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A SENSITIVITY ANALYSIS OF CHAOS AT HIGH ANGLE OF ATTACK

Péter **Gránásy**
GE Lighting Tungsram, Hungary
József **Rohács**
Technical University of Budapest, Hungary

Abstract: Modelling the aircraft motion at high angle of attack is necessary to obtain information on manoeuverability. Extended simulation based on a NASA wind tunnel test of HARV (High Alpha Research Vehicle) were made on the longitudinal motion of the aircraft. Investigating a single stable poststall equilibrium by sinusoidal excitation in thrust deflection several bifurcations and even chaos was found. At high angle of attack the aerodynamic coefficients itself contains some uncertainties. Considering this effect the drag, lift, moment coefficients and lift and moment derivatives due to pitch rate and alpha dot were changed with a linearly increasing relative error. Simulations were made on the bifurcation map to compare the effect of changes in the aerodynamic coefficients. The structure of the period doublings qualitatively remains the same. Small changes are observable at drag coefficient variation. The lift variation provides more deviation with vanishing some attractor and period doubling however the domain and character of chaotic attractor is basically unchanged. Modifying in the moment coefficient gives higher variation and the equilibrium got unstable. The system keeps its structure with small variation of derivatives. A conclusion could be draw the appeared nonlinear phenomena are robust against the aerodynamic contamination.

Nomenclature

c_d = drag coefficient, dimensionless
$\bar{c}$ = reference mean aerodynamic chord, m
c_L = lift coefficient, dimensionless
c_{L0} = static lift coefficient, dimensionless
c_{Lq} = lift derivative with respect to pitch rate
$c_{L\dot{\alpha}}$ = lift derivative with respect to $\dot{\alpha}$
c_m = pitching moment coefficient, dimensionless
c_{m0} = static pitching moment coefficient, dimensionless
c_{mq} = pitching moment derivative with respect to pitch rate
$c_{m\dot{\alpha}}$ = pitching moment derivative with respect to $\dot{\alpha}$
g = gravitational constant, m/s^2
l_x = distance between center of gravity and aerodynamic center along x-axis, m
l_z = distance between center of gravity and aerodynamic center along z-axis, m
l_{xe} = distance between center of gravity and engine thrust center along x-axis, m
I_y = moment of inertia about the pitch axis kgm^2
m = aircraft mass, kg
q = pitch rate, rad/s
$\bar{q}$ = dynamic pressure, N/m^2
S = reference area, m^2
T = thrust, N
u = x axis speed in body axis system
w = z axis speed in body axis system
α = angle of attack, deg or rad (AoA)
δ_{vp} = pitch vector thrust angle, deg or rad
ε = thrust deflection excitation amplitude, deg
θ = pitch angle, deg or rad
ω = thrust deflection excitation angular frequency, rad/sec
WT subscript denotes the wind tunnel test results

Introduction

High alpha manoeuvers got close to tactical applications. Flight tests proved their superiority over conventional ones. A precise modelling of its dynamics should provide basic and precise information for designing low cost and safe aircraft. However obtaining reliable data at high alpha is a difficult task[1-3]. It evokes an obvious question: if the wind tunnel testing, which is much more accurate than any analytical model for aerodynamic coefficients, is not precise how the dynamics could be predicted. This paper would like to be devoted for providing some key results with a kind of sensitivity analysis continuing some early studies[4].

Mathematical model

An analytical model of HARV (High Alpha Research Vehicle) was investigated[5]. This model is based on a series of wind tunnel tests and trigonometric (arcus tangent) interpolation was performed with the validity of -10 - 90 deg AoA. Dynamic effects are considered in term of derivatives.

The aerodynamic coefficients are highly nonliear. Furthermore the describing mathematical equations contain hysteresis effect and cross-products of variables. The following equations describe the longitudinal motion of an aircraft.

$$\dot{u} = -qw + \frac{X}{m} - g\sin\theta + \frac{T_x}{m} \qquad (1)$$

$$\dot{w} = qu + \frac{Z}{m} + g\cos\theta + \frac{T_z}{m} \qquad (2)$$

$$\dot{q} = \frac{c_m \bar{q} S \bar{c} + X l_z + Z l_x - T_x l_{xe}}{I_y} \qquad (3)$$

$$\dot{\theta} = q \qquad (4)$$

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where

$$X = \bar{q} S \bar{c} (c_L \sin\alpha - c_D \cos\alpha) \quad (5)$$

$$Z = \bar{q} S \bar{c} (-c_L \cos\alpha - c_D \sin\alpha) \quad (6)$$

$$c_L = c_{L0} + \frac{\bar{c}}{2V}(c_{L\dot{\alpha}} \dot{\alpha} + c_{Lq} q) \quad (7)$$

$$c_m = c_{m0} + \frac{\bar{c}}{2V}(c_{m\dot{\alpha}} \dot{\alpha} + c_{mq} q) \quad (8)$$

$$T_x = T\cos(\delta_{vp} + \varepsilon \cos\omega t) \quad (9)$$

$$T_z = T\sin(\delta_{vp} + \varepsilon \cos\omega t) \quad (10)$$

Some papers were published on the investigation of longitudinal motion of current model[4, 6]. Thrust - thrust deflection variables were the major scope of the studies. These studies determined the equilibria surface, bifurcation diagram in the thrust-thrust deflection parameter space, eigenvalues wandering along thrust deflection values.
Chaos was obtained by applying periodic forcing term in thrust deflection (10), (11) at a certain poststall equilibrium. Chaos is used as a long term aperiodic behaviour of a deterministic system which sensitive to initial conditions where long term aperiodic means never subsiding transient, deterministic is no stochastic input or uncertain parameter and sensitivity to initial condition is close solution exponentially diverge[7].
A stroboscopic map could be seen on Fig.1 for T=35kN, δ_{vp}=10deg and ε=2deg parameter values. The angular frequency is varying from 0.1 till 1 rad/sec. This map was done by recording, after transient subsided, the angle of attack at the same phase for 30 periods.

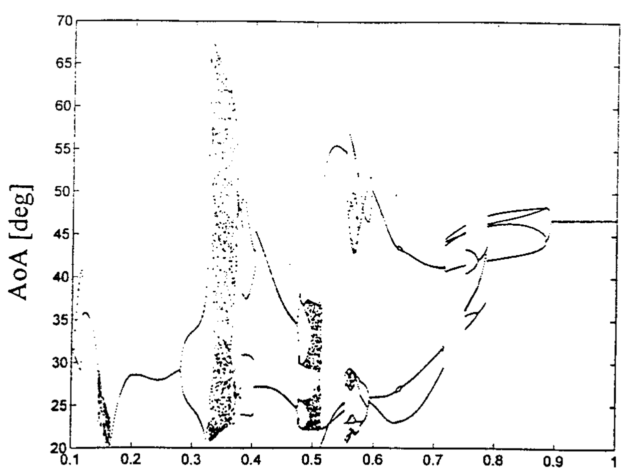

Fig.1 Stroboscopic map

The limit cycle behaviour is obvious for higher angular frequencies as it is expected from periodic forcing. Around 0.9 rad/sec the first period-doubling bifurcation happens as the two separate branches appear.
It is worth to note (although it is not scope of this paper) the appearing period-3 and 6 window as a coexisting attractor. It means there are two stable limit cycles (for this case) and the system converge to the limit cycle in whose attraction basin the initial condition was located. It could cause jump phenomenon.
Further period-doublings could be observed as the angular frequency decreases but chaos appears not in the cascade of bifurcations at 0.52 rad/sec. Other chaotic windows could be seen at 0.15-0.17 rad/sec and 0.33-0.37 rad/sec.

Sensitivity analysis of aerodynamic coefficients

Measuring aerodynamic data at high angle of attack are not accurate. It strongly depends on the history of the motion. It arises the question how any simulation could be reliable.
The equations (11), (12) and (13) try to model some uncertainties. It gives linearly increasing (measurement) error 0 at 0 deg AoA and 1%, 5%, -1%, -5% relative error for drag and lift coefficient, 0.01, 0.005, -0.01, -0.05 absolute error for moment coefficient at 90 deg AoA. Only one coefficient was changes in one simulation. The stroboscopic map of these 12 simulations will be compared with Fig.1.

$$c_D = c_{DWT}(1 \pm 0.01(\text{or } \pm 0.05)\ \frac{\alpha\ (\text{deg})}{90}) \quad (11)$$

$$c_{L0} = c_{L0WT}(1 \pm 0.01(\text{or } \pm 0.05)\ \frac{\alpha\ (\text{deg})}{90}) \quad (12)$$

$$c_{m0} = c_{m0WT} \pm 0.01(\text{or } \pm 0.05)\ \frac{\alpha\ (\text{deg})}{90} \quad (13).$$

Fig.2 gives the lift coefficient variation according to (12).

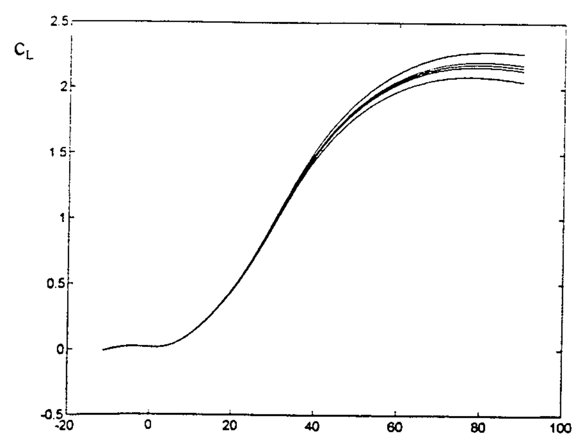

Fig.2 Lift coefficient variation

Fig.3, 4 and 5 shows how the equlibria move if the aerodynamic coefficient change. These eqilibria are quite close to the original system's. Error on the left side is positive at the third is zero and right side is negative. The system keeps its stability except moment's two outer points.

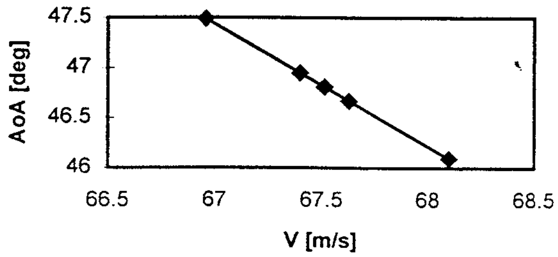

Fig.3 Equilibria sensitivity to drag

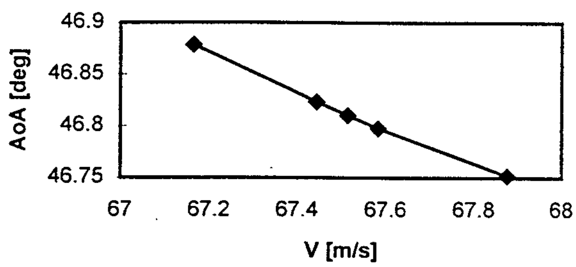

Fig.4 Equilibria sensitivity to lift

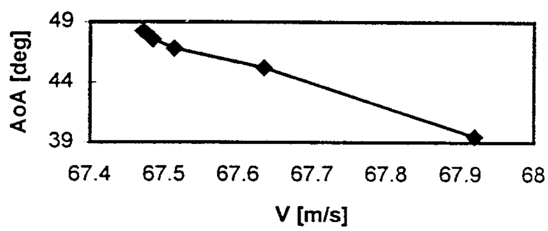

Fig.5 Equilibria sensitivity to moment

Drag coefficient

1% positive error

No major changes could be observed. Some bifurcations appear at slightly different place, not really visible.

5% positive error

A little more could seen form the period-6 window around 0.7 rad/sec. The bifurcations are shifted for lower frequencies but no major changes.

1% negative error

No visible change.

5% negative error

PD^{12} (period-doubling from 1cycle to 2 cycles) happens at lower frequency, at 0.88 rad/sec. Period-3 and 6 window is very small around 0.78 rad/sec.

Lift coefficient

1% positive error

Bifurcations appear at slightly higher frequencies but there is no other change.

5% positive error

The first period-doubling emerges around 0.93 rad/sec and shortly after several small chaotic window appear in the region of 0.73-0.86 rad/sec. Period-3 and 6 window is not visible anymore (it does not mean that it does not exist or the attraction basin has shrunken only the attraction basin has different shape). Other chaotic window is in the system around 0.56-0.58 rad/sec. At lower frequencies the structure remains the same.

1% negative error

PD^{12} happens at lower frequency and it is not so close to PD^{24}. Period-3 and 6 solution could be seen first around 0.72 rad/sec.

5% negative error

PD^{12} is shifted to 0.82 rad/sec and PD^{24} is not visible. All other bifurcations are at lower frequencies as could be seen on Fig.6. Small shaotic window is around 0.53-0.56 rad/sec.

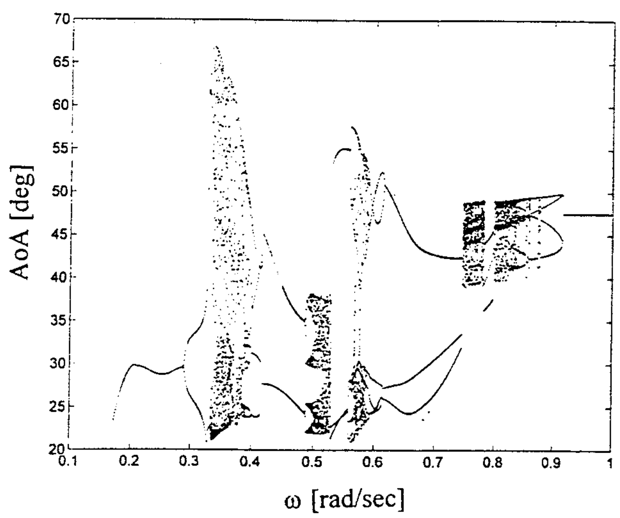

Fig.6 Stroboscopic map of 5% relative lift error

Moment coefficient

0.005 positive error

PD^{12} happens at 0.94 rad/sec and abruptly PD^{34}. Rather huge chaotic window appears at 0.88 rad/sec and it is visible till 0.74 rad/sec. Roughly, the structure of bifurcation remains the same but with bigger chaotic areas such as 0.34-0.41 rad/sec, 0.52-0.55 rad/sec and 0.57-0.59 rad/sec.

0.01 positive error

The equilibrium got unstable due to this change. Chaos evolves much earlier at 1.29 rad/sec and it is visible till 0.79 rad/sec. Like 0.005 positive error case the chaotic windows are increasing but at lower frequency the structure of bifurcations are recognisable.

0.01 negative error

The stroboscopic map is much more clean. PD[24] could be seen without PD[12] at 0.87 rad/sec. Period-3 and 6 window could be seen in the region of 0.72-0.74 rad/sec with period-1 solution. Like the original system period-3 is dominant in the region from 0.7 rad/sec. Chaos evolves in the domain of 0.51-0.54 rad/sec and structure remains the same for lower frequencies.

0.05 negative error

Like the outer moment point this equilibrium got unstable as well. The superposition to a limit cycle solution gives mathematically interesting structure such as Arnol'd tongue over 2 rad/sec.

Sensitivity analysis of aerodynamic derivatives

Like the aerodynamic coefficients the derivatives were modified but with much higher error. Lift and moment derivatives due to pitch rate and alpha dot were changed with a relative error 0 at 0 deg AoA and ±30% at 90 deg AoA as (14-17) formalise it.

$$c_{L\dot{\alpha}} = c_{L\dot{\alpha}WT}(1 \pm 0.3\frac{\alpha\ (\text{deg})}{90}) \quad (14)$$

$$c_{Lq} = c_{LqWT}(1 \pm 0.3\frac{\alpha\ (\text{deg})}{90}) \quad (15)$$

$$c_{m\dot{\alpha}} = c_{m\dot{\alpha}WT}(1 \pm 0.3\frac{\alpha\ (\text{deg})}{90}) \quad (16)$$

$$c_{mq} = c_{mqWT}(1 \pm 0.3\frac{\alpha\ (\text{deg})}{90}) \quad (17).$$

Lift derivatives due alpha dot and pitch rate

±30% positive error

Almost identical with the stroboscopic map of the original system.

Moment derivative due to alpha dot

30% positive error

The PD[12] happens at slightly lower frequency and the period-3 and 6 window is smaller.

30% negative error

It is very similar to the original system.

Moment derivative due to the pitch rate

30% positive error

The period-3 solution burst several times and it leads to chaos after several period-doubling bifurcation. Around 0.7 rad/sec period-3 gains back its stability and remain stable till 0.53 rad/sec where a small chaotic window appear. Chaos in the original system appearing at this angular frequency shifted to lower values but chaos at 0.33-0.37 rad/sec kept its position.

30% negative error

As could be seen on Fig.7, the PD[12] shifted to 0.87 rad/sec and period-4 solution emerges only for very short interval. Period-3 solution is visible from 0.75 rad/sec till 0.54 rad/sec with PD[36] at 0.62 rad/sec and back to period-3 at 0.57 rad/sec. At 0.54 rad/sec small chaotic window appear and back to period-4 with a cascade bifurcations. Earlier not visible period-8 solution could be seen around 0.38 rad/sec and the usual chaos in the region of 0.33-0.37 rad/sec remains at its place.

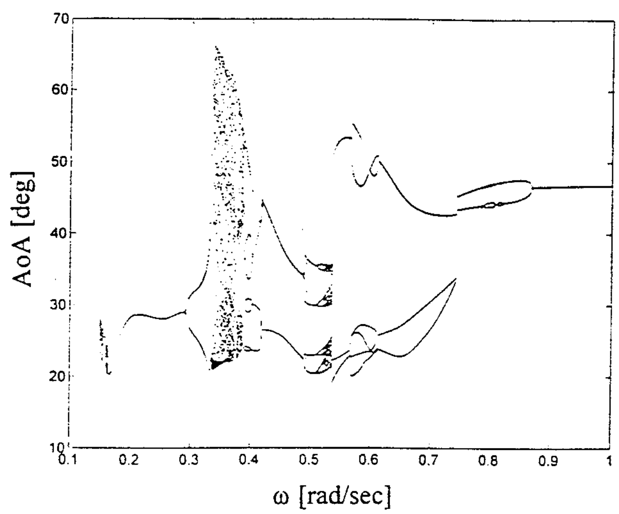

Fig.7 Stroboscopic map of 30% negative error in pitching moment derivative due to pitch rate

These results have similar characters like the early study on sensitivity analysis[4]: among the derivatives the moment derivative due to pitch rate has the biggest effects on dynamics.

Conclusions

A sensitivity analysis was performed on the dynamics of high alpha aircraft. Lift, drag , moment coefficients, lift and moment derivatives due to alpha dot and pitch rate were modified with linearly increasing error.

Several simulations were made from a stable poststall equilibrium with excitation in thrust deflection. Angular frequency of excitation was varied from 0.1-1 rad/sec. Stroboscopic maps were compared.

The structure of the solution remain the same although some bifurcation appeared at different frequency or disappeared. Chaotic windows were obtained very close to the original system's. The earlier found nonlinear phenomena are is robust against aerodynamic contamination.

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