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EFFECT OF LOAD FACTORS ON TURN MANEUVER OF AGRICULTURAL AIRCRAFT

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Abstract

In this paper a theoretical study of the turn maneuver of an agricultural aircraft is presented. The maneuver with changeably altitude is analyzed. The effect of load factors on the turn maneuver characteristics during the field-treating flights are analyzed. The mathematical model which describes the procedure of the correct climb and descent turn maneuver are used. For a typical agricultural aircraft numerical results and limitations of the climb, horizontal and descent turn maneuver are given. The problem of climb turning flight is described with the system of differential equations which describe the influence of normal and tangential load factors on derivation velocity, the path angle in the vertical plane and the rate of turn in function of the bank angle during turning flight. The system of differential equations of motion are solved on a personal computer with Runge-Kutta-Merson numerical method. Some results of this calculation in this paper are presented.

Symbols

a_c - acceleration vector of airplane mass center,
C_L - lift coefficient,
C_{Lmax} - maximum lift coefficient,
C_{Ls} - safety lift coefficient,
C_D - drag coefficient,
D - drag,
g - acceleration due to gravity,
L - lift,
n - normal load factor (coefficient),
n_x - tangential load factor (coefficient),
m - mass of airplane,
S - wing area,
T - thrust,
t - time of turn maneuver,
V - velocity,
V_s - safety speed,
W - weight of airplane,
x,y,z - coordinates of airplane mass center,
α - angle of attack,
α_s - wing-setting angle (angle between wing chord and axes of fuselage),
γ - path angle in the vertical plane (angle of flight path),
η - factor $\eta = \Delta h(2g/V_1^2)$,
ρ - density of air,
τ - factor $\tau = t(g/V_1)$,
ψ - path angle in the horizontal plane (angle of yaw),
φ - bank angle,
$\Omega = d\psi/dt$ - rate of turn

Introduction

Aerial treating has become an integral part of modern agriculture. However, agricultural flying has been plagued by a great number of accidents. The maneuvers of an agricultural aircraft are divided into those carried out while entering or while leaving the spraying line. Despite low height of the airplane while spraying, this part of the flight is considered to be the safest because of an appreciable flight velocity. Upon completing a spraying run the aircraft enters a turn maneuver procedure. Pilots assert that the procedure is the most dangerous maneuver and most of the accidents occur while this maneuver is being performed. There are many secondary factors that influence the safety of agricultural flying; some of them will be treated in this paper.

In accordance with the above mentioned considerations, two segments of the procedure turn are considered to be critical: climb while turning, and rolling. In these phases of the maneuver the aircraft may encounter stalling. The aim of this paper is to determine those factors that provoke the appearance of stall and to estimate their magnitude.

The analysis has been restricted to the climb at the first phase and the descent at the end of turn maneuver. A special types of the turn maneuver has also been analyzed, i.e. the case in which the x-direction forces are balanced. This case involves a balance between thrust and drag ($T = D$). Analytical solutions for the climb turn maneuver at a constant normal load coefficient have been obtained and analyzed.

The usual treat (spray) run is carried out along the larger dimension of the field. At the end of a treating run a turn procedure is made, and the pilot enters the following treating line (Figure 1).

The turn maneuver procedure is divided into five parts: (1) increase of power (the tangential load factor $n_x > 0$), (2) climb, roll and turn (up to $\psi = 60°$), (3) continuous turn maneuver with changeable values of normal and tangential load factors (up to $\psi = 95°$), (4) 180 degrees turn (with descent in the last phase of turn), and (5) decent to the next spraying line. Of course, the turn maneuver procedure can be implemented in a different way as well, with a different combination of parameter variation during its performation.

The following analysis discusses the reduction of velocity along part (2) of the turn procedure. The velocity at the end of part (2) is important as it determines the conventional stall margin as well as the available margin before aileron-stalling occurs while rolling.

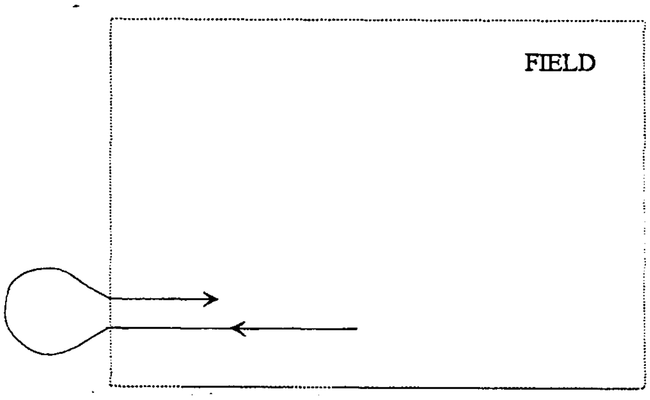

Figure 1: - Turn procedure

Mathematical Model

Using the stability-axes system for the nonsteady turning flight the following equations of motion may be written[1,2]:

$$m\frac{dV}{dt} = T\cos(\alpha - \alpha_s) - D - mg\sin\gamma$$

$$mV\frac{d\gamma}{dt} = [T\sin(\alpha - \alpha_s) + L]\cos\varphi - mg\cos\gamma \quad (1)$$

$$mV\frac{d\psi}{dt}\cos\gamma = [T\sin(\alpha - \alpha_s) + L]\sin\varphi$$

where are: γ path angle in the vertical plane, ψ path angle in the horizontal plane and φ bank angle.

By introducing tangential and normal load factors:

$$n_x = \frac{1}{mg}[T\cos(\alpha - \alpha_s) - D] \quad (2)$$

$$n = \frac{1}{mg}[T\sin(\alpha - \alpha_s) + L] \quad (3)$$

the system of equations of motion (1) are transformed into:

$$\frac{1}{g}\frac{dV}{dt} = n_x - \sin\gamma$$

$$\frac{V}{g}\frac{d\gamma}{dt} = n\cos\varphi - \cos\gamma \quad (4)$$

$$\frac{V}{g}\frac{d\psi}{dt}\cos\gamma = n\sin\varphi$$

The system of differential equations (4) describes the influence of normal and tangential load factors on derivation velocity V, the path angle in the vertical plane γ and the rate of turn $d\psi/dt$ as a function of the bank angle φ during turning flight.

Generally, the normal and tangential load factors are changed during turning flight. However, these factors are assumed to be constant for the approximate calculation during some parts of turn path (the first part of path is $0° < \psi < 60°$, the second part of path is $60° < \psi < 95°$ and the third part of path is $95° < \psi < 180°$, see Figure 2). The results of numerical calculation are not affected significantly by this assumption[3-7].

Substituting relation $dh/dt = V\sin\gamma$ into the system of differential equations of motion (4) we eliminate time from the system and obtain the following system of differential equations:

$$\frac{dV}{d\psi} = \frac{V\cos\gamma}{n\sin\varphi}(n_x - \sin\gamma)$$

$$\frac{d\gamma}{d\psi} = \frac{\cos\gamma}{n\sin\varphi}(n\cos\varphi - \cos\gamma) \quad (5)$$

$$\frac{dh}{d\psi} = \frac{V^2\sin\gamma\cos\gamma}{ng\sin\varphi}$$

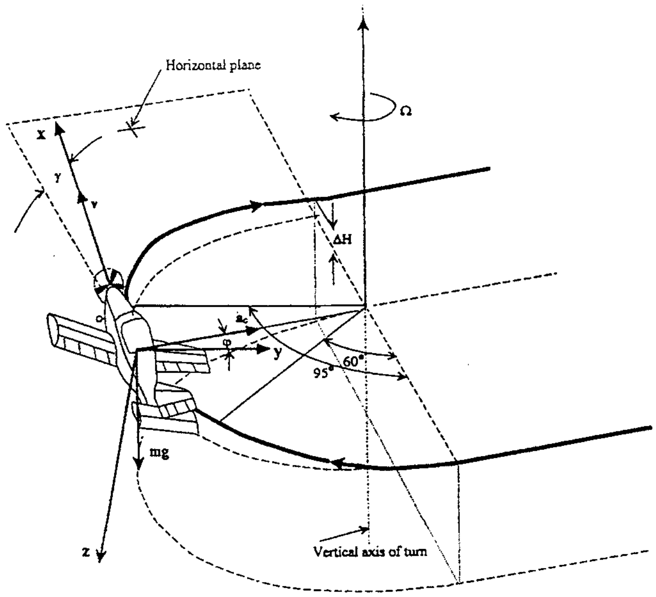

Figure 2: - The climb turn of agricultural aircraft

as a function of path angle in the horizontal plane (ψ).

A fourth equation, that results directly from the last equation of the system (4) can also be added to the system of equations. The equation yields the time to turn, t:

$$\frac{dt}{d\psi} = \frac{V\,cos\gamma}{ng\,sin\varphi} \tag{5a}$$

The system of differential equations of motion (5) are solved with numerical methods with normal and tangential load factors and bank angle as parameters. The first equation determines derivative of velocity, the second one determines derivative of path angle in the vertical plane, and the third one gives derivative of flight altitude as a function of path angle in the horizontal plane.

Using the usual assumptions: that $cos(\alpha - \alpha_s) \approx 1$, density of air $\rho \approx constant$ (small altitude change during turning) and $mg \approx constant$ (small change of airplane mass resulting from fuel consumption during turning), the tangential load factor can be expressed in form:

$$n_x = \frac{T\,cos(\alpha - \alpha_s) - D}{mg} \cong \frac{T}{mg} - \frac{D}{mg} \tag{6}$$

and, for small values of path angle in the vertical plane ($L \cong mg$), we obtain:

$$n_x \cong \frac{T}{mg} - \frac{C_D}{C_L}, \tag{7}$$

Thus, tangential load factor expresses the influence of airplane thrust (power plant) and polar on characteristics of turn maneuver.

A very interesting case of turning is for $n_x = 0$, i.e. when the thrust T is equal or almost equal to the aerodynamic drag D. The special feature of the case is that analytic solution can be obtained.

The system of equations of motion (4) in this case is transformed into:

$$\begin{aligned} \frac{1}{g}\frac{dV}{dt} &= -sin\gamma \\ \frac{V}{g}\frac{d\gamma}{dt} &= n\,cos\varphi - cos\gamma \\ \frac{V}{g}\frac{d\psi}{dt}cos\gamma &= n\,sin\varphi \end{aligned} \tag{8}$$

Dividing the first equation by the second one, one obtains a differential equation with separated variables:

$$\frac{dV}{V} = -\frac{sin\gamma}{n\,cos\varphi - cos\gamma}d\gamma \tag{9}$$

Integrating the equation from initial condition, i.e. initial values for $\gamma_1 = 0$ and V_1 to some arbitrary point γ and V the solution is obtained:

$$\frac{V}{V_1} = \frac{n\,cos\varphi - 1}{n\,cos\varphi - cos\gamma} \tag{10}$$

This equation describes speed variation during turn maneuver for given values of normal load factor and bank angle φ. Since the altitude as well as the angle of flight path γ increases during turn, it is evident that the airplane's speed continuously decreases at the same time. Hence, lift coefficient increases.

The highest altitude increase would have been obtained when the airplane speed were lowest, i.e. when the lift coefficient would have been increased to its maximal value. However, as the flight at $C_{L_{max}}$ is unsufficiently safe, the lift coefficient is limited, as it is the case with many other performances, to a safe value $C_{L_s} = (0.7 - 0.9)\,C_{L_{max}}$.

The flight duration up to a point on the turning path, determined by the angle γ, can be obtained from the second equation of the system (8) and by substituting the solution (10):

$$t = \frac{1}{g}\int_{\gamma_1=0}^{\gamma} \frac{V\,d\gamma}{n\,cos\varphi - cos\gamma}$$

$$= \frac{V_1}{g}(n\,cos\varphi - 1)\int_{\gamma_1=0}^{\gamma} \frac{d\gamma}{(n\,cos\varphi - cos\gamma)^2} \qquad (11)$$

Hence,

$$t = \frac{(V_1/g)}{n\,cos\varphi + 1}\left[\frac{sin\gamma}{n\,cos\varphi - cos\gamma} + \frac{2n\,cos\varphi}{\sqrt{n^2\,cos^2\varphi - 1}}arctg\left(\sqrt{\frac{n\,cos\varphi + 1}{n\,cos\varphi - 1}}tg\frac{\gamma}{2}\right)\right] \qquad (12)$$

For further analytic determination of the turn characteristics it is necessary to establish a relation between angle of yaw and angle of flight path. If we divide the third equation of the system (8) by the second one, we obtain:

$$cos\gamma\frac{d\psi}{d\gamma} = \frac{n\,sin\varphi}{n\,cos\varphi - cos\gamma} \qquad (13)$$

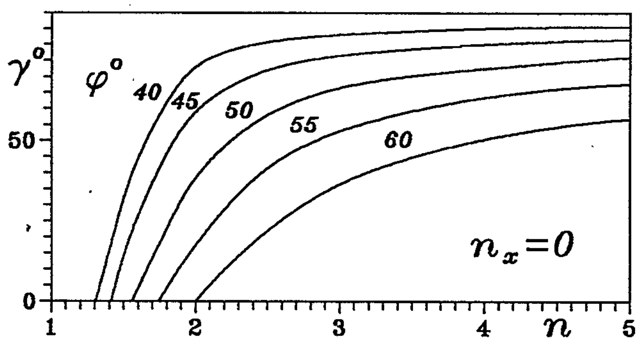

Figure 3: - Angle of flight path at the end of turn ($\psi = \pi$) as a function of normal load coefficient

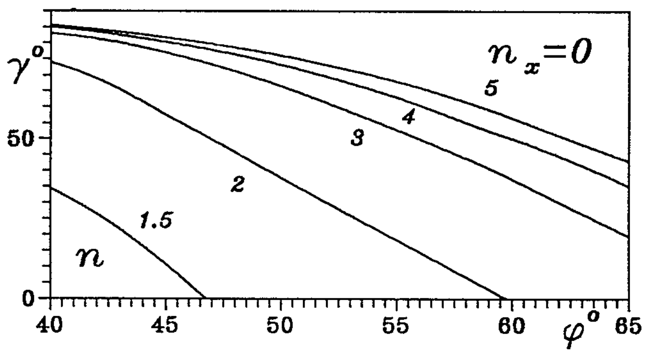

Figure 4: - Angle of flight path at the end of turn ($\psi = \pi$) as a function of the bank angle φ

The integral of the equation between limits $\gamma_1 = 0$ and γ is:

$$\psi = n\,sin\varphi\int_{\gamma_1=0}^{\gamma} \frac{d\gamma}{(n\,cos\varphi - cos\gamma)cos\gamma} \qquad (14)$$

which yields:

$$\psi = tg\varphi\left\{ln\left[tg\left(\frac{\gamma}{2} + \frac{\pi}{4}\right)\right] + \frac{2}{\sqrt{n^2cos^2\varphi - 1}}arctg\left(\sqrt{\frac{n\,cos\varphi + 1}{n\,cos\varphi - 1}}tg\frac{\gamma}{2}\right)\right\} \qquad (15)$$

The expression (15) gives the dependence of the angle of flight path γ_2 at the end of turn maneuver ($\psi = \pi$) on the bank angle φ and normal load coefficient n. The curves $\gamma_2 = f(n)$ are shown in Figure 3 for several values of bank angle φ. Points of intersection of this curves with abscissa ($\gamma_2 = 0$) correspond to a horizontal regular maneuver for which, as it is known: $n = 1/cos\varphi$.

From Figure 3 it is evident that the flight path becomes steeper when the normal load coefficient n increases, especially for relatively moderate values of the bank angle φ ($< 45°$). This is logical, as the increase of the normal load coefficient n results from kinetic energy. Reduction in the energy results in an altitude increase and, consequently, in an increase in flight path slope.

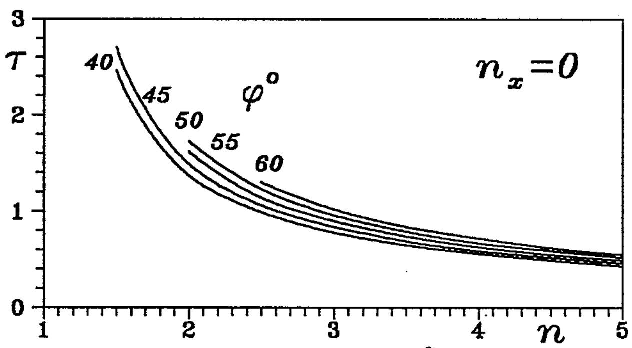

Figure 5: - Factor $\tau = t(g/V_1)$ as a function of the normal load coefficient

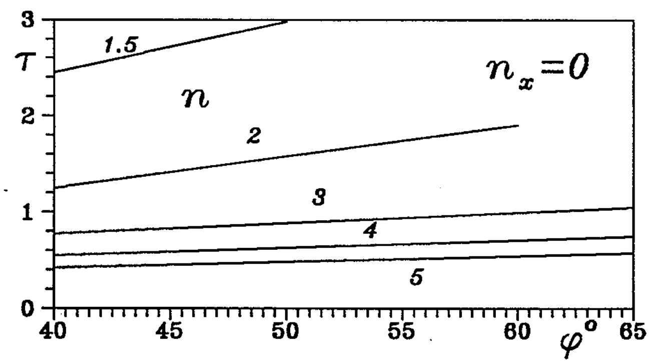

Figure 6: - Factor $\tau = t(g/V_1)$ as a function of the bank angle φ

Based on Equation (15), we may derive the relation $\gamma_2 = f(\varphi)$ for different values of normal load coefficient n and ($\psi = \pi$). These solutions are shown in Figure 4, from which one can see that the angle of flight path at the end of turn maneuver γ_2 drops rapidly with an increase of aircraft bank angle φ. This is pronounced especially at lower values of normal load coefficient n, what is in full agreement with the above conclusions.

Now, we can determine the time needed to carry out the turn from equation (12):

$$t = \frac{(V_1/g)}{n\,cos\varphi + 1}\left[\frac{sin\gamma_2}{n\,cos\varphi - cos\gamma_2} + \frac{2n\,cos\varphi}{\sqrt{n^2cos^2\varphi - 1}}arctg\left(\sqrt{\frac{n\,cos\varphi + 1}{n\,cos\varphi - 1}}tg\frac{\gamma_2}{2}\right)\right] \tag{16}$$

The time of turn t resulting from the above equation is shown in a nondimensional form $\tau = t(g/V_1)$ as a function of the normal load coefficient with the bank angle φ as a parameter by taking $\gamma = \gamma_2$ and $\psi = \pi$ (see Figure 5).

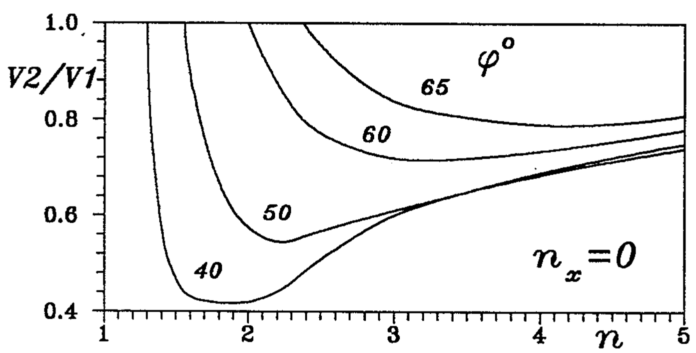

Figure 7: - Speed ratio V_2/V_1 as a function of normal load coefficient

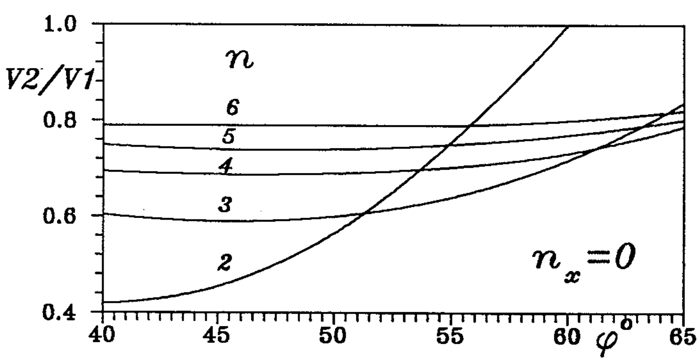

Figure 8: - Speed ratio V_2/V_1 as a function of the bank angle φ

From Figure 5 one can see a large decrease of time of turn maneuver when the normal load coefficient increases and less pronounced effect of the bank angle on the time of turn. This last conclusion is illustrated by low slope of curves, especially for the values of normal load coefficient which are greater than three (see Figure 6).

The equation (10) gives the aircraft speed at the end of turn maneuver:

$$\frac{V_2}{V_1} = \frac{n\,cos\varphi - 1}{n\,cos\varphi - cos\gamma_2} \tag{17}$$

Dependence $V_2/V_1 = f(n)$ for several values of bank angle φ is shown in Figure 7 and $V_2/V_1 = f(\varphi)$ for several values of the normal load coefficient in Figure 8.

It is necessary to check the speed at the end of turn. Since the lift is equal to:

$$L = nW = \frac{1}{2}\rho_2 {V_2}^2 C_{L_s} S \tag{18}$$

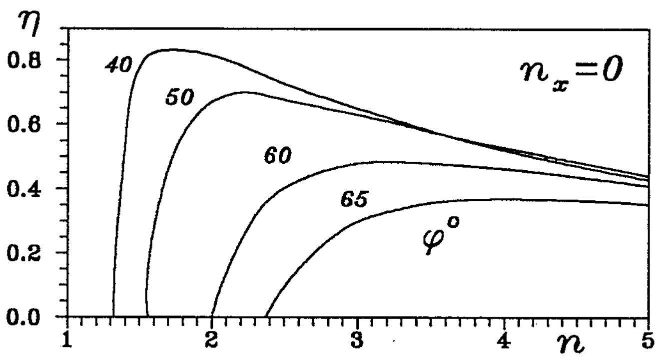

Figure 9: - Factor $\eta = \Delta h(2g/V_1^2)$ as a function of normal load coefficient

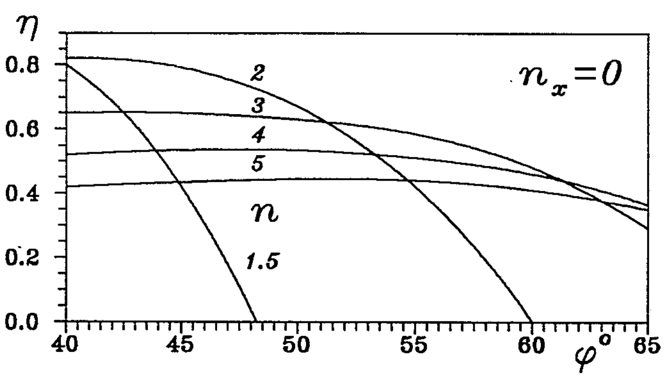

Figure 10: - Factor $\eta = \Delta h(2g/V_1^2)$ as a function of the bank angle and the normal load coefficient as parameters

the speed at the end of turn should be:

$$V_2 \geq V_s\sqrt{n} \qquad (19)$$

where V_s is the lowest allowable (safe) horizontal speed at altitude h_2 and which is equal to:

$$V_s = \sqrt{\frac{2mg}{\rho_2 C_{Ls} S}} \qquad (20)$$

By using the relation for equivalence of potential and kinetic energy we may obtain the altitude increase $\Delta h = h_2 - h_1$ at the turn:

$$\Delta h = \frac{V_1^2 - V_2^2}{2\,g} = \frac{V_1^2}{2\,g}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right] \qquad (21)$$

By substituting the relation for speed ratio V_2/V_1 given by equation (17), the altitude increase during turn maneuver may be written as:

$$\Delta h = \frac{V_1^2}{2\,g}\left[1 - \left(\frac{n\,cos\varphi - 1}{n\,cos\varphi - cos\gamma_2}\right)^2\right] \qquad (22)$$

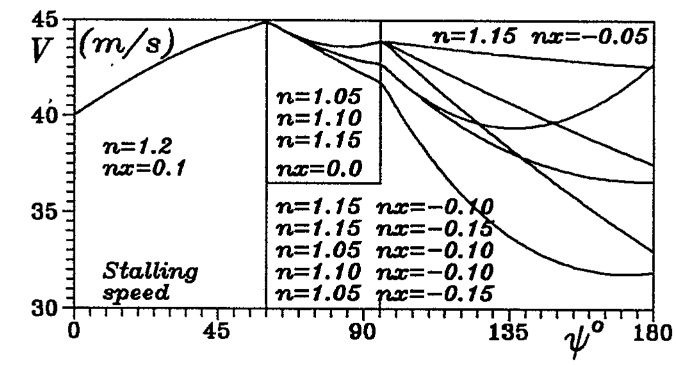

Figure 11: - Speed changes during turn maneuvers

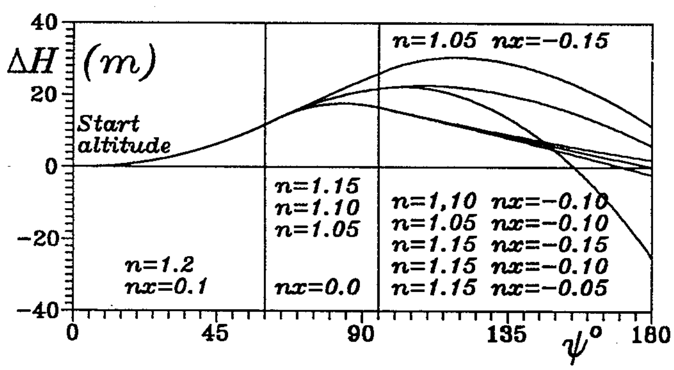

Figure 12: - Altitude changes during turn maneuvers

The altitude increase Δh shown in Figure 9 in a nondimensional form $\eta = \Delta h(2g/V_1^2)$ is given as a function of the normal load coefficient and the bank angle as parameters and in Figure 10 $\eta = f(\varphi)$ and the normal load coefficient as parameters.

This special case, when the tangential load factor $n_x = 0$, also confirms the above conclusions about the influence of main parameters on the agricultural airplanes' turn characteristics[3-7]; in the domain of optimum bank angles and for the typical values of normal load coefficient the altitude increase becomes higher with the normal load coefficient decreasing, but at the same time, the time of turn increases. And vice versa, with an increase of the normal load coefficient the altitude increase becomes smaller as well as the time of turn.

Numerical Results

The calculation results for the particular case of the agricultural aircraft climb turn maneuver when the tangential load coefficient is $n_x = 0$ is plotted in Figures 3 to 10 as a function of the normal load coefficient and of the bank angle. The results of the angle of flight path change, time of turn, initial and final speed ratio and altitude increase during turning maneuver are shown for different airplane bank angles within the range $\varphi = 40 - 65°$.

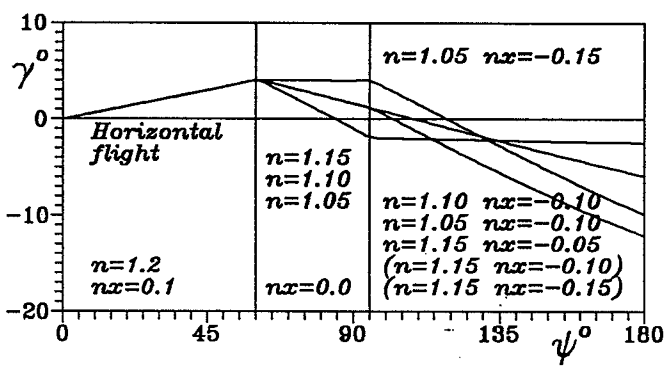

Figure 13: - Path angle changes during turn maneuvers

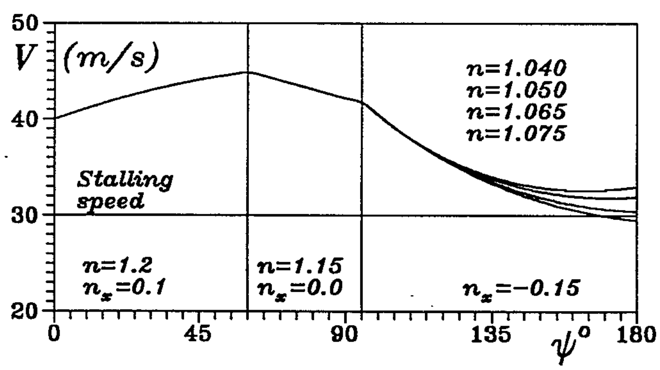

Figure 14: - Speed changes during turn maneuvers

The system of differential equations of motion (5) has been solved on a personal computer with Runge-Kutta-Merson numerical method[8]. Some results of the calculation are presented on Figures 11 to 16.

The calculation were performed involving the above given assumptions and using the typical values of the start velocity $40\,m/s$ and the bank angle $\varphi = 30°$ for this kind of aircraft (agricultural). The results which are given in Reference (3) indicate, that the normal and tangential load factors are constant during turning at the first part of path.

The representative results are: for the normal load factors $n = 1.2$, and for tangential one $n_x = 0.1$. For the second part of the path the tangential load factor is constant $n_x = 0.0$ and the normal load factor was varied from $n = 1.05, 1.10$ to 1.15 for the horizontal turn maneuver. At the third part of the path the values of tangential load factor are $n_x = -0.05, -0.10$ and -0.15, and for the normal load factor are $n = 1.05, 1.10$ and 1.15.

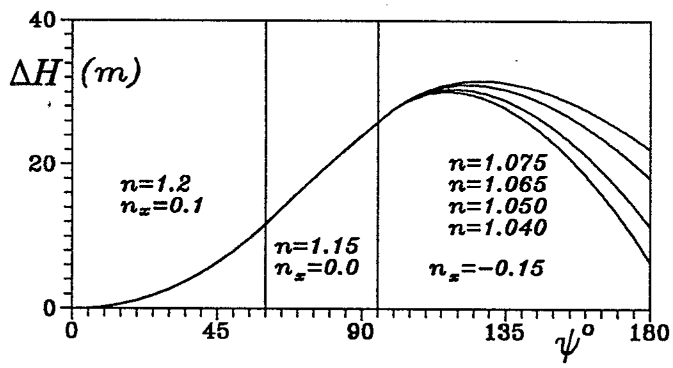

Figure 15: - Altitude changes during turn maneuvers

In the Figure 11, speed changes during turn maneuvers are given as a function of the path angle in the horizontal plane (angle of yaw) for various combination of the normal and tangential load factors. The altitude changes during turn maneuvers with reference to start altitude are given in Figure 12.

The path angle in the vertical plane (angle of flight path) are given in Figure 13 as a function of the path angle in the horizontal plane (angle of yaw) for various combination of the normal and tangential load factor values.

In the Figure 14 results of calculations of speed changes during turn maneuvers are presented with the reference to the stalling speed for the constant values of the normal and tangential load factors in first two part of path flight (in the first part $n = 1.2$ and $n_x = 0.1$ and in the second part $n = 1.15$ and $n_x = 0.0$).

Figure 15 shows the altitude changes during turn maneuvers as a function of the path angle in the horizontal plane (angle of yaw) with load factors in the last part of the turn amounting $n_x = -0.15$ and $n = 1.040, 1.050, 1.065$ and 1.075. Finally, the Figure 16 shows the flight path angle changes during turn maneuvers for the selected values of the load factors.

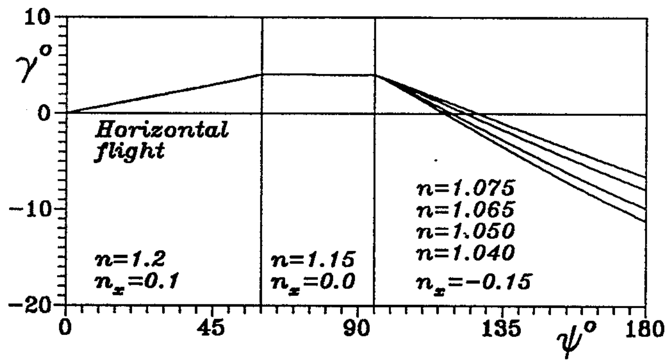

Figure 16: - Path angle changes during turn maneuvers

Concluding Remarks

This analysis, as well as some previous investigations[3-7] shows that the solutions for the turn maneuver are to be sought primarily for the case when the basic turn-defining parameters for this type of airplane have moderate values. Thus, some moderate and variable normal and tangential load factors values result in some moderate and acceptable altitude increase, speed and bank angles. Of course, the flight parameters at the entry in a new flight path, i.e. a new treating line shall take values that correspond to the optimal treating conditions. In other words, the new-path flight parameters shall duplicate the previous treating line flight condition (Figure 1).

Should the greatest altitude at the end of the turn maneuver be the aim, then the use of algorithm for particular analytic solution presented in this paper would be fully justified. However, this could be of greater interest for the case of an another type of aircraft such as fighting airplanes and not for agricultural airplanes.

The analysis and calculation of the climb (and descent) turn maneuver by involving the normal and tangential load factors provide general results. In this way, observation and analysis of behavior of all types of agricultural aircraft are possible. The analysis shows that the stalling speed and the start altitude at the beginning of the turn maneuver are limiting factors. For this kind of aircraft the stalling speed of approximately $30\,m/s$ is a typical value. Accordingly, at the end of the treating (spraying) run (beginning of turn maneuver) the velocity should exceed $40\,m/s$.

In this paper the criterion has been established by determining the margin of the speed over the stalling speed (safety speed) after the agricultural airplane increases its height above the ground by a given amount. The required altitude may be different for different types of airplanes and depends on their capability to perform rolling and turning maneuvers. The minimum margin of velocity allowed is determined from the requirements of the rolling maneuver.

However, experience has been showing, that the characteristics of the turn maneuver predominantly depend upon the manner in which every single pilot controls the airplane and performs the turn maneuver.

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