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NONLINEAR EVOLUTION OF A THREE-DIMENSIONAL WAVETRAIN IN A FLAT PLATE BOUNDARY LAYER

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Abstract

This paper presents results of an experimental study of the transition in boundary layers. The experiments were conducted in a low-turbulence wind tunnel. The process was triggered by a three-dimensional Tollmien-Schlichting wavetrain excited by a harmonic point source in the plate. Hot-wire anemometry was used to measure the signal and investigate the nonlinear regime of these waves. It was observed that the three-dimensional wavetrain behaved very differently from two-dimensional ones. In particular, it did not involve the growth of subharmonics or higher harmonics. The first nonlinear signal to appear was a mean flow distortion. This had a spanwise structure consisting of regions of positive and negative mean distortion distributed like longitudinal streaks, which became more complex as the nonlinearity developed. The structure in the direction perpendicular to the wall has also been studied. Initially the distortion profiles resembled Klebanoff modes, but further downstream it also became more complex. Elsewhere studies have revealed the existence of streak-structures in turbulent flows. It is conjectured that the current experiments may provide a link between early wave-like instabilities and some coherent structures of turbulent boundary layers.

1 Introduction

The laminar-turbulent transition point is often a crucial element in the design of wings. In spite of it, the prediction of the transition point is still a challenge in fluid mechanics. Transition is affected by a large number of parameter, such as free-stream turbulence, pressure gradient, conditions of the leading edge and degree of isotropy and homogeneity of the free-stream turbulence.

It has now been established that there are a number of different routes to transition. Among them the most studied is the Tollmien-Schlichting route. This route involves the exponential amplification of two dimensional waves, which, if the amplitude is large enough, give rise to three dimensional waves via a secondary instability mechanism (Herbert 1988). The three dimensional waves can be of two types depending on the kind of resonant interaction that occurs. With primary resonance the three dimensional waves have frequency equal to the fundamental two dimensional waves, whereas with parametric resonance the three dimensional waves are subharmonics of the fundamental. In both cases the wave system that arises, saturates in the form lambda vortices (Corke & Mangano 1989, Kachanov & Levchenko 1984, Kachanov 1987, Kachanov 1994). With subahrmonic resonance the vor-

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tices exhibit a staggered pattern whereas for fundamental resonance they are aligned in the streamwise direction. High amplitude fundamental waves tend to favour primary resonance. In any case, the lambda vortices give rise to high frequency oscillations by a mechanism as yet not established. Some researchers attribute the breakdown to the instability of an inflexional profile that would be created by the vortical structure. Others argue that the tip of the lambda vortices stretch under the action of the shear layer giving rise to high frequency oscillations, but there may be other plausible explanations.

Another route to transition is the so called by-pass transition. The scenario consists of the formation and transient growth of longitudinal streaks, that is, regions of higher and lower streamwise velocity (Henningson, Lundbladh & Johansson 1993). If the amplitude of the streaks is sufficiently large the appearence of high frequency disturbances is observed. It has been suggested that the high frequency disturbances are caused by spanwise inflexional instability of the streaks (Elofsson & Alfredsson 1997). The transient growth is caused by a mechanism different from that causing the growth of the Tollmien-Schlichting waves. In fact transient growth can occur under subcritical conditions, that is, conditions for which all Tollmien-Schlichting modes are stable. These route is generally connected with disturbances of amplitudes considerably larger than those of the classical Tollmien-Schlichting route.

Both routes have been shown theoretically and confirmed by experiments. The current work is primarily concerned with the Tollmien-Schlichting route to transition, but, as will be seen, the experimental observations do not display the appearence of subharmonic modes as would be expected from secondary instability theory. The results display the formation of longitudinal streaks. Possible connection of this streak structure with those observed in by pass transition have not been fully investigated, but, despite a few similarities, the process appears to be different.

Both theoretical and experimental studies of the Tollmien-Schlichting route to transition have concentrated on a rather restrictive case, namely, the situation when the primary wave driving the process is a plane monochromatic wave. While this is a very important study, and almost certainly a necessary step towards more generic situations, the case of more practical interest is the so called natural transition. In natural transition the waves arise from free-stream turbulence, wall vibration, acoustic waves, and others sources which are not controlled. In this context the waves that arise are not plane-monochromatic but are highly modulated involving a large number of two and three-dimensional modes.

Gaster suggested that a possible way to model the natural transition is to disturb the flow with a pulse emanating from a point source. The idealized pulse contains all frequencies of the spectrum including three-dimensional modes. Gaster tested the idea in his classical experiment (Gaster & Grant 1975) and observed that, owing to the selective amplification of modes in the boundary layer, the pulse generated a wavepacket in agreement with the linear instability theory (Gaster 1975). The nonlinear evolution was also investigated and the observations indicated that nonlinearity appeared at surprisingly low wave amplitudes, in fact, amplitudes for which monochromatic wavetrains would display linear behaviour. This was later confirmed by experiments in which the nonlinear evolution of wavepackets were compared with that of wavetrains (Gaster 1978, Gaster 1993). Later other experiments with wavepackets in boundary layers were carried out by Breuer & Haritonidis (1990), Cohen, Breuer & Haritonidis (1991), Cohen (1993) and Breuer, Cohen & Haritonidis (1997). In all the experiments the nonlinear evolution of the packets were connected to the appearence of waves with frequency close to the subharmonic of the fundamental.

Following Gaster, Shaikh (1993, 1994, 1997) studied a more generic type of disturbance, namely, that generated by a white noise excitations. Shaikh compared the evolution of disturbances generated from different white noise sequences. The sequences had the same spectral content, only the phase of the components relative to each other were different. He observed that the transition point was very sensitive to the phase composition of the disturbance, and concluded that the amplitude was not the only parameter affecting the process.

The works of Gaster and Shaikh pointed to the fact that the nonlinear mechanisms in the evolution of modulated waves were different from those observed in plane wavetrains. However the wavepackets of Gaster were too restrictive while the white noise sequences of Shaikh were too generic, making it difficult to interpret the results. By allowing the amplitude of the wavepackets to be complex, Medeiros (1996, 1996) generated experimentally a set of wavepackets with identical envelopes but with different phase relative to the envelope. With this he was able to show the influence on the phase of the evolution of modulated waves (Medeiros 1997*b*). Later he also showed experimentally that although the subharmonic resonance appeared to be present in the process, it alone could not explain the observations (Medeiros 1997*c*). The nonlinear mechanism observed involved the production of subharmonic waves. The fact that not only amplitude but also phase affects the transition process is of large practical importance because the transition prediction methods so far used have only accounted for the amplitude of the waves.

The wavepackets are modulated both in streamwise and spanwise directions. At first it appeared that the important ingredient was the streamwise modulation, rather than the spanwise modulation (Gaster 1984, Medeiros & Gaster 1994, Medeiros & Gaster 1995), however numerical simulations of two-dimensional wavepackets by Medeiros (1996) have shown that spanwise modulation is essential to the process. This has led to the investigation of the nonlinear evolution of waves that are only spanwise modulated, namely, a wavetrain emanating from a point source. This paper presents some results of these investigations. Preliminary results have been presented by Medeiros (1997*d*) and Medeiros (1997*a*)

Investigation of three-dimensional wavetrains has also been carried out by other researchers (Kachanov 1985, Mack 1985, Seifert 1990, Seifert & Wygnanski 1991, Wiegand, Bestek, Wagner & Fasel 1995), but these were restricted to the linear stage.

2 Experimental Results

The experiments were conducted in the low turbulence wind tunnel of the University of Cambridge, Cambridge, UK [1]. The boundary layer developed over a flat plate. The pressure gradient was controlled with the help of a flap at the trailing edge of the plate. With this set up a fairly small pressure gradient was obtained, figure 1. Also, the profiles measured were close to the theoretical Blasius profile, figure 2. The measurements covered the region ± 200mm in the spanwise direction from the centreline with profiles 2cm apart.

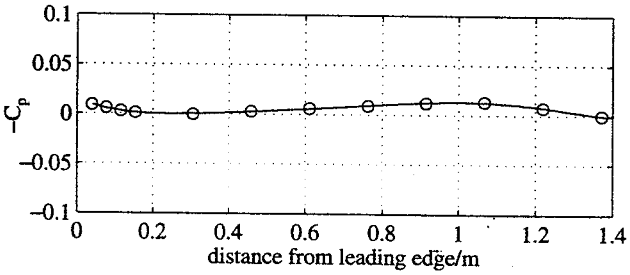

Figure 1: The static pressure distribution over the plate

The disturbances were introduced in the flow via a loudspeaker embedded in the plate and communicated to the flow through a .3mm hole, located on the centreline of the plate 203mm from the leading edge. The velocity records were measured with a 2.5μm gold plate tungsten hot-wire connected to a constant

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[1] This tunnel is now located at Queen Mary and Westfield College, London University, London, UK

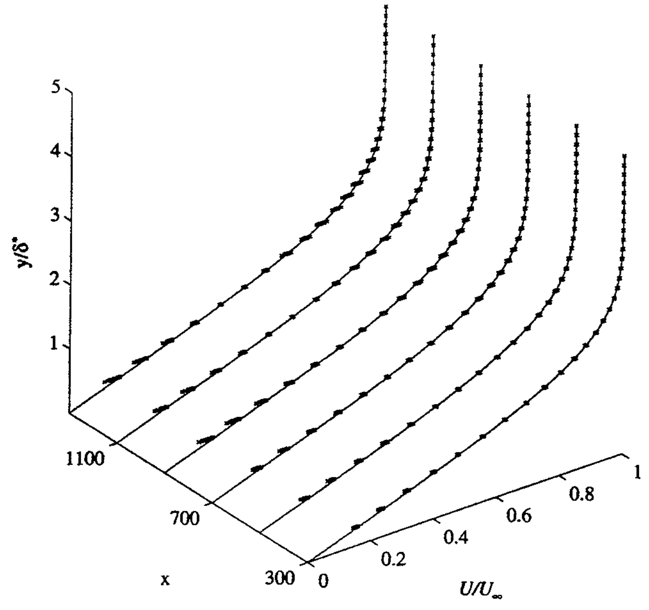

Figure 2: The mean flow. Lines represent the theoretical Blasius profile. Asterisks represent the measured values

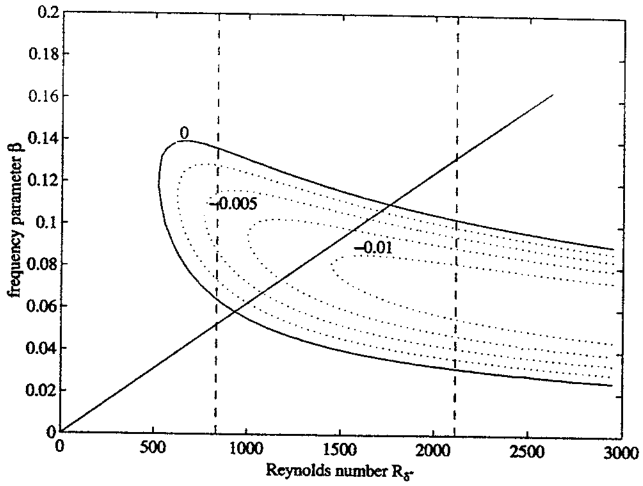

Figure 3: The instability diagram showing the path of the 200Hz Tollmien Schlichting wave.

temperature anemometer. The hot-wire was mounted on a three-dimensional traverse gear. The streamwise and spanwise positions of the wire could be directly measured from the traverse mechanism. The distance from the wall had to be obtained indirectly by measuring the velocity profile and comparing to the theoretically obtained. The procedure has been used in similar experiments in this tunnel (Shaikh 1993) and has proved to be fairly accurate. More details of the experimental set up are given by Medeiros (1996).

In experiments with wavetrains the flow is usually disturbed by a continuous harmonic source. In the current series of experiments a long but finite 200Hz wavetrain is excited from a point source. The linear evolution a two dimensional mode with frequency 200Hz is shown by the straight line on the instability diagram, figure 3. The excitation was introduced at R_δ about 800 and measurements were taken up to R_δ^* 2100, as indicated by the dashed lines in the figure. One can see that the excitation was introduced upstream of branch I of the neutral curve and that measurements were taken beyond branch II, after which the Tollmien-Schlichting waves decay. The waves cross branch II at R_δ^* around 1700.

The evolution of the disturbances observed experimentally along the centreline is shown in figure 4. The measurements were taken at a nondimensional distance of $0.52\delta^*$. Using finite wavetrains, the disturbance is an event that can be repeated. Therefore ensemble averages can be taken in order the get a clearer signal. The records displayed here, as well as those shown in other figures, were obtained from 64 ensembles. The first important observation is that wave amplitudes grow up to R_δ^* about 1700 and thereafter decay, consistent with the linear theory, figure 3. A mean flow distortion is also observed that is not predicted by the linear theory. Initially the distortion is negative, but further downstream switches to positive. It is remarkable that the change in the trend of the mean flow distortion occurs close to where the disturbance crosses branch II. The mean flow distortion is made very clear by the use of finite length wavetrains. It could have remained undetected if a continuous wavetrains were used. It is possible that the use of continuous wavetrains have prevented these mean flow distortions from being observed in previous experiments with three-dimensional wavetrains.

Figure 5 shows the evolution of a wavetrain of considerably smaller amplitude. Here

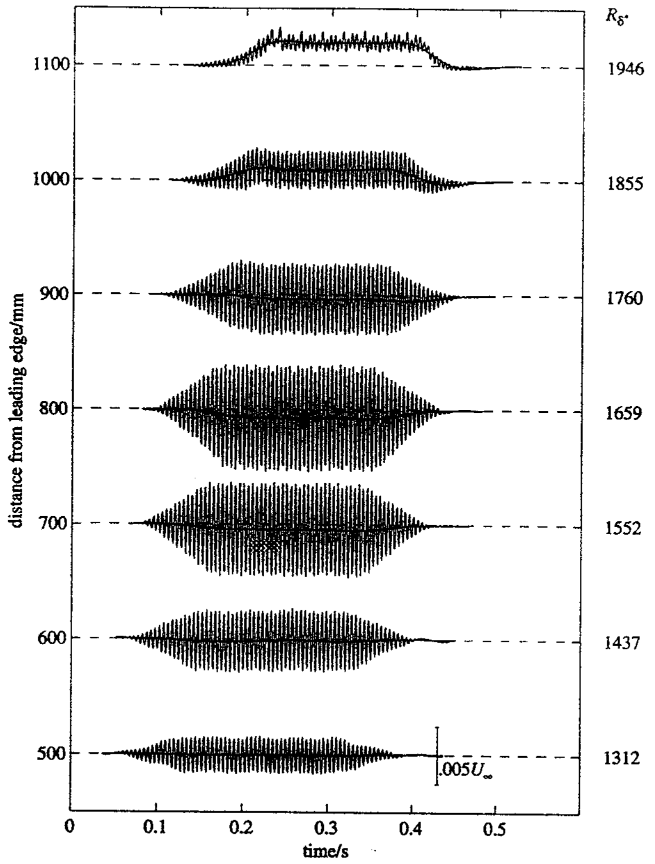

Figure 4: The evolution of the 200Hz three-dimensional wavetrain along the centerline of the plate at a distance of $0.52\delta^*$ from the wall.

again the nonlinear mean flow distortion is observed, despite the low amplitude of the oscillations. The oscillations are so small that slightly downstream of branch II they are very difficult to measure. The mean flow distortion, on the other hand, can be observed up until the end of the flat plate. This phenomenon may come as a surprise, but it is possible that the decay of the mean flow distortion is governed by the recovery response of the boundary layer which for a laminar boundary layer is very slow. Also interesting is that the change in the sign of the mean flow distortion for the very small disturbance occurs at the same location as for the relatively large disturbance.

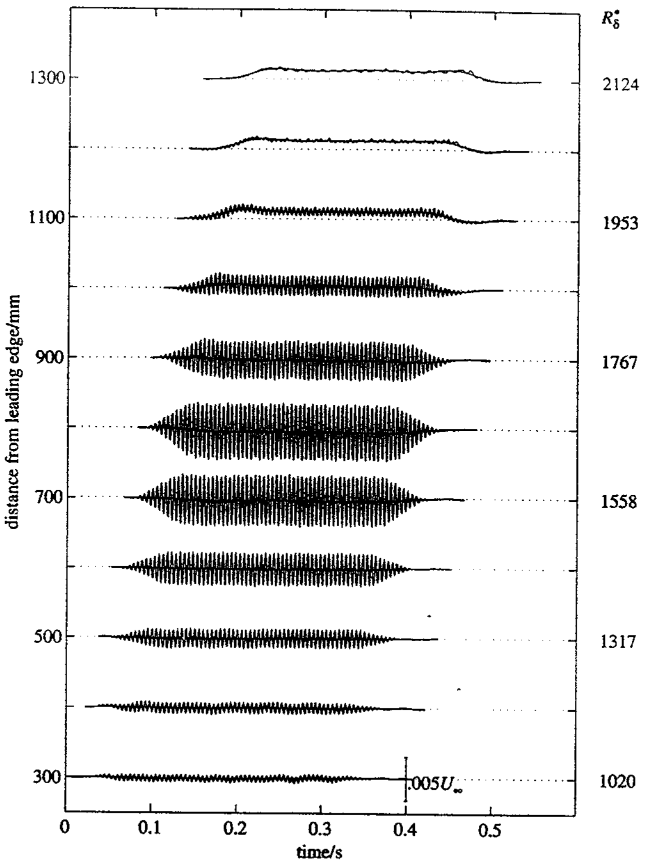

Figure 5: The evolution of a very small amplitude 200Hz three-dimensional wavetrain along the centerline of the plate at a distance of $0.52\delta^*$ from the wall.

Measurements were also taken off the centerline of the flow in order to study the spanwise structure of the wave system. Measurements were taken at different spanwise locations 10mm apart from each other covering the entire width of the disturbance field. This provided a view of the disturbance as it crosses a streamwise location, figures 6, 7 and 8. Initially the disturbances display swept back wavecrests with a smooth modulation across the span. Further downstream , the signal develops strong distortions and the appearence of streaks is observed.

The structure of the streaks is made clearer if the oscillating part of the signal is digitally filtered, figure 9. A picture of the evolution of the streak structure as it evolves downstream is shown in figure 10. Initially there is a central region of negative mean flow distortion together with two lateral regions of positive mean

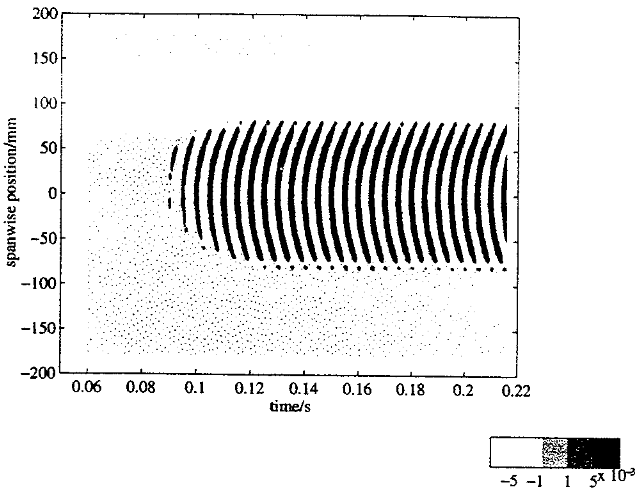

Figure 6: Spanwise structure of the disturbance field 700mm from the leading edge and $0.52\delta^*$ from the wall.

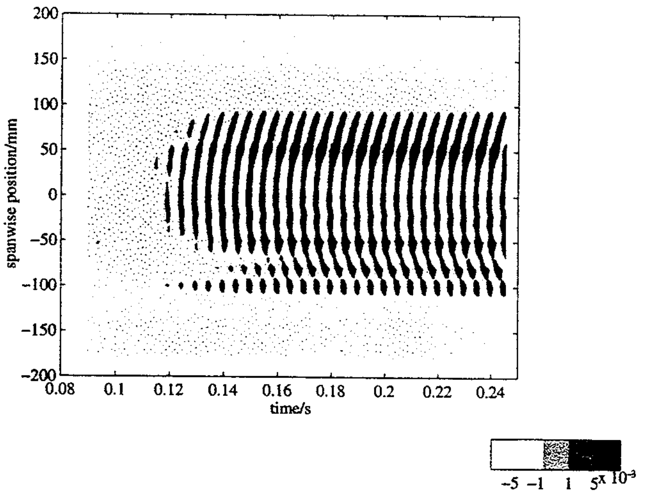

Figure 7: Spanwise structure of the disturbance field 900mm from the leading edge and $0.52\delta^*$ from the wall.

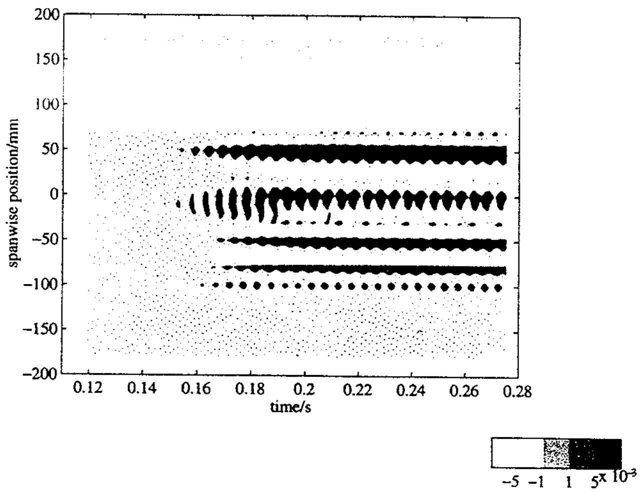

Figure 8: Spanwise structure of the disturbance field 1100mm from the leading edge and $0.52\delta^*$ from the wall.

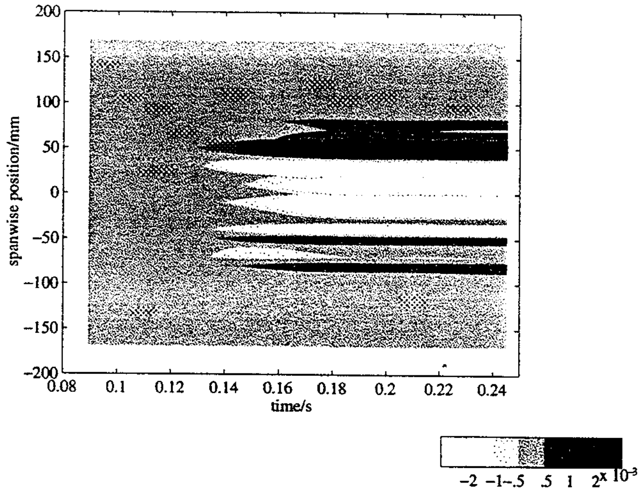

Figure 9: Spanwise structure of the mean flow distortion (streaks) 900mm from the leading edge and $0.52\delta^*$ from the wall.

flow distortion. This structure suggests the existence of a pair of counter-rotating concentrations of vorticity which would push down high momentum fluid in the lateral regions and lift up low momentum fluid in the central region. However, it is as yet unclear whether these mechanisms are actually taking place. The concentrations of vorticity are probably too weak to be considered vortices and perhaps the lift up/push down effect is too small to affect the flow. As the waves evolve, the structure becomes more complex. At x=1000mm the appearence of a region of positive mean flow distortion right at the center of the wavetrain is observed. This corresponds to the change in the sign of the mean flow distortion observed in figure 4. From station x=1000mm onwards the structure doesn't experience remarkable changes, apart from the broadening of the central positive mean flow distortion.

It is interesting to look at the evolution in the frequency domain which was carried out by Medeiros (1997*a*). However, the spanwise resolution of the experiment is relatively low. The

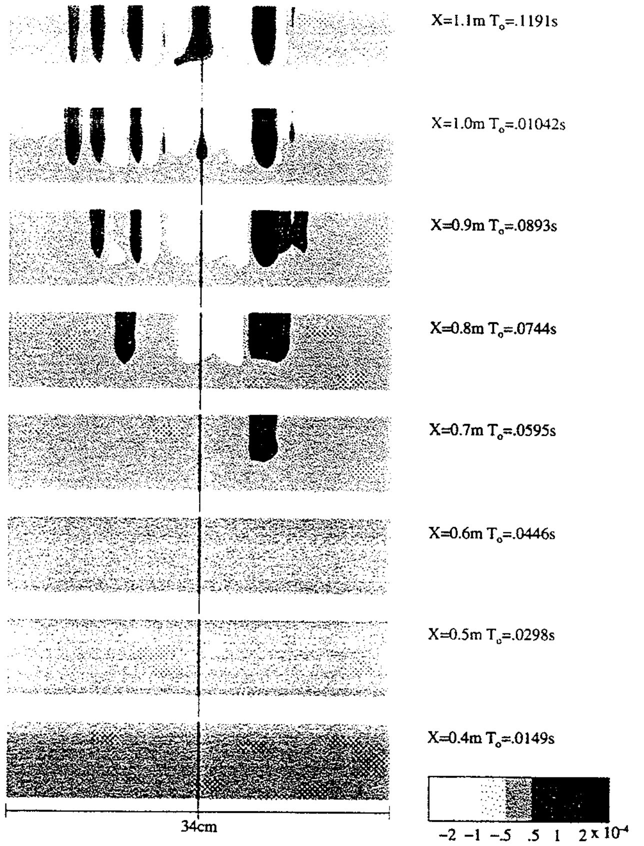

Figure 10: Downstream evolution of the streaks

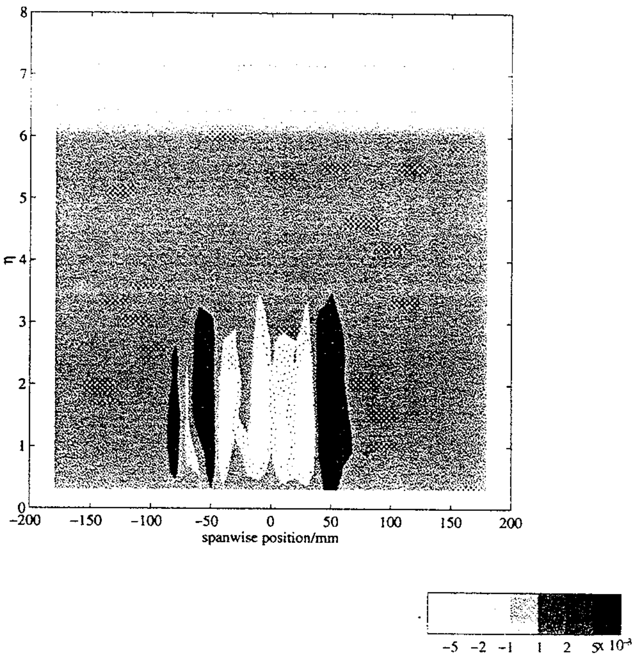

Figure 11: The mean flow distortion distribution on a plane perpendicular to the flow direction 900mm from the leading edge.

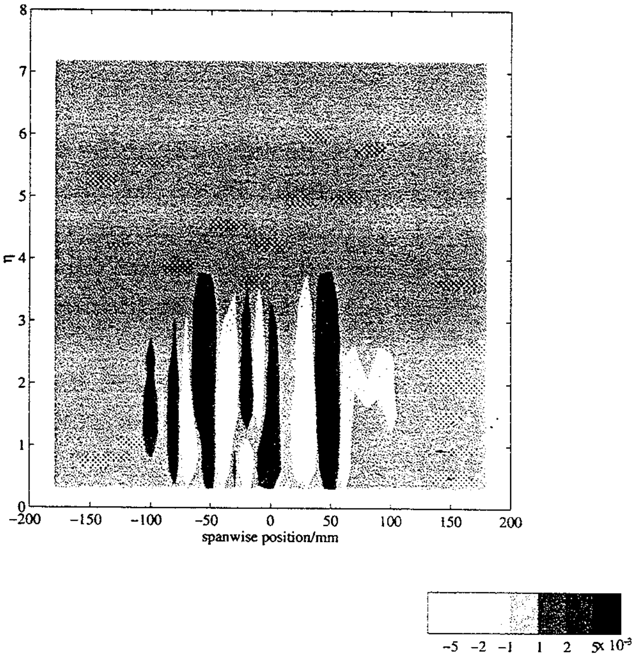

Figure 12: The mean flow distortion distribution on a plane perpendicular to the flow direction 1100mm from the leading edge.

results became difficult to interpret because alias effects could not be ruled out. Therefore, care should be taken in analyzing those results. What is clear is that initially the nonlinear mechanism generates only two region of positive mean flow distortion and one central region where the distortion is negative, while further downstream the spanwise wavenumbers become significantly larger.

Also important is to investigate the structure of the mean flow distortion in the direction normal to the wall. Figures 11 and 12 show contour plots of the mean flow distortion on planes perpendicular to the flow direction 900mm and 1100mm. The mean flow distortion is concentrated inside the boundary layer. In the external part of the flow no sign of the mean flow distortion is observed. At x=900mm the structure is basically composed of two regions of positive mean flow distortion and a central region of negative mean flow distortion. Whereas the positive lumps are fairly concentrated the negative region spreads over a larger portion. Moreover, the negative region appears to be composed of several lumps. The profile resembles that of a Klebanoff mode with a single maximum inside the boundary layer, particularly for the positive streaks. The maximum is located between $\eta(= y\sqrt{\frac{U_\infty}{\nu x}})$ 1 and 2, which is also similar to that of Klebanoff modes. At x=1100mm the negative central region splits into a number of regions and lumps of positive mean flow distortion arise. At this stage the central part of the disturbance the signal is too complex for any definite conclusion concerning the location of the maximum.

The evolution of the profile along the streamwise direction may be more meaningful than the analysis of the signal at a particular streamwise station, figure 13. The figure shows how the disturbance field slowly evolves from a relatively simple structure into a much more complex one at x=1100mm.

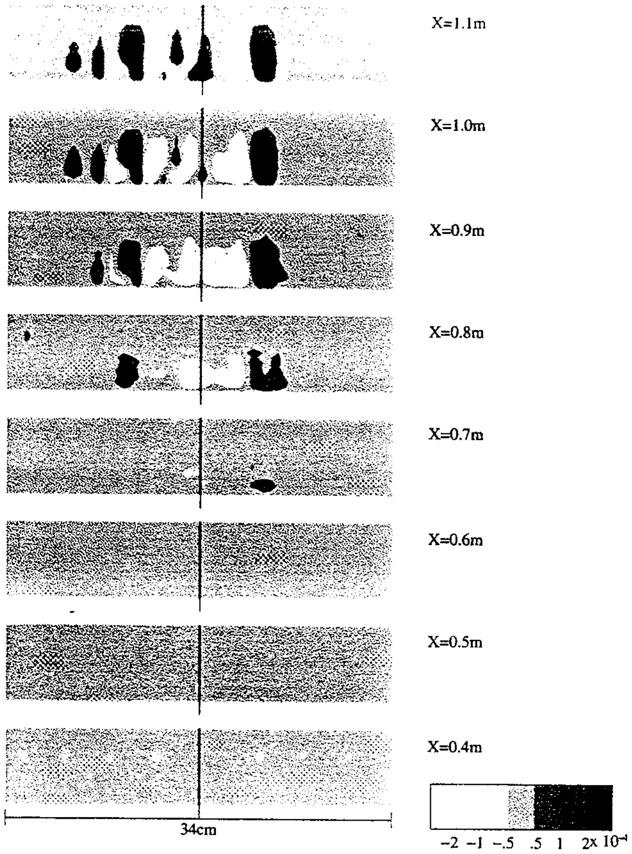

Figure 13: Evolution of the mean flow distortion.

An overall view of the transfer of momentum from the low velocity streaks to the high velocity streaks is given by the distribution of the displacement thickness variation over the entire disturbance field, figure 14. In this picture the details of the complex distribution in the direction perpendicular to the wall are lost, and an averaged view of the distortions results. The figure shows a somewhat more symmetric pictures of the flow than that of figure 10. For the positive lateral regions the displacement thickness distribution indicates a structure similar to that suggested by figure 10. The central region, on the other hand, indicated that the central negative region splits into three regions separeted by two newly generated positive regions.

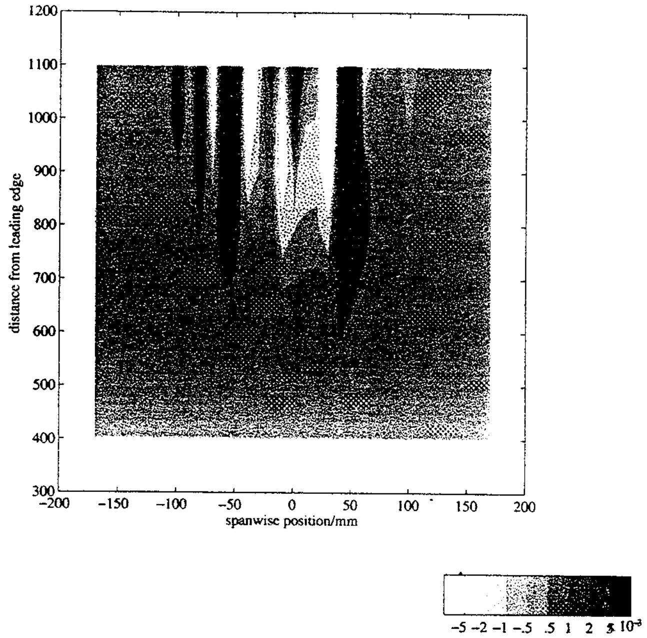

Figure 14: Distribution of the displacement thickness variation over the plate relative to displacement thickness of the Blasius profile.

3 Conclusion and Discussion

The work studies the nonlinear evolution of wavetrains emanating from a point source in a flat plate boundary layer. The first interesting result was that the first indication of nonlinear behaviour was not a subharmonic signal, but a mean flow distortion that formed longitudinal streaks. This mean flow distortion may have arisen from self interaction of the modes via the Reynolds stresses term, but this conjecture is yet unproven. Initially the mean flow distortion displayed a relativelly simple spanwise structure with a central region of negative mean flow distortion and two lateral regions of positive mean flow distortion. It appeared that the strength of the nonlinear interaction at this stage was stronger off the centerline of the flow. This might be linked to the fact that for some frequencies the three-dimensional wavetrains also display amplitude maxima off the centerline in the linear regime (Wiegand et al. 1995). This would be consistent with these being generated from the Reynolds stresses.

The structure of these distortions in the direction perpendicular to the wall was also investigated. Initially they resemble klebanoff modes with one amplitude peak at a position η between 1 and 2.

The relativelly simple structure gives rise to a fairly complicated flow field further downstream. The more complicated field appears to originate at a position close to the second branch. The more complicated structure arises in the central portion of the wavetrain, where the initial negative mean flow distortion was formed. There, regions of positive mean flow distortion arises and the profiles of the distortions do not display the Klebanoff mode shape. Despite the complexity, an overall view shown by the displacement thickeness variation over the plate, suggests that the central negative region splits into three. The positive mean flow distortion regions do not change considerably along the process, except in amplitude.

Streaks have also been observed in by pass transition. However, there the streaks tend to keep their spanwise spacing, as oppose to what is observed in the current experiment. Longitudinal streaks have been observed in turbulent flow. In same cases they appear to be a key ingredient of the mechanism of production of turbulence. It has also been shown that modulated waves give rise to transition at smaller amplitudes in comprison with plane wavetrains. It is possible that the modulation of the waves provide a short cut between the early wavelike behaviour and the vortical structures observed in turbulent flows. These conjecture is currently being investigated. The possibility that the initial mean flow distortion be generated from the self interaction of waves is also being investigated.

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