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AN IMPROVED METHOD FOR THE DESIGN AND CALCULATION OF AERODYNAMIC CHARACTERISTICS OF AIRFOILS WITH THE DOMINANT TURBULENT BOUNDARY LAYER AT SUBSONIC AND LOWER TRANSONIC SPEEDS

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Abstract

In this paper, the most important aspects of computation techniques applied in a computer program TRANPRO, used for the airfoil design, optimization and analysis are presented. Speed domain in applications ranges from small subsonic to lower transonic values, with the corresponding Reynolds numbers usual for the aviation purposes. This program is a higher and more sophisticated version of the well known program Trandes, whose upgraded modes for airfoil design and analysis are used in TRANPRO as subroutines. The new computer program has several important advantages, among which a specific inverse-direct approach in airfoil design is the most relevant. In the inverse phase of the design mode, trailing edge closure is achieved automatically, while in Trandes, the user was supposed to redefine Cp input by himself. In the direct phase (which does not exist in Trandes), the airfoil shape is additionally adjusted to strictly satisfy required value of a chosen important aerodynamic parameter, if such a necessity exists. Turbulent boundary layer calculation applied in TANPRO overcomes the problems of the original Carlson/Nash-Macdonald model, which generally underestimates the profile drag values. Improvements were also made in the wave drag calculations.

Symbols

C_L	airfoil lift coefficient
C_{DP}	airfoil profile drag coefficient
C_{DW}	airfoil wave drag coefficient
C_D	airfoil total drag coefficient
$C_{M\,1/4}$	airfoil quarter-chord moment coefficient
Cp	pressure coefficient
M	Mach number
Re	Reynolds number
MRe	Re x 10^6
c	absolute airfoil chord length (here it is always equal to unity)
C	factor of convergence in inverse design mode
K_{C_M}	factor of convergence in direct design mode
(x /c)	relative chordwise coordinate
(t /c)	relative thickness
$\bar{x}_c$	relative chordwise position from which airfoil contour modification begins in direct design mode
Cp_{base}	base pressure coefficient used in separation correction model
Cpl_{max}	maximum value of pressure coefficient on the lower surface of the airfoil in the vicinity of the trailing edge
α	angle of attack
α_i	angle of attack at which the separation correction is initialized
Cp_{sep}	pressure coefficient at separation point
$Xsep_i$	separation point position corresponding the α_i angle of attack
$Xsep(\alpha)$	separation point position for angles of attack higher than α_i
$CCp(Xsep, \alpha)$	base pressure coefficient correction factor
$\Delta CCp(Xsep, \alpha)$	base pressure coefficient correction factor gradient
$CCp(Xsep_i, \alpha_i)$	base pressure coefficient correction factor at α_i angle of attack
α_i^*	non-standard α_i
$Xsep_i^*$	separation point position corresponding the α_i^* angle of attack
CCp^*	base pressure correction factor at α_i^*
δ^*	boundary layer displacement thickness
θ	boundary layer momentum thickness
R_θ	Reynolds number defined by boundary layer momentum thickness as characteristic length
H	boundary layer shape factor
M_e	local Mach number at the outer edge of the boundary layer
u_e	local velocity at the outer edge of the boundary layer
ρ_e	local density at the outer edge of the boundary layer

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Introduction

Contemporary approach in aerodynamic aircraft design most often implies the need for a quick, efficient and, most of all, reliable numerical analysis of the aerodynamic characteristics of airfoils that might be selected for the lifting surfaces. In many cases, a necessity of a new airfoil design for a particular aircraft project would cause unacceptably high expenses, without the use of such computer programs that can, by properly done analyses, reduce the number of possible airfoils to several best which should be experimentally verified for the final selection. Such computer programs generally work in two possible modes: the airfoil design mode, that creates its geometry shape for the specified design conditions, and the analysis mode, used for the determination of its characteristics in off-design cases.

This paper describes in brief the most important aspects of the computation techniques applied in a computer program TRANPRO[2], used for the airfoil design, optimization and analysis. In this computer program, enhanced and upgraded modes both for the inverse airfoil design and airfoil analysis of a rather well known program Trandes[1], are used as subroutines. In airfoil design case, unlike Trandes, which uses purely inverse approach, in TRANPRO an inverse-direct technique is applied. Firstly, the trailing edge closure is achieved automatically (inverse part), while in original Trandes the user was supposed to solve that problem by himself, which was one of its key problems. Secondly, the new program gives the user a possibility to select a very strict value of a dominant aerodynamic factor, such as $C_{M\,1/4}$ or C_L (direct part) which is reached by the final airfoil shape in predefined tolerance limits. In both programs, inverse design is done by prescribing the Cp distribution around the initial airfoil shape for given airflow conditions and α, while the final airfoil can differ quite a lot from the one used as input, except in the nose region which is kept constant. In case of TRANPRO, the final airfoil is reached by a single initiation of the program without any further influence by the user, while in Trandes, the user had to redefine Cp input "manually" and restart the program as many times as necessary until airfoil with desired characteristics is obtained. By that, the experience level of the user of TRANPRO is not the crucial factor for the successful finalisation of the airfoil design, which was unfortunately the case with many early computer programs based on inverse techniques.

In the airflow analysis, the zonal approach is applied. The inviscid part of the flow is calculated by a very successful model, introduced in [1], based on the solving of full perturbation potential equation on a series of rectangular grids, enabling very quick convergence of the solution. Although the application of potential method places the upper speed limit in the domain of lower transonic values, it may be considered high enough for many practical applications. The final solution is obtained within seconds on advanced computer systems.

The viscous part of the flow is simulated by the dominating turbulent layer (TBL) calculation with the transition point fixed close enough to the leading edge, both on the upper and lower airfoil surface. In TRANPRO, an integral TBL calculation is applied, which is a modified form of the Carlson/Nash-Macdonald[1,10] algorithm used in the original Trandes program. The latter underestimates the profile drag values to the order of 10 to 15% compared with the corresponding experimental values, in the whole applicable speed domain. Modification described in this paper reduces this kind of discrepancy to the average of only several percent, and also introduces a successful algorithm for TBL calculations at higher angles of attack, at which intensive separation is inevitable. Carlson reported that in some later versions of Trandes massive separation correction was included, but this approach, according to the experience of the author of this paper, tends to give rather strange behavior of the TBL parameters in the separated region.

Finally, wave drag calculation applied in TRANPRO is not sensitive to possible problems associated with the use of the uniform distribution of rectangular grids, which also happened to be one of the problems in the original program.

Airfoil design and optimization procedure

As a first step in airfoil design procedure, in both programs[1,2], the user is supposed to prescribe the input Cp distribution which should correspond to the expected final airfoil. By that, the estimated shock wave position (for transonic cases), as well as the values of C_L and $C_{M\,1/4}$ are directly predefined, while the total drag coefficient can only be roughly estimated in advance. Also, the initial airfoil shape must be defined as an input, of which only a desired part of the nose section is retained on the final airfoil, while the rest of the contour is calculated according to the input values of Cp distribution, α, M, Re and some other program parameters which are of minor importance for the paper. For the given initial conditions, the displacement surface is obtained by potential flow calculation, while the final airfoil shape is determined by subtracting the local δ^* values from the displacement surface.

Unfortunately, the satisfactory final airfoil shape is very rarely obtained in a first design attempt in case of early generation programs, such as the original Trandes is. Very often, "γ" or "U" airfoil shapes, like those shown in Fig. 1., with the trailing edge closure problems, appear as the results. In those cases, it was originally suggested that the Cp input should be reconsidered, or that upper and lower surfaces could be translated in a way to close the trailing edge properly, and than blended to a new nose shape of some other existing airfoil. Two main problems arise by this. Like first, the desired relative thickness approximately determined by the initial airfoil is often very roughly altered, which is unacceptable especially in the case of transonic airfoils. Secondly, blending to a new nose shape can hardly be done in a way to satisfy the uniqueness of the first and second order derivatives on both sides of the blending points.

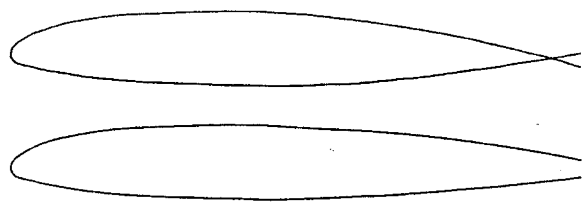


Figure 1.

In TRANPRO, an automatic modification of prescribed Cp distribution is done, until the trailing edge closure is achieved, without any relevant affecting of the desired airfoil relative thickness. This is the inverse mode of the airfoil design procedure. Also, if necessary, it is possible to define a desired value of a relevant aerodynamic parameter, which is achieved by "elastic bending" modification of the mean line in the prescribed domain of the airfoil. This is the direct mode of the design procedure, which is also done automatically. The whole design procedure is accomplished in a single program run.

The Inverse design mode

In case that the achieved trailing edge closure is not satisfactory after the first calculation step, the inverse design mode in TRANPRO starts the corrective procedure. By this procedure, initial - user-defined Cp input should be altered automatically in such a way that its general form is retained as much as possible. It can be assumed that alternations in Cp input should be enlarged proportionally as the trailing edge coordinates are approached, where closure problem exists. The sign of correction (+/-) should be chosen so that the process is initially convergent, while the magnitude of local corrections must be such that the necessary number of corrective calculation steps (all steps in this mode after the initial, or step number "1") is as small as possible.

All mentioned can be summarized by the general functional dependence:

$$\Delta Cp_i^{(n+1)} = f\left[\left(\frac{t}{c}\right)_{t.e.}^{(n)}, \left(\frac{x}{c}\right)_i, C, Cp_i^{(n)}\right] \quad (1)$$

where:

- $\Delta Cp_i^{(n+1)}$ - represents the alternation in Cp input at the "i"-th calculation point on the airfoil in (n+1) calculation step;
- $Cp_i^{(n)}$ - is the input value of pressure coefficient at the "i"-th point in the (n)-th calculation step;
- $(t/c)_{t.e.}^{(n)}$ - represents the relative trailing edge thickness (negative in case of "γ" airfoils) at the end of the (n)-th calculation step;
- $(x/c)_i$ - relative chordwise distance of the "i"-th calculation point ;
- C - factor of convergence in inverse design mode.

According to that, the following relations proved to be very reliable in practice:

$$\left[Cp_i^{(n+1)}\right]_{up} = \left[Cp_i^{(n)}\right]_{up} \cdot \left[1 + C \cdot \left(\frac{x}{c}\right)_i \cdot \left(\frac{t}{c}\right)_{t.e.}^{(n)}\right] \quad (2)$$

for the upper airfoil surface, and:

$$\left[Cp_i^{(n+1)}\right]_{low} = \left[Cp_i^{(n)}\right]_{low} \cdot \left[1 - C \cdot \left(\frac{x}{c}\right)_i \cdot \left(\frac{t}{c}\right)_{t.e.}^{(n)}\right] \quad (3)$$

for the lower airfoil surface.

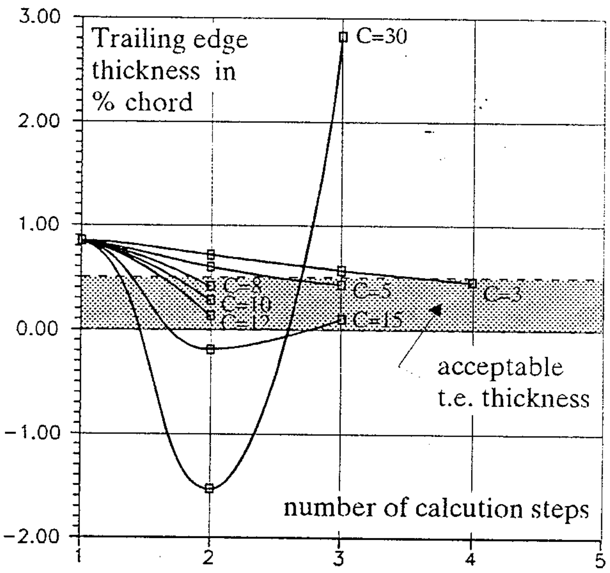


Figure 2.

With such formulation of equations (2) and (3), sign of the trailing edge thickness directly determines the correct sign of the whole correction process. Convergence rate is dictated by factor "C", whose optimum value for many practical purposes ranges in the domain C = 8 ÷ 12.

Figure 2. presents an example of the influence of this factor on the convergence rate for typical cases of airfoil design, encountered in practical applications of TRANPRO by the author. As it can be seen, good results can be obtained in much wider range of this parameter than recommended, but extremely low or high values can lead to too slow convergence or divergence of the process. In this example, the allowed trailing edge thickness was between 0% and 0.5% chord, but this algorithm can very easily satisfy more strict constraints.

Example in Fig. 3. shows the order of the difference between the initial and final Cp input (trailing edge properly closed), when initial input parameters are defined by a user of "average" experience.

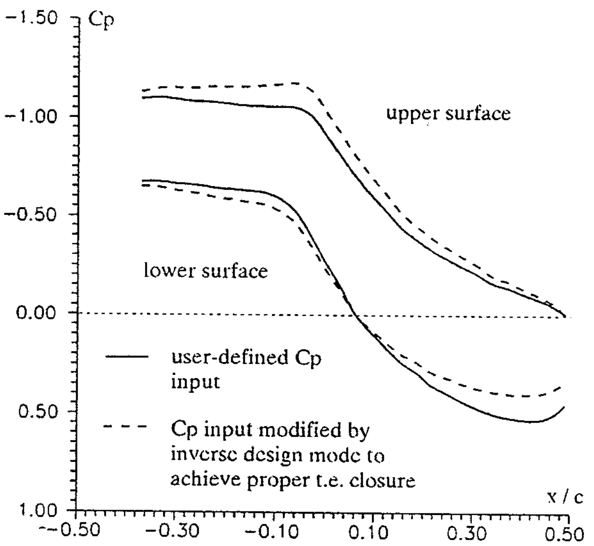


Figure 3.

The Direct design mode

Direct design mode is initiated automatically after the trailing edge solution has converged. Since the indirectly predefined values of C_L and $C_{M\,1/4}$ (by Cp distribution) have been changed in the first stage of airfoil design, the aim of this mode is to bring one of them back to the desired value.

Such a demand may very often appear in practice. For instance, in case of the design of airfoils for self-stable airplane wings, or helicopter rotor blades, the usual request is that $C_{M\,1/4} = 0$ (as close as possible), while the lift coefficient should posses some reasonably high positive value at the same time. In that case, some tolerance limit for $C_{M\,1/4}$ must also be defined as program input parameter, besides the prescribed value.

The general idea applied in this mode is to achieve the an aim by altering the rear portion of mean line of the airfoil obtained at the end of the inverse design stage. This part of mean line will be treated as "elastically deformable", while the rest of it will remain unchanged. The magnitude of "elastic deformation" of mean line will be in direct proportion with the discrepancy between the actual value of, let us suppose $C_{M\,1/4}$, and the desired value.

Such variation of mean line shape can be done in many ways. Here, four simple possibilities will be discussed, that have proven very efficient in operational airfoil design cases when quarter-chord moment coefficient was considered. They are based on the idea that the "deformable" part of mean line can be treated as a cantilever beam, on which four elementary load types may act: force or moment at its end, or triangular or uniform continual loads over the whole section that is to be modified. The four types of loading can determine the notation of their applications as "F", "M", "T" and "U", respectively.

Parameters which are used in this design mode are defined as follows:

- desired $C_{M\,1/4}$ value (input parameter)
- achieved $C_{M\,1/4}$ value in the (n)-th calculation step, denoted by $C_{M1/4}^{(n)}$
- allowable tolerance limits of $C_{M\,1/4}$ defined by $(\pm)\,\Delta C_{M1/4}$ (input parameter)
- length of the rear portion of mean line that will be modified defined by chordwise coordinate x_c from which modification starts (best results are obtained when last 20-40% are redefined, although there is a possibility to modify the whole airfoil mean line)
- relative vertical displacement of the mean line at the trailing edge $\Delta\bar{z}_{t.e.} = (\Delta z_{t.e.}/c)$
- empirical equation that relates desired $C_{M\,1/4}$ and $\Delta z_{t.e.}$ which enables the quickest convergence rate.

Such equation, of the form:

$$\Delta\bar{z}_{t.e.} = \frac{C_{M\,1/4} - C_{M\,1/4}^{(n)}}{K_{C_M}} \tag{4}$$

proved to be very reliable when convergence factor K_{C_M} is defined properly for each of four possible types of modification and a given relative $\bar{x}_c$.

For practical purposes, it is quite acceptable to neglect the rotation of airfoil upper and lower surface coordinates when shape of the mean line is varied. This means that Δz_i of the mean line is also Δz_i of airfoil surfaces for a given station x_i in the correction domain, while their Δx_i are assumed to be zero.

Keeping that in mind, and after some simple mathematics, the following equations both for airfoil upper and lower surface modifications were obtained:

(I) for "F" modifications:

$$\Delta \bar{z}_i = \frac{\Delta \bar{z}_{t.e.}}{2}\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)^2\left[3-\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)\right] \quad (5)$$

(II) for "M" modifications:

$$\Delta \bar{z}_i = \Delta \bar{z}_{t.e.}\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)^2 \quad (6)$$

(III) for "T" modifications: (7)

$$\Delta \bar{z}_i = \frac{\Delta \bar{z}_{t.e.}}{11}\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)^2\left[20-10\cdot\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)+\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)^3\right]$$

(IV) for "U" modifications: (8)

$$\Delta \bar{z}_i = \frac{\Delta \bar{z}_{t.e.}}{3}\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)^2\left\{-1+4\cdot\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)+\left[1-\left(\frac{\bar{x}_i-\bar{x}_c}{1-\bar{x}_c}\right)\right]^4\right\}$$

According to the authors experience, the value of the factor K_{C_M} must be defined strictly in order to obtain safe convergence of the solution in this design mode. Recommended values of this parameter, as a function of the modification type and the $\bar{x}_c$ coordinate, are given in Table1. Values for other stations should be obtained by proper interpolation. The convergence reliability evaluations are given in the last row.

Table 1.

Recommended values of K_{C_M}					
type	$\bar{x}_c$ 0.0	$\bar{x}_c$ 0.2	$\bar{x}_c$ 0.4	$\bar{x}_c$ 0.6	$\bar{x}_c$ 0.8
F	0.7	1.5	2.0	4.0	6.0
M	1.0	2.0	3.0	5.0	8.0
T	0.7	1.5	2.0	4.0	6.0
U	0.7	1.5	2.0	4.0	6.0
Cre	D	C	B	A	A

Cre - convergence reliability evaluations:

- A - highly reliable
- B - reliable
- C - reliable with reserve
- D - possible problems

In some applications, when $\bar{x}_c \leq 0.2$, the user might have to make a few test runs in order to determine other optimum values of K_{C_M} for particular airfoils.

Following the same logic, instead of the quarter-chord moment coefficient, desired value of the lift coefficient can be achieved very successfully. In that case, only recommended K_{C_L} will be different.

Finally, after all four modifications have been done for the selected values of $\bar{x}_c$, the airfoils should be tested at the off-design regimes, including desired ranges of α, M and Re. The final choice is than made by the user's decision, considering the values of $(C_L/C_D)_{max}$, $(C_L^3/C_D^2)_{max}$, C_{Dmin} etc.

Besides the very high efficiency of the presented inverse-direct algorithm in operational applications, it can also be used in education of students, engineers and scientists who are getting acquainted with this kind of problems. In the process of gaining experience in airfoil design, TRANPRO will suggest the user what is eventually not correct in the definition of the input parameters, and enable him to learn very quickly. In case of some older generation programs which required "manual" correction of Cp input, gaining experience was a long lasting, and, in many cases, very frustrating job.

Example of airfoil design

Suppose that the basic design parameters are defined as "design case" in Fig. 4., while the initial airfoil is defined by Fig. 5(a).

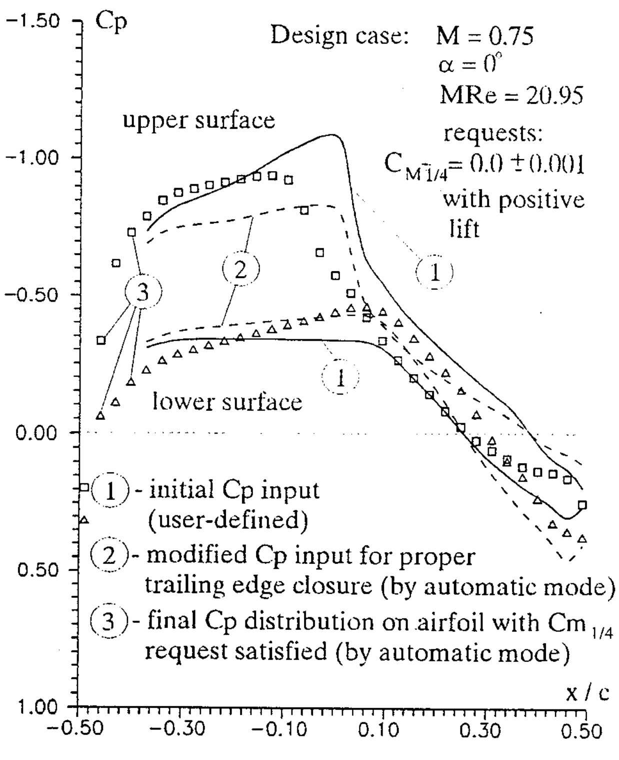


Figure 4.

We can also assume that the Cp input is given by curves (1) in Fig. 4. , which is obviously not quite correct for what is expected from the final airfoil (by this, a case of an "inexperienced" user is simulated thus giving the TRANPRO a rather tough "homework"). For the simplicity, we will only discuss the case of the "M" modification with $\overline{x}_c = 0.7$, and with C = 12 in the inverse design mode.

The trailing edge convergence with the prescribed tolerance $0.0\% \leq (t/c)_{t.e.} \leq 0.5\%$ was achieved in the third calculation step of the inverse design mode (Table 2.). Prescribed zero moment coefficient value, with the positive lift at the same time, was accomplished in the fourth step of the direct mode (Table 3.). All seven steps of the design procedure require, for instance on PC Pentium type computers, approximately 10 seconds of CPU time.

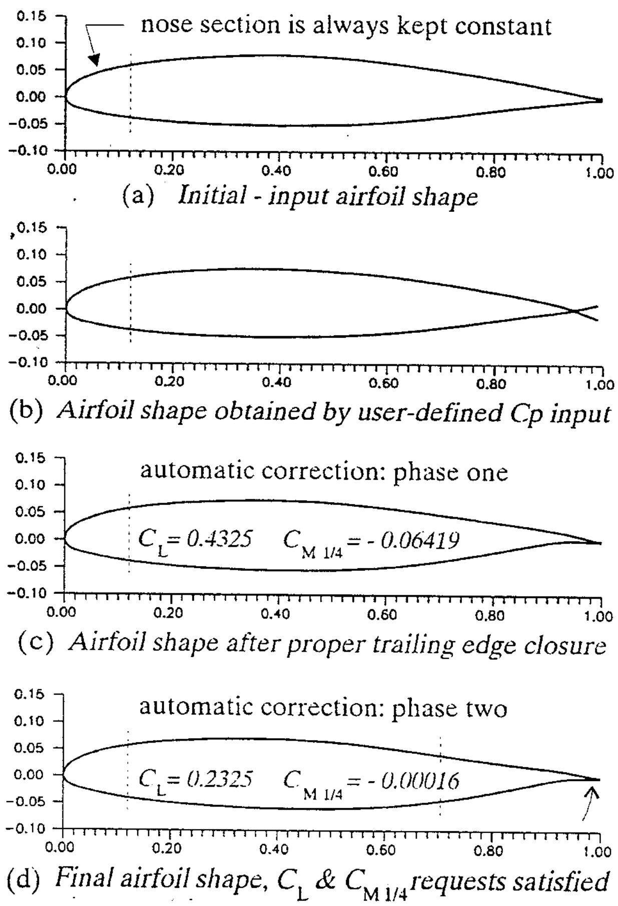


Figure 5.

As it can be seen in Fig. 4., Cp distribution on the final airfoil shows that the shock wave position is some 10% in front of the initially prescribed. This is the consequence of the fact that the initial Cp was most probably defined paying attention only to the maximizing of the lift characteristics and ignoring the moment coefficient requirement, which is unlikely to expect from a more careful user. Still, the final solution has been achieved in a reasonably fast convergence process of the global design procedure. Such a user can either accept the new airfoil, try with the other direct design phase options, or reconsider his Cp input in a very short time, after analyzing the solution(s) suggested by TRANPRO.

Table 2.

Inverse design mode	
Calculation step	$(t/c)_{t.e}$
1 (obtained by input Cp)	- 3.193% chord
2	- 1.004% chord
3	+ 0.106% chord

Table 3.

Direct design mode		
Calculation step	C_L	$C_{M\ 1/4}$
1 (inv. step "3" airfoil)	0.4325	- 0.06419
2	0.2525	- 0.00622
3	0.2361	- 0.00117
4	0.2325	- 0.00016
In all steps $(t/c)_{t.e} = +0.106\%$ chord = const.		

Presented inverse-direct algorithm is, unfortunately, unable to deal completely with a special kind of improprieties in input Cp defined by user. This is the problem of local waviness in prescribed Cp distribution, which is not in the function of airfoil design, but is rather a result of the users lack of experience. Such an example is also given in Fig. 4., where on Cp curve (1) for the upper surface close to the trailing edge, a distortion exists. Through the design process, it will be mapped to all successive airfoil shapes as a small but noticeable surface downward deflection near the trailing edge, even though the Cp distribution is modified several times as explained (Fig. 5.(b) - (d)).

This problem is only partially damped by a special mode for the airfoil surface smoothing[1], which in most cases can not eliminate this impropriety entirely. That mode is not "allowed" to make any substantial changes in airfoil shape, since otherwise it would interfere too much with the actual design process and possibly neutralize the effects of the other design features. On the other hand, the mentioned problem can easily be overcome by the assistance of some of the existing computer graphics packages, if they are applied for more precise input Cp predefining.

Airflow calculations

For the calculation of airflow parameters, both in the cases of airfoil design and optimization and for the airfoil analysis, in TRANPRO the zonal approach is applied. This approach is still very successfully used in many operational engineering applications. Its advantage over more complex methods lies mostly in extremely high time and computer resource efficiency, while the accuracy of the results can be brought to a very satisfactory level. The latter statement will hopefully be proven, from the aspect of this paper, in the following considerations.

Calculation of the inviscid part of the flow

The inviscid part of the flow is calculated over the displacement surface of the airfoil, i.e. the airfoil contour increased by the smoothed local distribution of δ^*. In TRANPRO this calculation is done by the same general algorithm as applied in Trandes. It is based on the solution of the full nondimensional perturbation potential $\bar{\phi}$ equation, which in physical x - z space for the unit airfoil length, takes the form:

$$\left(a^2-u^2\right)\bar{\phi}_{xx}+\left(a^2-w^2\right)\bar{\phi}_{zz}-2uw\bar{\phi}_{xz}=0 \qquad (9)$$

while, applied in calculation space ξ - η, changes to:

$$\left(a^2-u^2\right)f\left(\bar{\phi}_\xi\right)_\xi+\left(a^2-w^2\right)g\left(\bar{\phi}_\eta\right)_\eta-2uw\,fg\bar{\phi}_{\xi\eta}=0 \qquad (10)$$

where f and g are mapping coefficients. Specially, in the local supersonic domain, where Jameson's rotated finite difference s - n scheme is used, the governing equation takes the form:

$$(1-M^2)\bar{\phi}_{ss}+\bar{\phi}_{nn}=0 \qquad (11)$$

in which:

$$\bar{\phi}_{ss}=\frac{1}{V^2}\left[u^2f(f\bar{\phi}_\xi)_\xi+2uw\,fg\bar{\phi}_{\xi\eta}+w^2g(g\bar{\phi}_\eta)_\eta\right] \qquad (12)$$

$$\bar{\phi}_{nn}=\frac{1}{V^2}\left[w^2f(f\bar{\phi}_\xi)_\xi-2uw\,fg\bar{\phi}_{\xi\eta}+u^2g(g\bar{\phi}_\eta)_\eta\right] \qquad (13)$$

Very quick convergence of the solution is obtained by calculating the flow on a series of rectangular grids, starting with 13 x 7, then 25 x 13, 49 x 25 and 97 x 49. Very often the final solution is obtained on 49 x 25 grid, so the finest grid need not be applied, which reduces the computation time remarkably.

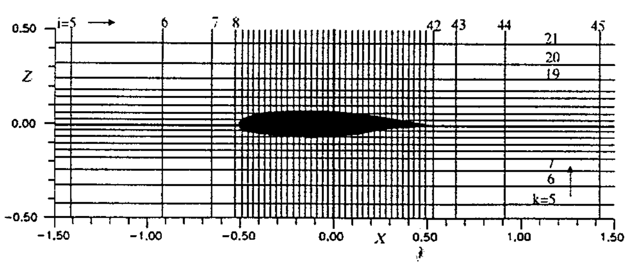


Figure 6. *Grid 49 x 25 in the vicinity of the airfoil*

Boundary layer and profile drag calculations

In this paper only turbulent boundary layer (TBL) case with transition point fixed close to the leading edge will be discussed. In Trandes, the Carlson/Nash - Macdonald integral TBL calculation is used, while in TRANPRO, a modified(3,4) version of this model is applied. The momentum integral equation:

$$\left(\frac{d\theta}{dx}\right)^{[n]}=-\frac{(\theta^+)^{[n-1]}}{u_e^+}\frac{du_e}{dx}\left(H+2-M_e^2\right)+\frac{1}{\left(\zeta^{[n]}\right)^2} \qquad (14)$$

is solved for the momentum thickness θ. In (14), "e" denotes the values on the outer edge of the TBL, while $[n]$ denotes a certain iteration cycle value. Parameter ζ is defined by:

$$\zeta^{[n]}=F_C\left[2.4711\cdot\ln\left(F_R\,R_\theta^{[n-1]}\right)+4.75\right]+1.5G^{[n-1]}+ \\ +\frac{1724}{(G^{[n-1]})^2+200}-16.87 \qquad (15)$$

in which: $F_C=1+0.066\left(M_e\right)^2-0.008\left(M_e\right)^3$ (16)

$$F_R=1-0.134\left(M_e\right)^2+0.027\left(M_e\right)^3 \qquad (17)$$

where G is the Clauser parameter. Shape factor $H=\delta^*/\theta$ is calculated by:

$$\bar{H}^{[n]}=\frac{1}{1-G^{[n-1]}\left(1/\zeta\right)} \qquad (18)$$

and $H^{[n]}=\left(\bar{H}^{[n]}+1\right)\left[1+0.178\left(M_e^+\right)^2\right]-1$ (19)

In modified model(3,4), the Clauser parameters G and β_p are related by :

$$G^{[n]}=6.1\sqrt{\beta_p^{[n]}+1.81}-4.1 \qquad (20)$$

while in the original Carlson/Nash-Macdonald model, the usual equation of Nash(1,10) is applied. Once θ is determined, TBL displacement thickness is calculated by $\delta^*=H\cdot\theta$. Finally, the distribution of δ^* is smoothed(1,2) over the airfoil, and so the airfoil displacement surface is obtained.

For profile drag calculations, the Squire-Young formula with the separate trailing edge values for upper and lower surface is used:

$$C_{DP}=2\cdot\left[\theta_{(te)U}\left(\frac{u_{e(te)}}{u_\infty}\right)_U^{\frac{\bar{H}_{(te)U}+5}{2}}+\theta_{(te)L}\left(\frac{u_{e(te)}}{u_\infty}\right)_L^{\frac{\bar{H}_{(te)L}+5}{2}}\right] \qquad (21)$$

Since the TBL on airfoils is of the non-equilibrium type, while the equation of Nash was derived for the equilibrium TBL, boundary layer values and profile drag calculated by Carlson/Nash-Macdonald model are being underestimated by this method.

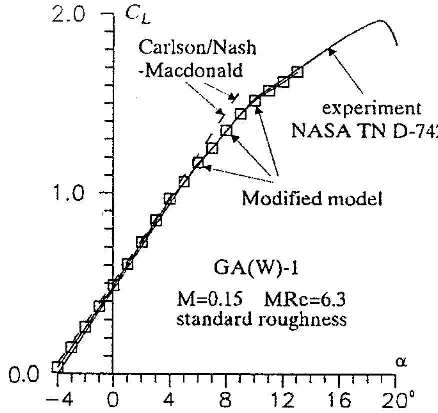


Figure 7.

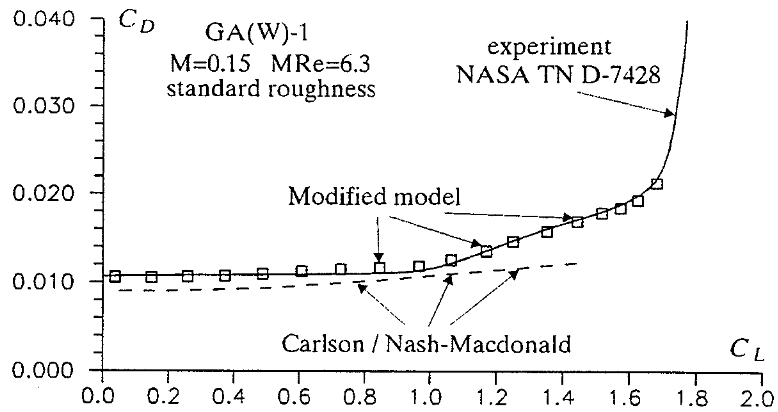


Figure 8.

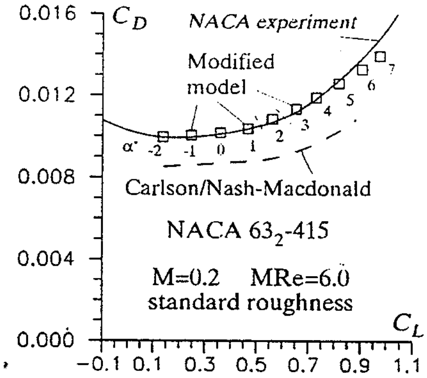


Figure 9.

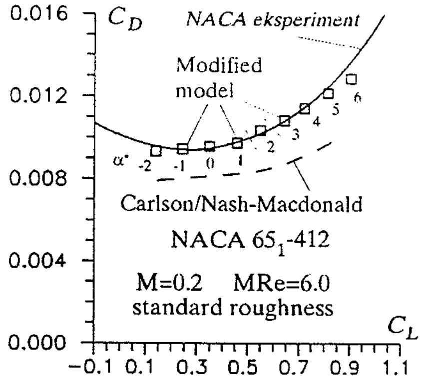


Figure 10.

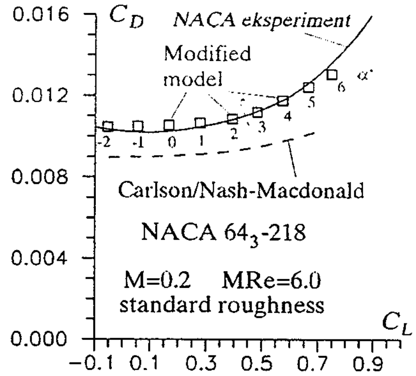
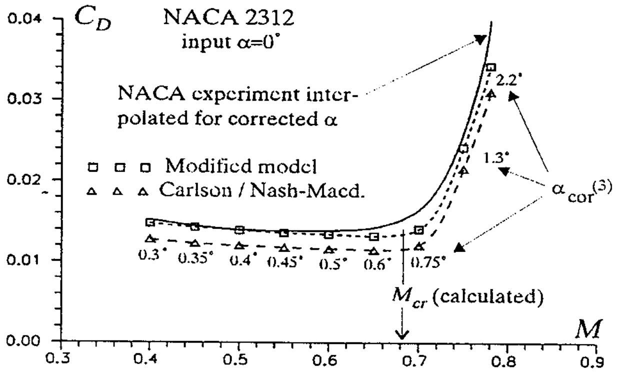
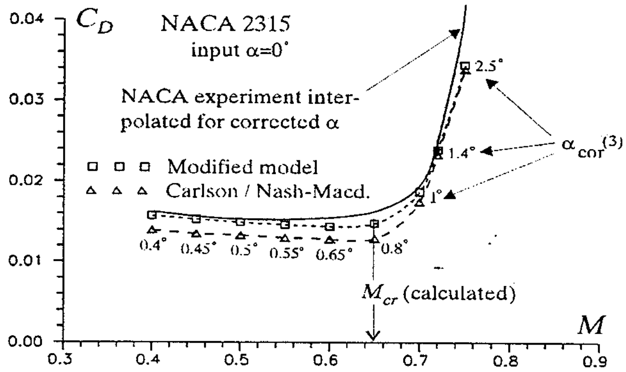


Figure 11.





$\alpha_{cor}^{(3)}$ - in some cases angle of attack correction is necessary for calculations on rectangular grids

Figure 12.

Figure 13.

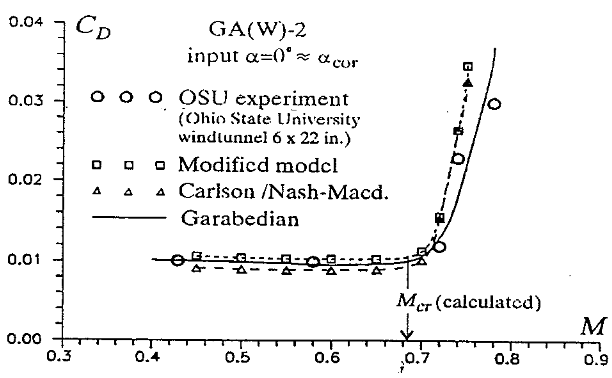


Figure 14.

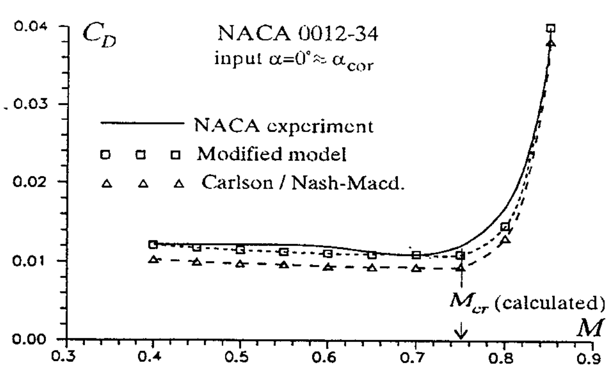


Figure 15.

The solution to this problem has been found in introducing the equation (20) within the modified TBL calculation model applied in TRANPRO, instead of the equation of Nash, used in the original model. Although purely empirical, this equation has been proven[3,4] very successful in use. For angles of attack which correspond to the close-to-minimum profile drag domain at smaller speeds, the mean relative error was reduced, in a test sample including 25 arbitrarily chosen airfoils, from 13.9% in case of Carlson/Nash-Macdonald model, to only 1.5% when modified model was applied (see Table 4.), compared with the appropriate NACA/NASA experimental data (in which transition was also fixed in the range of 6-8% from the leading edge).

For small speed cases, this can also be observed in Figures 7 - 11. (around-minimum drag domain), while for higher subsonic speeds, at which improvement is also noticeable, examples are given in Figures 12 -15. Although some "boundary layer perfectionists" may argue about the use of equation (20), the concrete results prove that it's TBL non-equilibrium effects averaging can give even better final results than the application of some much more compound methods.

In TRANPRO, the problem of TBL weak separation at smaller angles of attack is automatically accounted for[3,4]. In case of moderate separation at higher angles of attack (also see Figures 7.-11.), a special algorithm has been introduced. It is based on the inverse-approach treatment of the separated region:

(A) The restricted TBL equations set[3,4], including equation (20), will also be applied in the separated region.

(B) For the considered airfoil (or certain categories and/or subcategories of airfoils), the angle of attack α_i, from which moderate separation correction is to be included ("i"), should be defined (see examples in Table 5.).

(C) The relative position of separation point $Xsep_i$ corresponding the α_i should be defined (the case of "new" airfoils is described in [3]).

(D) Relation between Xsep and $\alpha > \alpha_i$ should be established. For instance, in case of all airfoils in Table 5. (except the NACA 230xx category) it is given by:

$$Xsep(\alpha) = Xsep_i - 0.03 \cdot (\alpha - \alpha_i) \qquad (22)$$

(E) In the separated region, the initial δ^* distribution is (indirectly) initially prescribed by assigning the Cp distribution, linearly varying from Cp_{sep} at the separation point (obtained by direct TBL calculation up to this point), to Cp_{base} at the trailing edge, which is determined by:

$$Cp_{base} = CCp(Xsep,\alpha) \cdot Cpl_{max} \qquad (23)$$

in which Cpl_{max} represents the maximum Cp value close to the trailing edge at the lower airfoil surface, and $CCp(Xsep,\alpha)$ is the base pressure coefficient correction-factor. If the initial δ^* distribution is assumed correctly by these factors, final solution will correspond to the actual values.

(F) $CCp(Xsep,\alpha)$ should be defined either as a function of a or Xsep in the following manner:

$$CCp(Xsep) = CCp(Xsep_i) + \Delta CCp(Xsep)(Xsep - Xsep_i) \qquad (24)$$

or $$CCp(\alpha) = CCp(\alpha_i) + \Delta CCp(\alpha)(\alpha - \alpha_i) \qquad (25)$$

where $\Delta CCp(Xsep)$ or $\Delta CCp(\alpha)$ represent the base pressure coefficient correction-factor gradient, and $CCp(Xsep_i,\alpha_i)$ is the base pressure coefficient correction-factor at α_i.

(G) In case of some airfoils, transition from attached flow (or weak separation) to modified separation calculation use can not be done through α_i that belongs to introduced separation correction algorithm. In those cases, the non-standard values of α_i^*, $Xsep_i^*$, and CCp^* may be determined (see examples in Table 5., although that should not necessarily be done in practical applications, since it covers very narrow range of angles if attack).

Results obtained for the moderate separation applied in the sample case of the group of airfoils listed in Table 5. correspond very well with the appropriate experimental data. That can also be observed in figures 8 - 11 and in [3,4]. In this case, the logic of the model has been proven on airfoils for which experimental data already exist. When applied to the newly designed airfoils, the same logic should be used, while the characteristic values should be initially assumed (the knowledge of their values for some other airfoils of the same category may be useful). Since the final solution is unique (within very narrow tolerance limits), smooth lift and drag curves in the domain of moderate separation will indicate that characteristic parameters of the method have been chosen properly.

Above presented moderate separation correction can cover the range of angles of attack which correspond the values up to the lift coefficients of the order (0.7-0.9) C_{Lmax}. Very high angles of attack must be treated by massive separation correction, which is under development for the particular use in TRANPRO.

For the comparison, in initial Trandes version, moderate separation correction was only roughly suggested and defined, and it was almost practically unusable. In later versions of that program, the massive separation model was included, but, according to the knowledge of the author of this paper, rather strange values of TBL parameters in the separated region are obtained by its use.

Table 4.

Applied model ->	Carlson / Nash-Macdonald				Modified model			
Airfoil	C_L	C_D	$C_{D\,(exp)}$	ΔC_D	C_L	C_D	$C_{D\,(exp)}$	ΔC_D
NACA 2412	.1270	.00825	.0098	15.8%	.1220	.00967	.0098	1.3%
NACA 2415	.1273	.00901	.0103	12.5%	.1200	.01054	.0103	2.3%
NACA 4412	.3618	.00856	.0099	13.5%	.3530	.01002	.0099	1.2%
NACA 4415	.3796	.00939	.0107	12.2%	.3746	.01095	.0107	2.3%
NACA 4418	.3872	.01034	.0115	10.1%	.3819	.01179	.0115	2.5%
NACA 63_2 - 215	.0734	.00841	.0098	14.2%	.0719	.00986	.0098	0.6%
NACA 63_3- 218	.0630	.00907	.0102	11.2%	.0602	.01057	.0102	3.6%
NACA 64_2 - 215	.0720	.00837	.0097	13.7%	.0699	.00981	.0097	1.1%
NACA 64_3 - 218	.0578	.00899	.0102	11.9%	.0533	.01048	.0102	2.7%
NACA 65_2 - 215	.0652	.00838	.0101	17.0%	.0588	.00981	.0101	2.9%
NACA 65_3 - 218	.0602	.00901	.0103	12.5%	.0566	.01049	.0103	1.8%
NACA 63_1 - 412	.2604	.00804	.0097	17.1%	.2578	.00944	.0097	2.7%
NACA 63_2- 415	.2524	.00860	.0099	13.1%	.2461	.01005	.0099	1.5%
NACA 63_3 - 418	.1276	.00913	.0108	15.5%	.1228	.01064	.0108	1.5%
NACA 64_1 - 412	.3667	.00812	.0097	16.3%	.3624	.00953	.0097	1.7%
NACA 64_2 - 415	.2462	.00856	.0100	14.4%	.2404	.01001	.0100	0.1%
NACA 64_3 - 418	.2415	.00912	.0106	14.0%	.2341	.01057	.0106	0.3%
NACA 65_1 - 412	.2509	.00802	.0093	13.8%	.2437	.00942	.0093	1.3%
NACA 65_2 - 415	.2432	.00862	.0101	14.6%	.2369	.01001	.0101	0.9%
NACA 65_3 - 418	.2478	.00925	.0107	13.6%	.2430	.01064	.0107	0.6%
NACA 63_2 - 615	.3154	.00878	.0102	13.9%	.3077	.01021	.0102	0.1%
NACA 63_3 - 618	.3146	.00937	.0108	13.2%	.3051	.01097	.0108	1.6%
NACA 65_3 - 618	.3279	.00960	.0110	12.7%	.3097	.01102	.0110	0.2%
NACA 23012	.1319	.00819	.0099	17.3%	.1287	.00959	.0099	3.1%
GA(W)-1	.4997	.00945	.0109	13.3%	.4882	.01097	.0109	0.6%
Mean relative error by model	Carlson / Nash-Macdonald			**13.9%**	Modified model			**1.5%**

Table 5.

Airfoil designation	α_i* deg..	$Xsep_i$* ≈	CCp *	α_i deg.	$Xsep_i$	CCp(Xsep,α)	Xsep(α) ≈
1	2	3	4	5	6	7	8
NACA 2412	/	/	/	2	0.90	1.45 + 0.25 (α -2)	0.90 - 0.03 (α - 2)
NACA 2415	/	/	/	2	0.90	1.10 + 0.25 (α - 2)	0.90 - 0.03 (α - 2)
NACA 4412	/	/	/	2	0.90	0.90 + 0.15 (α - 2)	0.90 - 0.03 (α - 2)
NACA 4415	/	/	/	2	0.90	0.95 + 0.15 (α - 2)	0.90 - 0.03 (α - 2)
NACA 4418	/	/	/	2	0.90	1.00 + 0.15 (α - 2)	0.90 - 0.03 (α - 2)
NACA 632 - 215	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 633- 218	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 642 - 215	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 643 - 218	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 652 - 215	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 653 - 218	/	/	/	3	0.90	1.10 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 631 - 412	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 632- 415	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 633 - 418	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 641 - 412	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 642 - 415	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 643 - 418	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 651 - 412	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 652 - 415	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 653 - 418	2	0.90	0.85	3	0.90	1.10 + 0.10 (α - 3)	0.90 - 0.03 (α - 3)
NACA 632 - 615	/	/	/	3	0.90	0.65 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 633 - 618	/	/	/	3	0.90	0.75 + 0.15 (α - 3)	0.90 - 0.03 (α - 3)
NACA 653 - 618	/	/	/	1	0.90	0.60 + 0.15 (α - 1)	0.90 - 0.03 (α - 1)
GA(W)-1	/	/	/	5	0.93	0.555 + 0.075 (α - 5)	0.93 - 0.03 (α - 5)
NACA 23012	/	/	/	3	0.96	1.4 + 0.30 (α - 3)	0.96 = const.

Wave drag calculation

In case of supercritical free-stream (M_∞) Mach numbers, total drag coefficient is obtained as:

$$C_D = C_{DP} + C_{DW} \qquad (25)$$

where for C_{DP} calculations equation (21) in combination with modified TBL model is used. This can be done because the Mach number considered in this paper is confined to smaller transonic values, at which weak shock wave will not induce massive separation (here the considered angles of attack will be close or equal to $\alpha = 0°$).

The C_{DW} is obtained as a difference of form (pressure) drag coefficients C_{Df} at supercritical and subcritical regime for the same C_L. In order to overcome the inherent problems of C_{Df} calculations on uniformly spaced rectangular grids which may lead to incorrect C_{DW} (also one of the possible problems of original Trandes), in TRANPRO the following approach is applied:

(A) For several subcritical M_∞ values and constant α, by integrating the calculated Cp distributions with respect to *dx* and *dz*, normal and tangential coefficients $C_{N(-)}$ and $C_{T(-)}$ should be obtained (here "(-)" and "(+)" subscripts will denote subcritical and supercritical conditions respectively). Then for each of those Mach numbers coefficient C_A is calculated as:

$$C_{A(-)} = C_{T(-)} + C_{N(-)} \tan\alpha \qquad (26)$$

(B) For a desired supercritical M_∞, keeping the same α as in (A), values $C_{N(+)}$, $C_{T(+)}$ and $C_{A(+)}$ are calculated in the same manner as in subcritical cases.

(C) Although C_L initially increases as transonic domain is entered, hypothetical $C_{A(-)}$ behaves in the same manner, so their mutual linear dependence established in subcritical domain can also be assumed at M_∞ from (B). For the C_L obtained in (B), and from the known function C_L->$C_{A(-)}$ obtained in (A) for a given α, by linear extrapolation a fictive "subcritical" value of $C_{A(-)fic}$ is calculated. That value corresponds to the C_L of the supercritical calculation, assuming a hypothetical case that no shock wave exists.

(D) A difference denoted as C_{A,C_L} is then calculated:

$$C_{A,C_L} = C_{A(-)} - C_{A(-)fic} \qquad (27)$$

(E) The wave drag coefficient is obtained as:

$$C_{DW} = C_{N(+)} \sin\alpha + C_{A,C_L} \cos\alpha \qquad (28)$$

This algorithm is completely free from any problems associated with C_{Df} calculations on rectangular grids, that exist in original Trandes algorithm[1]. This is the reason that it has been applied for the calculation of wave drag in supercritical domain of both "modified" and "Carlson-Nash-Macdonald" model curves in Figures 12. - 15. A sample case of total drag C_D calculation is given in Table 6.

Table 6.

NACA 2312 ; calculation for α = 0° ; Modified TBL model; α – correction applied[3]							
M	C_L	C_{DP}	$C_{A(-)}$	$C_{A(+)}$	$C_{A(-)fic}$	C_{DW}	C_0
0.40	0.2260	0.0148	0.00129	/	/	/	0.0148
0.45	0.2348	0.0143	0.00140	/	/	/	0.0143
0.50	0.2428	0.0139	0.00167	/	/	/	0.0139
0.55	0.2529	0.0136	0.00200	/	/	/	0.0136
0.60	0.2664	0.0134	0.00244	/	/	/	0.0134
0.65	0.2853	0.0132	0.00295	/	/	/	0.0132
0.70	0.3037	0.0133	/	0.00436	0.00353	0.0008	0.0141
0.75	0.3250	0.0158	/	0.01253	0.00415	0.0084	0.0242
0.78	0.3494	0.0162	/	0.02298	0.00487	0.0181	0.0343
Extrapolation function: $C_{A(-)} = 0.0294\ C_L - 0.0054$							

References

(1) L.A.Carlson: *"Trandes: A Fortran Program for Transonic Airfoil Analysis or Design"*, NASA CR- 2821, June 1977.

(2) I. Kostić: "*TRANPRO - Airfoil Design, Optimization and Analysis Program*" (1997 upgrade), Faculty of Mechanical Engineering, Belgrade 1997.

(3) I. Kostić: *"Adaptive Approach in Integral TBL Modeling for Subsonic and Transonic Flow Around Airfoils"*, Ph.D. Thesis, Belgrade 1997.

(4) I. Kostić, : *"Application of an Improved Integral Turbulent Boundary Layer Model With Moderate Separation Correction on NACA Series and Low Speed Supercritical Airfoil Derivatives"*, ICAS-96-7.3.1, 20th ICAS Congress, Sorrento, Italy, 1996.

(5) A. D. Young: *"Boundary Layers"*, BSP Prof. books, 1989.

(6) T. Dragović: *"Aerodynamics of Aircraft Design"* Belgrade, 1997.

(7) R.J.McGhee, W.D.Beasley: *"Low Speed Aerodynamic Characteristics of a 17-Percent Thick Airfoil Section Designed for General Aviation Applications"*, NASA TN D-7428

(8) F.M. White: *"Viscous Fluid Flow"*, McGraw-Hill, 1974.

(9) I.H.Abbott, A.E.Doenhoff: *"Theory of Wing Sections & Summary of Airfoil Data"* , Dover Publications, NY. 1959.

(10) J.F.Nash, A.G.J.Macdonald: *"The Calculation of Momentum Thickness in a Turbulent Boundary Layer at Mach Numbers up to Unity"* , Aero. Res. Council C.P. No.963, 1967.