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UNCONVENTIONAL FLIGHT ANALYSIS

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Abstract: The flights out of conventional flight or load envelopes like motion of aircraft at high angle of attack, or after loosing the conventional control are the unconventional flights. The paper describes the problems and methods of analysis of the unconventional flight situations. It gives some recommendations for generation and development of the highly non-linear models of aerodynamic coefficients, shows the effects of non-linearities on the dynamics of the high angle manoeuvrable aircraft. The simulation of the aircraft unconventional motion is realised by application of the system of stochastic non-linear differential equations. The fully stochastic process of the aircraft real motion is approximated by Markov chain.

Nomenclature

a, b, c, d = constant parameters,
 c_A = reference mean aerodynamic chord, m
 C_F, C_D, C_L = aerodynamic force, drag and lift coefficients, dimensionless,
 C_M, C_m = moment and pitching moment coefficients, dimensionless,
 C_{F_0}, C_{M_0} = force and moment coefficient at initial, zero state conditions, at zero angle of attack, static force and moment coefficient, dimensionless,
 C_{F_α}, C_{M_q} = force and moment coefficient derivatives, partial derivative of the given force and moment coefficients with respect to variables α and q indicated in the sub-subscript, (eq. 4.),
 F, D, L = aerodynamic force, drag and lift, N,
 f, f_x, f_y, f_z = function vectors,
 g = gravitational constant, m/s^2 ,
 l_x, l_z = distances between the center of gravity and aerodynamic center along x and z axes, m,
 l_{xe} = distance between the center of gravity and engine thrust center along x axis, m,
 I_y = moment of inertia about the pitch axis, kg/m^2 ,
 M = mass of aircraft, kg,
 M = aerodynamic moment, Nm
 P, P_t = probability vector and transition probability matrix,
 $p \in R^k$ = the parameter vector characterising the state of the aircraft,
 p = probability density function,
 q = pitch rate, rad/s,
 \bar{q} = dynamic pressure, N/m^2 ,

S = reference area, wing area, m^2 ,
 t = time,
 T = thrust, N,
 T_x, T_z = thrust components along x and z axes, N,
 $u \in R^m$ = input (control) vector,
 V, u, w = aircraft velocity, m/s, x and z axis speed in body axis of system, m/s,
 V = volumetric number, dimensionless,
 $W \in R^s$ and $\xi \in R^q$ = noise vectors (in simplified case the Wiener and Gaussian noise vectors respectively),
 X, Z = aerodynamic forces along x and z axes, N,
 x = state variable,
 $x(t)$ = stochastic process, $x(t) = X$, at the moment t ,
 $x \in R^n$ = state vector,
 $y \in R^r$ = output (measurable) signal vector
 $z \in R^l$ = vector of environmental characteristics (vector of service conditions),
 α = angle of attack, degree or rad, (AoA),
 δ = magnitude of the dimensionless crossflow velocity in the body fixed axis system, deflection angle, degree or rad
 δ_{vp} = pitch vector thrust angle, degree or rad,
 η = white noise standard vector,
 ε = amplitude of excitation, degree,
 θ = pitch angle, degree or rad,
 ω = frequency of excitation, thrust deflection,
 ω, μ = random variables assigning the position of vectors p and z within admissible space Ω_p, Ω_z described by density functions $f_p(\bullet), f_z(\bullet)$,
 σ_x, σ_y = noise transfer matrices,
 τ_x, τ_y, τ_w = time-delay vectors.
AoA = angle of attack, degree or rad,
DOF = degree of freedom (6DOF \rightarrow six DOF),

Introduction

The modern fighters will be able to flight beyond the conventional flight and/or load envelopes. The future large passenger aircraft can be applied only in case of considerable increasing the flight safety, because they will not have static stability due to too decreasing the large tail required. These special and unconventional flights, like motion of aircraft at high angle of attack, poststall regimes, flight in poor weather condition, motion of aircraft after loosing the control, before accident, etc. are investigated not very well yet. However the flights are unconventional today will conventional tomorrow. Therefore the discussion on the measurement methods, calculation of non-linear aerodynamic characteristics, real flight situation modelling, control in special flight situations, accident investigation, crash analysis, etc. can generate a big step in development of the new aircraft through the better understanding the unconventional flight situations.

The Department of Aircraft and Ships at the Technical University of Budapest started a new long period research project¹ with goals the

- real flight situation modeling,
- application of flight data to the real flight situation and accident investigations,
- study the flights after loosing the control (before accident and crash),
- investigation of the aircraft motions at high angle of attack,
- application of the methods of statistical flight dynamics,
- examination of the effects of aerodynamic and structural non-linearities on aeroelasticity,
- development of new control methods and systems.

The experience in theoretical and practical investigation of the unconventional flight situation had led up to the organisation of the first international conference on the given topics.

The paper describes the problems and methods of analysis of the unconventional flight situation. It gives some recommendations for generation and development of the highly non-linear models of aerodynamic coefficients. The effects of non-linearities on the aircraft poststall motion are demonstrated by some interesting results of investigation. The simulation of the aircraft unconventional motion is realised by application of the system of stochastic non-linear differential equations. The real motion of aircraft in the space as a stochastic process is approximated by Markov chain. So, the motion of aircraft loosed the conventional control system is recommended to describe by transition probability matrix.

The results of investigations can be applied to the aircraft accident investigations, crash analysis, control system design, etc..

The problems

The unconventional flight is a flight beyond the conventional, well studied flight and load envelopes. The examination of such flight can be based on the investigation of the real flight through the changes in aircraft motion variables.

The aircraft motion in three dimensional space can be described by solving the aerodynamic (gasdynamic), inertial and elastic equations². These equations are coupled through the aircraft shape and structure.

In general case, the aircraft motion seems to be describable easily for an engineer if the variation of its state vector \mathbf{x} chosen appropriately is expressed as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) . \quad (1)$$

In fact, the variation of state vector \mathbf{x} is influenced by the variation in the instantaneous values of a number of factors (service conditions, methods of maintenance and repair applied, the realised management, the real, time-depending flight characteristics, the atmospheric conditions, etc.). These influences can be given in terms of stochastic processes, random variables or random space (turbulence of atmosphere). Moreover, state vector \mathbf{x} can not generally be measured directly. Instead of it, some output signal vector \mathbf{y} can be measured. Consequently, the controlled motion of the aircraft or their technical conditions, their dynamics can be described only by a much more complicated model than giving in (1), namely by the following general set of non-linear, stochastic differential equations developed by us for real flight situation modelling^{3, 4}:

$$d\mathbf{x} = f_x[\mathbf{x}(t), \mathbf{x}(t-\tau_x), \mathbf{p}(\mathbf{x}, \mathbf{z}, \omega, \mu, t), \mathbf{z}(\mu, t), \mathbf{u}(t), \omega, \mu, t)]dt + \sigma_x(\mathbf{x}, \mathbf{p}, \mathbf{z}, \omega, \mu, t) d\mathbf{W} , \quad (2a)$$

$$\mathbf{y} = f_y[\mathbf{x}(t), \mathbf{x}(t-\tau_y), \mathbf{p}(\mathbf{x}, \mathbf{z}, \omega, \mu, t), \mathbf{z}(\mu, t), \mathbf{u}(t), \omega, \mu, t)] + \sigma_y(\mathbf{x}, \mathbf{p}, \mathbf{z}, \omega, \mu, t) \xi , \quad (2b)$$

$$\mathbf{u}(t) = f_u[\mathbf{x}(t), \mathbf{x}(t-\tau_u), \mathbf{p}(\mathbf{x}, \mathbf{y}, \omega, \mu, t), \mathbf{z}(\mu, t), \mathbf{u}(t), \omega, \mu, t)] , \quad (2c)$$

$$\mathbf{x}(t=t_0) = \mathbf{x}_0(t=t_0, \omega_0, \mu_0) ,$$

$$\mathbf{y}(t=t_0) = \mathbf{y}_0(t=t_0, \omega_0, \mu_0) .$$

The system of equations (2) contains the all problems connected with the study and description of the

unconventional flights. There are the changes in the structural and system parameters, unknown aerodynamic functions, effects of aerodynamic and structural parameter uncertainties and non-linearities on the flight dynamics, time-delay effects, etc.

The research project worked out by the Department of Aircraft and Ships at the Technical university of Budapest defines the four basic problems¹:

- A.) considerable changes and deviations in the characteristics, non-linearities and gap in the aerodynamic coefficients,
- B.) investigation of future aircraft (increasing the speed, dimensions, load, maneuverability, flexible aircraft motion, thrust vectoring, effects of non-linearities on the aircraft dynamics, gust effects, etc.),
- C.) investigation of accidents, development of the flight data recorders, real flight situation modeling, motion of aircraft after loosing the control,
- D.) design the new control systems.

The solving of these problems can reach only after development of the new models, methods for measurement, data-processing, modeling, like

- new types of aerodynamic coefficient models,
- motion after loosing the conventional control,
- statistical flight mechanics,
- on board monitoring and diagnostic systems,
- real motion simulation,
- accident investigation,
- flight safety investigation, etc.

In following part of this paper some results of investigation made in field of first three problems will be described.

Aerodynamic Models

The time-dependent aerodynamic equations describe the instantaneous aerodynamic effects on the aircraft assumed in form of coefficients of aerodynamic forces and moments depending on the state of the flow field surrounding the aircraft, like air viscosity, motion variables, e.g. linear and angular velocities, aircraft shape and real geometrical characteristics changed by deflection of the control elements and deformation of aircraft, and they may have a sensitive dependence on initial conditions:

$$C_F = F / \bar{q}S, \quad C_M = M / \bar{q}Sc_A. \quad (3)$$

The aerodynamic coefficients can be determined from coupled solution of the aerodynamic, inertial and elastic equations. Of course, the equation should be solved for

each changes in initial condition and realised control, e.g. for widely differing motion histories. The computing of the aerodynamic coefficients costs too much, and accuracy of the results may not be acceptable for description of the aircraft real motion. In reality, the determined aerodynamic coefficients are corrected or evaluated on the basis of laboratory, wind tunnel and flight experience.

In the references and aircraft technical descriptions the aerodynamic coefficients are given in forms of different functions of different characteristics, like angle of attack, velocity, angle of control surface deflection, etc. The aerodynamic coefficients have the considerable non-linearities including the hysteresis (Fig. 1), too.

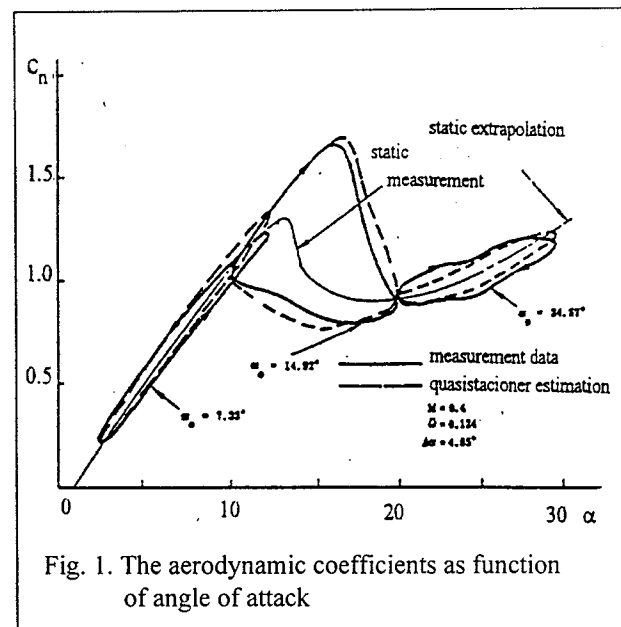


Fig. 1. The aerodynamic coefficients as function of angle of attack

The mathematical description of the aerodynamic coefficients are called as aerodynamic models. First models were based on the work of Bryan⁵, who used two principal assumptions: the aerodynamic force and moments depend only on the instantaneous values of the motion variables, and their dependence is a linear. For example the pitching moment can be given in a Taylor series about the reference states:

$$C_m(t) = C_{m_0} + C_{m_v} V(t) + C_{m_q} q(t), \quad (4)$$

where the C_{m_v} , C_{m_q} are the moment coefficient derivatives:

$$C_{m_v} = \left(\frac{\partial C_m}{\partial V} \right)_{V=V_0}, \quad C_{m_q} = \left(\frac{\partial C_m}{\partial q} \right)_{q=q_0}, \quad (5)$$

The effects of the past history of the aircraft motion on the current aerodynamics and the more realistic interaction between the aerodynamic coefficients and angle of attack as well as the control surfaces effects are taken into account by models introduced by Glauert⁶:

$$C_L(t) = C_{L_0} + C_{L_\alpha} \alpha(t) + C_{L_{\alpha^2}} \alpha^2(t) + C_{L_{\dot{\alpha}}} \dot{\alpha}(t) + C_{L_{\delta}} \delta(t) + \dots \quad (6)$$

Here $C_{L_{\dot{\alpha}}} = \left(\frac{\partial C_L}{\partial \dot{\alpha}} \right)_{\dot{\alpha}=\dot{\alpha}_0}$ and the time lag addition is proportional to $\dot{\alpha}$.

The next family of the aerodynamic models was developed by Tobak⁷ replacing the Bryan's function by a linear functional in form of the linear superposition integral like:

$$C_M(t) = C_M(0) + \int_0^t C_{M_\delta}(t-\tau) \frac{d}{d\tau} \delta(\tau) d\tau + \frac{l}{V} \int_0^t C_{M_q}(t-\tau) \frac{d}{d\tau} q(\tau) d\tau \quad (7)$$

The functions describing the dependence of aerodynamic coefficient and derivatives on all of the past values of the motion variables correspond to Volterra's description of a functional:

$$C_M = G[\delta(\xi), q(\xi)] \quad (8)$$

where ξ is a running variable in time ranging over interval zero to t .

Generally the whole past history of motion variables is unknown. Therefore the functional (8) can be replaced by a functional describing the dependence on the past in form of analytical functions in neighbourhood of $\xi = \tau$ reconstructed from the Taylor series expansions of the coefficients about $\xi = \tau$.

$$C_{M_8}[\delta(\xi), q(\xi); t, \tau] = C_{M_8}(t, \tau; \delta(\tau), \dot{\delta}(\tau), \dots, q(\tau), \dot{q}(\tau) \dots) \quad (9)$$

This approach gives possibility to take into account the considerable nonlinearities, time lag, and hysteresis, too.

The all of the aerodynamic coefficients are based on these models described. In case of the considerable nonlinearities in aerodynamic coefficients near the critical angle of attack at stall and poststall domain the aerodynamic models often are given in form of table, only, and the applied values are determined from table by interpolation.

In our research program we used a different type of the aerodynamic models. We found that the simple models like (5) can not be used for investigation the aircraft motion at high angle of attack and under quick changes in excitation or disturbance. Therefore we can recommend to apply the analytical models like

$$c_F = b_0 + \sum_{i=1}^n b_i \arctan((\alpha - c_i) d_i) \quad (10)$$

developed specially for approximation the experimental data got in wind tunnel investigation^{8,9}. They can be used in full AoA region from -10 degrees. to 90 degrees. The analytical models used by us were defined for different speed and elevator deflection with linear approximation between them. One example of these analytical models is represented in figure 2. NASA's representation of given derivative involves eight arcus tangent functions.

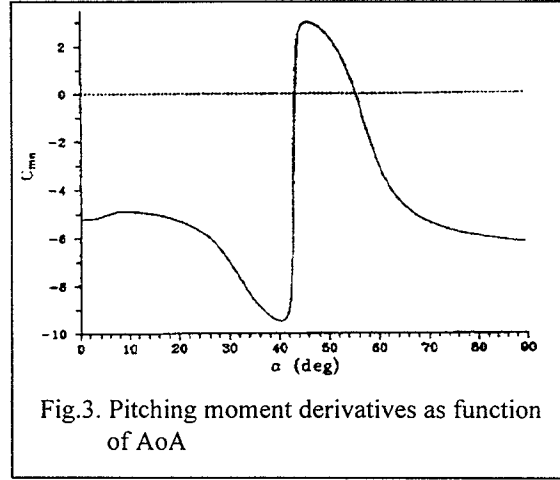


Fig.3. Pitching moment derivatives as function of AoA

Thrust Vected Aircraft at High Angle of Attack

The research program was concentrated on the high angle motion of the thrust vectored aircraft. We have investigated the longitudinal motion, only. The applied system of equation can be given in the following form:

$$\dot{u} = -qw + \frac{X}{M} - g \sin\theta + \frac{T_x}{M} \quad (11)$$

$$\dot{w} = qu - \frac{Z}{M} + g \cos\theta + \frac{T_z}{M} \quad (12)$$

$$\dot{q} = \frac{C_m \bar{q} S c_A + X l_z + Z l_x - T_x L_{xe}}{I_y} \quad (13)$$

$$\dot{\theta} = q \quad (14)$$

where

$$X = \bar{q} S (C_L \sin\alpha - C_D \cos\alpha),$$

$$Z = \bar{q} S (C_L \cos\alpha + C_D \sin\alpha),$$

$$C_L = C_{L_0} + \frac{c_A}{2V} (C_{L_{\dot{\alpha}}} \dot{\alpha} + C_{L_q} q),$$

$$Z = \bar{q} S (C_L \cos \alpha + C_D \sin \alpha) ,$$

$$C_L = C_{L0} + \frac{c_A}{2V} (C_{L\dot{\alpha}} \dot{\alpha} + C_{Lq} q) ,$$

$$C_m = C_{m0} + \frac{c_A}{2V} (C_{m\dot{\alpha}} \dot{\alpha} + C_{mq} q) ,$$

$$T_x = T \cos \delta_{vp} , \quad T_z = T \sin \delta_{vp} .$$

The described system of equation and analytical models of aerodynamic coefficients were filled up by data of F/A-18 aircraft^{8, 10}. The system of equation was solved by different numerical methods (Runge-Kutta, Adams-Moulton) with different step sizes^{9, 10, 11}. The software Matlab and ACSL were used in simulation. The getting results were stable and the same at steps 10^{-2} and 10^{-6} . The developed models and software were tested by simulation of the special motions of vectored aircraft figures of which were published in the references.

According to the bifurcation theory, the asymptotic behaviour of a non linear differential system can be analysed by studying the steady states and periodic orbits when the system parameters are slowly varying.

The steady states are characterised by determined equilibrium points and eigenvalues. In first case, the equilibrium surface can be found from system of equation (11) - (14) when left side of equations equal to zero. In our case the equilibrium points correspond to the trim positions.

The figure 3. shows the equilibrium surface obtained in the thrust-thrust deflection parameter space. Further information can be gained from bifurcation curves given on figure 4.

As it can be seen the poststall motion of the thrust vectored aircraft can be divided into five different poststall domain. A chosen flight level at $V=0.3$ M and $H=15000$ ft ($T=22.7$ kN, $\delta_{vp}=0^\circ$) is the origo of the

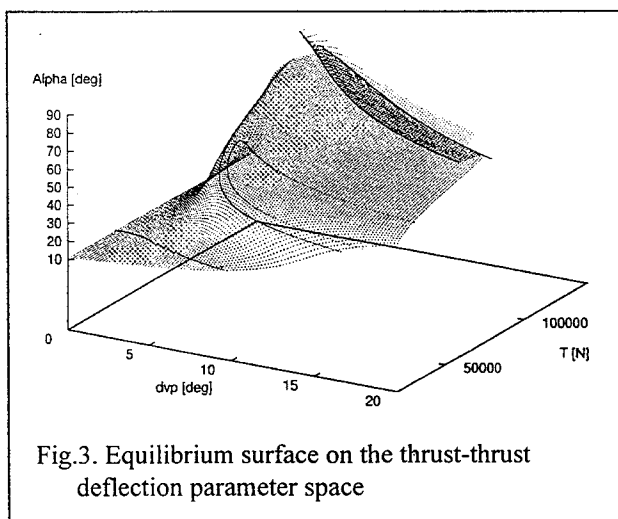


Fig.3. Equilibrium surface on the thrust-thrust deflection parameter space

figure. By increasing the thrust and thrust-deflection, the system reads the first Hopf bifurcation (H_1) (a small amplitude limit cycle appears at the bifurcation point). Further increasing thrust and thrust deflection there is no stable state of the aircraft. Over this region a before and poststall oscillation emerges. By another Hopf-bifurcation the system gains back its stability in the poststall regime. At high thrust and thrust-deflection the saddle-node bifurcation (SN) emerge creating jump phenomena, note folding on figure 3. The bifurcation were followed by continuation method.

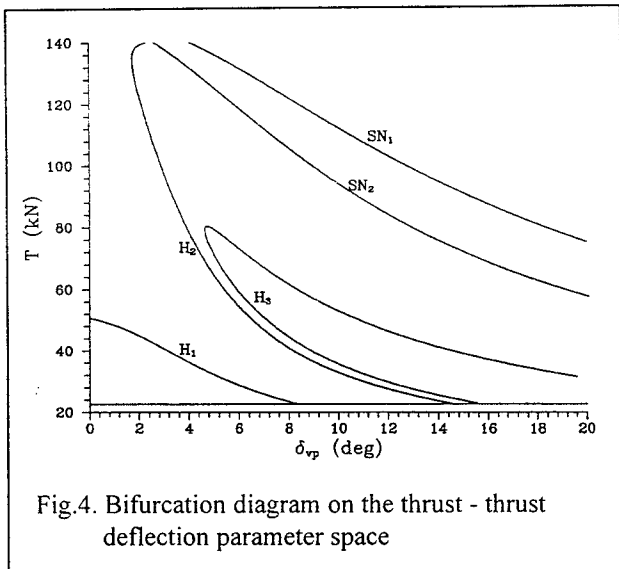


Fig.4. Bifurcation diagram on the thrust - thrust deflection parameter space

The very non-linear character of poststall dynamics had been studied by application of the bifurcation theory and chaos. The input was generated in the thrust deflection (not in the thrust), as usual non-linear approach would have been.

$$T_x = T \cos(\delta_{vp} + \varepsilon \cos \omega t) , \quad T_z = T \sin(\delta_{vp} + \varepsilon \cos \omega t) \quad (15)$$

The figure 5. demonstrates the typical result of our investigation. This is a stroboscopic map which is a generalised Poincaré-map recording only one variable

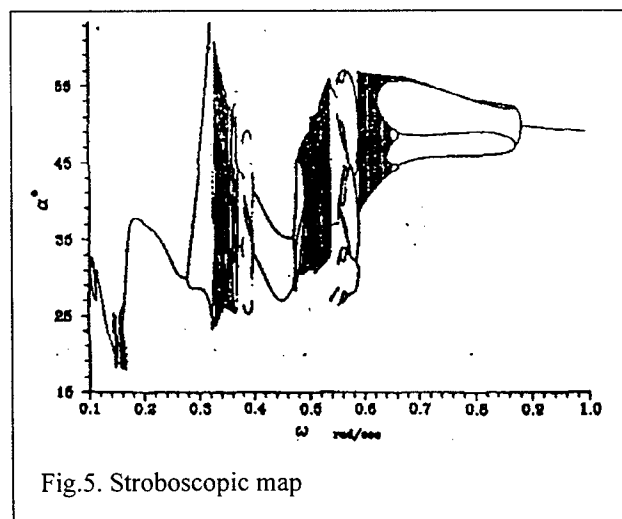


Fig.5. Stroboscopic map

(AoA) in one cycle at the same phase varying one parameter (angular frequency). In this case a certain poststall (AoA = 46 deg) equilibrium point was chosen for initial condition. 35 kN thrust and 10 deg thrust deflection defines this slightly stable state. The realised thrust deflection amplitude was 2 degree. The figure shows that around 0.9 rad/sec another non-linear phenomenon appears, so called period doubling bifurcation. At this point the time period becomes twice long (no sudden catastrophic change). Decreasing the frequency a cascade of period doubling bifurcation happen leading to chaos around 0.65 rad/sec. A Other chaotic window could be found around 0.35rad/sec.

The chaos can be represented by chaotic attractors which are the phase plots of the system outputs after periodic excitations. The figure 6. shows the 3D chaotic attractor got in case of 2 degrees trust deflection amplitude and 0.33 rad/sec angular velocity.

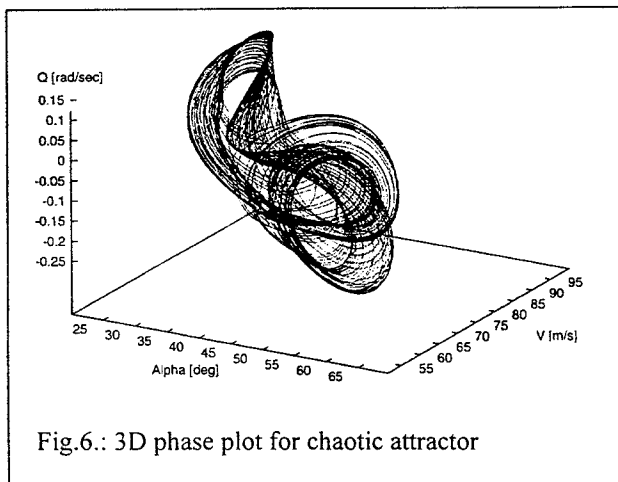


Fig.6.: 3D phase plot for chaotic attractor

So, the our investigation gave the very interesting conclusion, the closed deterministic system (11) - (14) can result to the chaos. The chaotic motions appeared at the different frequency of excitation depend on the applied types of the aerodynamic models, but have not fundamental sensitivity to the changes in the values of the aerodynamic model components.

Uncontrolled Aircraft Motion

The program for investigation of the motion of aircraft loosed the conventional control system started two years ago. The free flying model had built for practical investigation purposes. The relatively light model can not equipped by an onboard data collection and recorder system. Therefore the new telemetric method was developed for measurement of the motion variables. It is based on making the three dimensional digital video records with further data-processing.

The mean goal of the subprogram in the unconventional flight project is the description of the aircraft motion after loosing the conventional aerodynamic control system. It is a really new theoretical and practical task should be solved for increasing the flight safety of the future large passenger aircraft. The question is the prediction what will be happened with the aircraft after loosing the control, before bumping against the ground.

The general model (2) can be assumed in the form of the set of the following stochastic (random) differential equation

$$\dot{x} = f(x, t) + \sigma(x, t)\eta(t) , \quad (16)$$

called as diffusion process. Of course this equation as the set of equations can be rewritten in the vector form. The first part of right hand of equation described the direction of the changes of the stochastic process passing through the $x(t) = X$ at the moment t , while the second part shows the scattering the random process. In case of uncontrolled aircraft motion the disturbance generated by air turbulence.

The equation (16) in mathematics is called as Markov process. Such type of process can be fully described by giving its transition probability density function

$$p(x_2, t_2 | X_1, t_1) , \quad (t_2 > t_1) , \quad (17)$$

which characterises the distribution probability of the continuous random process, $x(t)$, at the moment t_2 , if it is passing through the $x(t) = X$ at the time, t_1 .

The transition probability density function can be described by application of the Fokker - Planck - Kolmogorov equations like:

$$\frac{\partial p(x_2, t_2 | X_1, t_1)}{\partial t_2} = -\frac{\partial}{\partial x_2} \left[f(x_2, t_2) p(x_2, t_2 | X_1, t_1) \right] + \frac{1}{2} \frac{\partial^2}{\partial x_2^2} \left[\sigma^2(x_2, t_2) p(x_2, t_2 | X_1, t_1) \right] . \quad (18)$$

or

$$\frac{\partial(x, t)}{\partial t} = -\frac{\partial}{\partial x} \left[f(x, t) p(x, t) \right] + \frac{1}{2} \frac{\partial^2}{\partial x^2} \left[\sigma^2(x, t) p(x, t) \right] . \quad (19)$$

The statistic flight mechanics¹² has worked out already several methods for application of this type of models. For example the statistical linearisation through the proof of the sensitivity function matrix to the flight mechanic models (11) - (14) and generating out the set of equation for the moments of the investigated stochastic process can used for study the scattering of the process depending on the changes in the initial condition. However the flight after loosing the control has a mach more complicated

picture depending on the unknown aerodynamics characteristics not studied yet in this high angle of regions and disturbance generated by air turbulence.

According to the equations (18), (19) defining the Markov process the following definition can be made:

$$p(X_2, t_2 | X_1, t_1) = \sum_{X(t)} p(X_2, t_2 | x, t) p(x, t | X_1, t_1),$$

$$(t_2 \geq t \geq t_1), \quad (20)$$

which equation is called as Chapman - Kolmogorov - Smoluchovski.

This equation gives possibility for approximation of the investigated non-linear stochastic process of continuous time and state space with the Markov chain of continuous time and discrete state space.

The space of the motion variable can be divided into several subspace as it shown in figure 7. Here N marks the normal, conventional flight, S1, S2, S3 characterise the different states of uncontrolled flight, while C indicates the catastrophic situation.

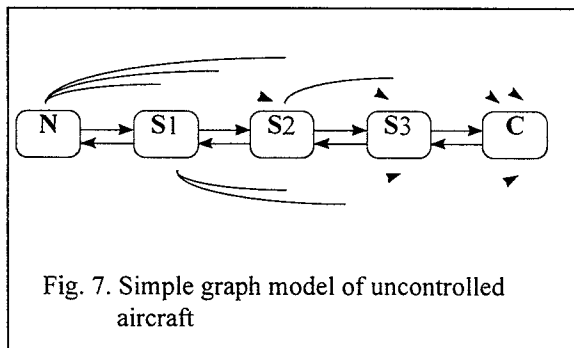


Fig. 7. Simple graph model of uncontrolled aircraft

In reality the flight after loosing the conventional control is a very complex process having a lot of alternative ways. For example the aircraft at stall can turn down for right or left side. Only this one event generate doubling the graph model given in figure 7. However it can not represented by two parallel graph models, by two parallel way, because during the further motion the aircraft can transit from one branch into second. So, we have to use the complex graph model including the different branches with ramification and joints.

The Markov chain can be described by the transition probability. Let mark the transition probabilities by β_{ij} . The β_{ij} are the transition probability densities giving the probability of moving the aircraft from state S_i to state S_j if it has been already in state S_i . As it is known this type of process can be approximated by Markov process, if

- the transition of process from one state into other is going during the negligible small time,

- the probability of transfer from one state into other through the one or more states is a negligible small value, and
- the time of staying the aircraft in different states can be approximated by exponential distribution.

In this case the process can be described by model:

$$\dot{\mathbf{P}}(t) = \mathbf{P}_t(t)\mathbf{P}(t), \quad (21)$$

where $\mathbf{P}(t)=[P_i(t)]$ is a vector of probabilities of cases when the aircraft is in states S_i ($i = N, S1, S2, S3, C$).

Now the project is concentrated on the defining the applicable graph model and on the estimation of the transition probability matrix.

Conclusions

The large research project named Unconventional Flight Analysis was organised by the Department of Aircraft and Ships at the Technical University of Budapest. The project is supported by international co-operation, too.

The project has several different subprograms, including

- the development of the aerodynamic models,
- investigation of the high angle motion of thrust vectored aircraft,
- study the motion of aircraft after loosing the control,
- development of the methods of measurement the uncontrolled aircraft motion, and statistical flight mechanics,
- examination of the effects of deviation in the system parameters and non-linearities on the non-linear flight mechanics, etc.

This paper defines the terminology of the unconventional flight, described the problems and tasks of the unconventional flight analysis, and shows some interesting and important results of the given research project.

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