.15	-17.25
11.19	11.38	-1.72
15.65	16.66	-6.45
22.07	27.73	-25.64
33.28	34.28	-2.99
36.73	38.18	-3.96

Table 2 - Comparison between the measured and the numerical frequencies before updating the model.

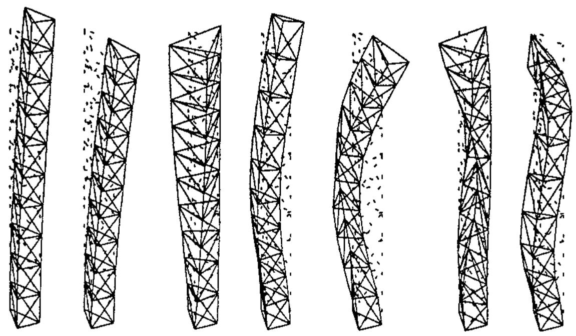

Figure 9 - The first 7 measured mode shapes.

Since the numerical-experimental correlation is not accurate enough, a model updating operation has been performed using the structural optimisation solution (SOL200) of MSC/NASTRAN.

Measured Freq. [Hz]	Numerical Freq. [Hz]	Err. [%]
2.97	2.95	+0.67
4.39	5.10	+1.63
11.19	11.14	+0.45
15.65	15.79	-0.89
22.07	22.37	-1.36
33.28	33.45	-0.51
36.73	36.45	+0.76

Table 3 - Comparison between measured and numerical frequencies after updating the model.

Table 3 shows a comparison between the measured frequencies and the numerical ones after the optimisation. The orthogonality check with respect to the mass matrix and the MAC test have also been performed to verify the consistency of the dynamic model.

The degree of correlation has been considered sufficient and the updated model has been used to investigate the applicability of the damage detection method.

Many structural damages were taken into consideration. In the following, only the results concerning structural damage located in the second bay of the main longeron, obtained reducing the cross area of the plastic tube by 10%, will be reported.

Without damage Freq. [Hz]	With damage Freq. [Hz]	Diff. [%]
2.97	2.91	2.02
11.19	11.17	0.18
15.65	15.70	-0.32
33.28	33.19	0.27
36.73	36.36	0.99

Tab.4 - Comparison between the measured frequencies with and without damage.

To improve the efficiency of the damage location algorithm, since only 5 modes in the adopted modal basis are influenced by the damage considered (bending modes in the *XY* plane and torsion modes), only these have been used during the damage localisation and identification process. Table 4 shows the frequency changes due to the structural damage for these 5 modes.

To apply the *Modal BAE* method previously described, the global and elements stiffness and mass matrices have been computed by means of MSC/NASTRAN, altering the normal modes solution (SOL103) by replacing the numerical modal basis with the measured one.

Figure 10 shows the final **D** matrix for the elements of the main longeron only.

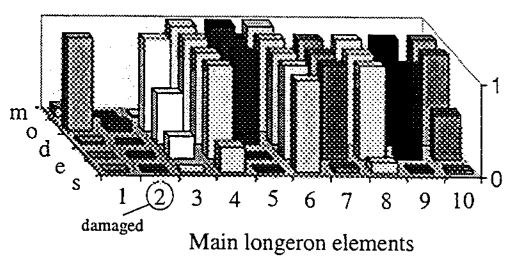

Figure 10 - The **D** matrix using 5 mode shapes.

The localisation method identifies as most probable damaged element the one actually damaged, even though in this case the terms of the **D** matrix column related to that element and corresponding to the fourth considered mode, are not equal to zero. This could be due to the fact that the localisation method has been applied directly using the experimental mode shapes, without any kind of smoothing operation: the disturbances always included in the measured data may influence the final result. Another possible reason could be the imperfect correlation between the numerical and the measured fourth mode, as can be seen in Figure 11, where the MAC matrix between the numerical and the measured modes without damage is shown.

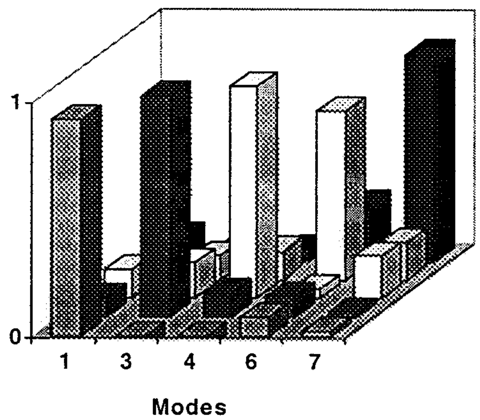

Fig. 11 - The MAC matrix : numerical versus measured modes without structural damage.

Finally, as in the second numerical example reported previously, the **D** matrix shows many terms close to zero, but only the column corresponding to the actual damaged element is entirely composed of terms close to zero.

Conclusions

Starting from the original *BAE* method presented in (10) the paper reports a modal formulation applicable to the structural damage localisation phase.

Two numericai examples and one experimental validation are reported to demonstrate the applicability of the proposed approach. The main difficulty in the application of the method, both in the original and in the modal formulation, is related to the difficulty of measuring the mode shapes with an accuracy comparable to that obtainable in the measurement of frequencies. Nevertheless, these difficulties are the same encountered in the application of any damage detection method based on this kind of modal data. The use of modal matrices instead of spatial ones offers some advantages, especially when using commercial finite element codes.

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