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IDENTIFICATION OF AIRCRAFT NON-LINEAR DYNAMICS USING VOLTERRA SERIES

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Abstract

An approach for a systematic identification of aircraft non-linear dynamics by means of a Volterra functional series is presented in this paper. Volterra functional series is an intuitively satisfying representation for continuous non-linear time-invariant dynamic systems. The rigorous mathematical formulation of Volterra functional series have motivated a variety of different functional representations in order to extend the range of systems modelling, as well as to overcome the difficulties in determining the Volterra kernels. Recently, with the advances in the theory of computational neural networks, it has been developed a particular network architecture that is shown to be equivalent to a discrete Volterra series. This methodology facilitates, in principle, the kernel calculation of any order. The neural network approach, to achieve a Volterra series, is applied for the case of an aircraft non-linear longitudinal dynamics. Results have shown that the approach performs well and provides suitable approximation of the non-linear behaviour of aircraft longitudinal dynamics. The easy implementation of this kernel identification methodology also contributes for further applications in aircraft non-linear analysis and control design.

Introduction

Non-linear dynamic responses of aircraft in flight present major modelling difficulties. As a dynamic system, an aircraft is a complex aggregate of elastic bodies allowing relative motion and subjected to a complicated system of external non-linear loads and inertial effects. A complete model that can calculate all these effects is still not practical for industrial applications. Indeed, much of the aircraft modelling schemes serving either research or industry, present many simplifications and linearisations. These limitations inhibit the design and analysis of aircraft dynamic behaviour that could account for non-linearities such as severe manoeuvres at higher angles of attack or at transonic regimes.

An alternative to overcome such modelling problems is to use techniques from non-linear systems identification theory. Aircraft dynamics identification has reached today a high level of development, mainly due to advanced measurement and data processing techniques[1]. It has been observed that the understanding of aerodynamic phenomena and their modelling remain a great challenge. Approaches for aircraft dynamics identification have commonly treated the aerodynamic loading using linear parameter methods[2]. Therefore, the role of system identification methods is still decisive to characterise aircraft non-linear dynamics. This situation motivates the research for new approaches to provide systematic ways of identifying aircraft non-linear dynamic behaviour.

Identification strategies are used to establish the properties of a dynamic system by the measurement of its input and output time histories. While the use of non-linear identification strategies for general dynamic systems has been of great interest for many researches, only few preliminary studies of non-linear aircraft flight dynamics identification have appeared in the literature[1]. One of the reasons for that is the inherent feature of aircraft responses affected by the aerodynamic environment that provides unique transient response behaviour. This limits the application of certain non-linear identification techniques.

Mathematical techniques in non-linear identification, such as the Volterra functional theory[3] of non-linear systems, furnishes a rigorous formulation. Supported by this mathematical formality, Tobak and Schiff[4] have proposed the indicial response functional to compose aerodynamic response histories. Although, the functional theory allows non-linear representation of the aerodynamic loads acting on aircraft, the complexity of its formulation limits practical use.

Volterra[3] has also shown that expansions of the definition of the Taylor series for a function can be generalised to functionals. The resulting functional series is the so-called, *Volterra series*. Volterra[3] has also proved that any continuous, causal, time-invariant non-linear system can be modelled as an infinite sum of multi-dimensional convolution integrals of increasing order; that is, the Volterra series itself. The great drawback of Volterra series, however, is the calculation of its *kernels*[5,6]. Attempts to develop a systematic approach for Volterra kernel identification have not provided substantial progress so far in the field of non-linear identification.

Related to expansions of the Volterra type of functional series, other methodologies have been developed[5]. It is the case of the Wiener methods[6,7] that provide potential identification schemes for non-linear dynamic systems. One of the Wiener methods is based on the expansion of a non-linear functional into a series of mutually

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orthonormal polynomial functionals called G-functionals. Although the Wiener methods provide a systematic approach to non-linear identification problems, the excessive number of coefficients required to identify the functional series, even for lower-order non-linear systems, makes this technique impractical and difficult to apply.

Other techniques for non-linear dynamic systems identification are based on block-oriented models[5,8,9]. These approaches represent systems by means of cascade structures of combinations of linear dynamic and non-linear static subsystems. The Hammerstein model[8,9] is a block-oriented representation of non-linear systems, in which a static non-linearity is followed by a linear dynamic subsystem. Similarly, system models that consist of a cascade of a linear dynamic subsystem, a static non-linearity, and another linear dynamic subsystem, or the LNL systems[10], furnish another possible approach in non-linear identification by combining the ideas from Wiener and Hammerstein cascade models.

These techniques have been developed strictly for random processes, in particular for white Gaussian inputs, in order to systematically obtain the parameters of the identified models for the associated class of non-linear systems. An important drawback associated to the aforementioned methods is the difficulty in determining the large number of identification parameters. These features suggest that the application of block-oriented modelling by the current methods for aircraft non-linear dynamics is questionable.

Recently, studies using *artificial neural networks*[11] for identification have shown great potential for non-linear systems modelling, either on its own or assisting other types of approaches. Neural networks are information processors based on the concepts derived from neuro-biology. They are composed of processing units arranged into layers and connected between them. The application of neural network to identify Volterra functional series has been proposed by Wray and Green[12]. It has been proved that particular network architecture can be equivalent to a Volterra series representation of a dynamic system. Moreover, the kernels of any order can also be extracted from the network parameters.

The aim of this paper is to present an investigation on a systematic identification approach for Volterra functional series representations of aircraft non-linear dynamics. The identification procedure is carried out by means of a supervised network training scheme using standard back-propagation algorithm[11]. The approach is used to identify an aircraft non-linear longitudinal dynamics representation from data obtained in simulations of the aircraft longitudinal equations of motion accounting for non-linear coupling effects and linearised aerodynamic reactions. The training pattern comprises motion-induced horizontal velocity response history to variations of the aircraft elevator angle. Generalisation tests and the extraction of the first two Volterra kernels are also presented.

Volterra Series

Volterra series[3] are functional forms developed as a generalisation of the Taylor series expansion for a function. The basic premise of the Volterra functional series approach is that an exact description for a continuous non-linear, physically realisable (or causal), time-invariant system is provided by an infinite series of multi-dimensional convolution integrals of increasing order expressed as,

$$
\begin{aligned}
y(t) = h_0 + &\int_{-\infty}^{\infty} h_1(\tau_1)u(t-\tau_1)d\tau_1 + \\
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 + \ldots + \\
&\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\ldots,\tau_n)u(t-\tau_1)\ldots u(t-\tau_n)d\tau_1\ldots d\tau_n + \ldots
\end{aligned}
\quad (1)
$$

where $y(t)$ is the system response, $u(t)$ is the input to the system, and h_n is the n^{th}-order Volterra kernel.

The Volterra kernels are functions of the variables τ_i and each one represents a measurement of the systems non-linearity. Therefore, as requirement for an adequate Volterra series representation for a non-linear system, accurate calculation of the kernels is necessary.

The zeroth-order Volterra kernel, h_0, is a constant equal to the zero-input response of the system. The first-order Volterra kernel represents the linear response of the system to a unit impulse input, while the higher-order kernels are the non-linear responses of the system to multiple (with respect to the kernel order) unit impulse inputs. The higher-order kernels are measures of the non-linearity, or the relative influence of a previous input on the current response, that characterises the temporal effect to the non-linear system.

For causal systems, if any of the $\tau_1,\ldots,\tau_n$ is less then zero, then the kernel $h_n(\tau_1,\ldots,\tau_n)$ is zero, and the lower limits of the integrals in Equation (1) can be set equal to zero. Moreover, with no loss of generality, it is possible to assure that each kernel $h_n(\tau_1,\ldots,\tau_n)$ in Equation (1) is symmetric with respect to any permutation of $\tau_1,\ldots,\tau_n$.

It is convenient to present the Volterra series for causal, finite memory, T, time-invariant, and discrete time systems, in order to relate the formulation to the subsequent numerical kernel calculation; that is,

$$
\begin{aligned}
y(t) \cong h_0 + &\sum_{\tau_1=0}^{T} h_1(\tau_1)u(t-\tau_1) + \\
&\sum_{\tau_1=0}^{T}\sum_{\tau_2=0}^{T} h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2) + \ldots + \\
&\sum_{\tau_1=0}^{T}\cdots\sum_{\tau_n=0}^{T} h_n(\tau_1,\ldots,\tau_n)u(t-\tau_1)\ldots u(t-\tau_n) + \ldots
\end{aligned}
\quad (2)
$$

Various methods to assess the Volterra kernels have been developed, as it can be seen in the review paper of

Billings[5] or in the work of Schetzen[6]. Some of the discussed and reviewed approaches in the aforementioned research works are: *(i)* kernel estimation of a finite-order system using multiple pulse inputs and repeated experiments; *(ii)* kernel approximation by an expansion of orthogonal functions, with coefficients determined by gradient-type algorithms and pattern recognition methods; *(iii)* discrete Volterra kernels determination in terms of multi-dimensional *z*-transforms using high-order correlation functions and coloured Gaussian inputs.

In contrast with the generality features of Volterra functional series and their approximation properties, one difficulty is that the required kernel order may need to be very large to achieve a specified accuracy over the given set of inputs to outputs. The determination of even the first- and second-order kernels may involve large set of parameters. In addition, identification of higher-order non-linear systems based on Volterra series generally leads to severe numerical problems for the determination of the respective high-order kernels[5,6].

An alternative approach for systematic determination of Volterra kernels of any order has been proposed by Wray and Green[12]. By employing the theory of artificial neural networks, Wray and Green[12] have achieved particular network architecture that is equivalent to a Volterra series. The approach provides a way to extract the kernels of all dimensions of a non-linear system that can be realised by an artificial neural network. The developments of the approach are outlined in the next section.

Neural Networks

Artificial neural networks[11] are information processing systems with the capability of learning through examples. Based on concepts derived from neuro-biology, neural networks are composed by a set of interconnected processing units, called *neurons*. The neurons process the signals presented to the neural network by accumulating each stimulus and by transforming the total value using a function; that is, the *activation function*. The stimuli to and from, a neuron are modified by the real value called *synaptic weight*, which characterises the respective connection between neurons.

A typical representation for a generic neuron j, where $x_1, x_2, \ldots, x_p$ are the stimulus signals, $w_{j1}, w_{j2}, \ldots, w_{jp}$, are the synaptic weights, θ_j is a bias value, v_j is the activation potential, o_j is the neuron output signal, and $\varphi(.)$ is the activation function (generally adopted as a non-linear sigmoidal funtion), is:

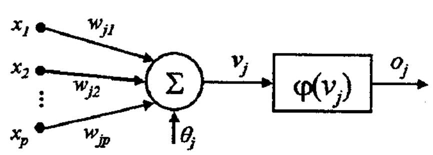

Figura 1 – Typical neuron representation.

Then, from Figure 1, one can observe that:

$$v_j = \theta_j + \sum_{i=1}^{p} w_{ji} x_i \quad (3)$$

and

$$o_j = \varphi(v_j) \quad (4)$$

Network *architecture* is the name given to the arrangements of neurons into layers and how they are connected. Typical neural networks have the following architecture: (1) *input layer* – where the input stimulus is presented to the network; (2) *hidden layers* – internal layers of a network, and (3) *output layer* – the last layer of the network, where the outputs are given. Such typical network architecture is commonly referred as a *multi-layer neural network*.

To perform a desired task, the synaptic weights of a network must be initialised and modified by a *training* algorithm. In *supervised training* algorithms, the weights are altered in accordance with a proper error-correction rule (e.g., *back-propagation* algorithms) based on the difference between desired and actual network outputs.

Neural network equivalent to Volterra series

In their paper, Wray and Green[12] have presented a methodology of extracting the n^{th}-order Volterra kernels by means of neural networks with specific architecture. For a SISO dynamic system, the neural network equivalent to a Volterra series is illustrated in Figure 2, where N is the number of time-delays of the input series, M is the number of hidden neurons, $u(t\text{-}j)$, for $j=0,\ldots,N$, is the input u at time-delay j, w_{ji} is the synaptic weight in the connection between input u at time-delay j and the hidden neuron i, c_i is the synaptic weight in the connection between the hidden neuron i and the output neuron, and $y(t)$ is the network output at current time.

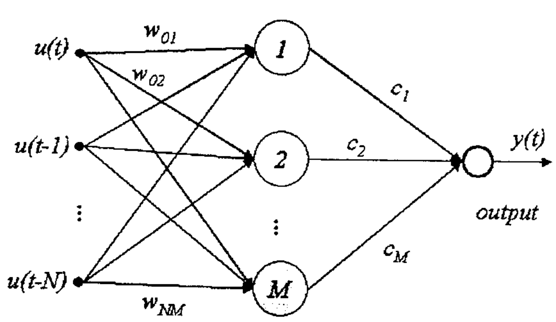

Figure 2 – Network architecture equivalent to a finite memory, discrete Volterra series.

Such architecture also considers that the hidden neurons present sigmoidal activation function and linear activation function in the output unit. The premise of Wray and

Green[12] approach is that the activation of the hidden neurons can be approximated by a n^{th}-order polynomial function. In this case, the network output can be given as,

$$y(t)=\sum_{j=1}^{M}c_j\left(\sum_{k=0}^{n}a_{kj}v_j^k\right) \qquad (5)$$

where a_{kj} are the polynomial coefficients and,

$$v_j=\theta_j+\sum_{i=0}^{N}w_{ji}u(t-i) \qquad (6)$$

An important detail is the fact that if the hidden neurons present different bias values, then the respective polynomial representation will be also different.

By expanding Equations (5) and (6) and by grouping appropriate terms, it can be shown that the network (*cf.* Figure 2) is equivalent to a finite memory, discrete Volterra series[12]. Moreover, if the network can be trained to represent a dynamic system, the kernels up to the n^{th}-order in Equation (2), can be extracted according to the following expression:

$$h_n(\tau_1,\tau_2,\ldots,\tau_n)=\sum_{j=1}^{M}c_j\,a_{nj}\,w_{\tau_1 j}\,w_{\tau_2 j}\cdots w_{\tau_n j} \qquad (7)$$

Here, the activation functions of the network hidden neurons shown in Figure 2, are adopted as ($\beta = 0.8$),

$$\varphi(v)=\left(\frac{2}{1+e^{-2\beta v}}\right)-1 \qquad (8)$$

Using the *least squares method*[13], the n^{th}-order polynomial approximation of the activation function given in Equation (8) can be obtained.

Non-Linear Identification via Volterra Series

To illustrate the approach a Volterra series representation of an aircraft longitudinal dynamics using a neural network is identified. The formulation used to simulate the aircraft non-linear dynamics is obtained from the work of Etkin and Reid[14]. Here, it is assumed decoupled longitudinal motion and linearised aerodynamic reactions. The only non-linear effects to be considered in the simulation are those originated by the coupling of pitch angular velocity with both horizontal and vertical scalar velocities. The resulting small disturbances set of aircraft longitudinal equations of motion are given by,

$$\Delta\dot{u}=\frac{X_u\Delta u+X_w w+X_q q+X_{\delta_e}\delta_e}{m}-g\cos\theta_0\theta-qw \qquad (9a)$$

$$\dot{w}=\frac{Z_u\Delta u+Z_w w+Z_q q+Z_{\delta_e}\delta_e-mg\sin\theta_0\theta+mq\Delta u}{m-Z_{\dot{w}}} \qquad (9b)$$

$$\dot{q}=\frac{M_u\Delta u+M_w w+M_{\dot{w}}\dot{w}+M_q q+M_{\delta_e}\delta_e}{m} \qquad (9c)$$

$$\dot{\theta}=q \qquad (9d)$$

where, u, w are the respective horizontal and vertical scalar velocities of aircraft centre of gravity, q is the pitch angular velocity, θ is the pitch angle, θ_0 is the reference pitch angle, δ_e is the elevator angle, X_i, Z_i, M_i, are derivatives of the resultant aerodynamic reactions with respect to the i^{th} variable, m is the aircraft mass, g is the gravity acceleration, I_y is the inertia moment, dot symbols represent $d(\,)/dt$, and Δ means small perturbations.

The data used are obtained from simulations of the longitudinal dynamics of the Boeing 747-100 aircraft[14]. The adopted regime of flight is the cruising in horizontal path at an altitude of approximately 12,000 *metres* and at a Mach number of 0.8. Here, the horizontal scalar velocity response, $u(t)$, due to variations in the elevator angle, $\delta_e(t)$, is considered in the identification process.

A supervised training process is used to obtain the neural network equivalent to the finite memory, discrete Volterra series. This identification process, shown in Figure 3, commences with an initialisation of the synaptic weights using random uniform distribution (-1.0 to 1.0). Back-propagation algorithm[11] is used to adapt the weights. To speed up the process, adaptive learning rate and momentum are also incorporated to the algorithm[11].

The training process demands that a broad range of motion-induced aircraft responses be used in order to enable the identified model to capture the non-linear nature of the system. Normally, in non-linear systems identification schemes[5], input signals must present specific statistical properties so that the main dynamic features of the systems can be explored during the process. Indeed, many techniques[5] have been developed strictly for white Gaussian input forms. In the case of aircraft identification, such approaches are questionable, mainly due to the features of the external aerodynamic loads involved in this problem.

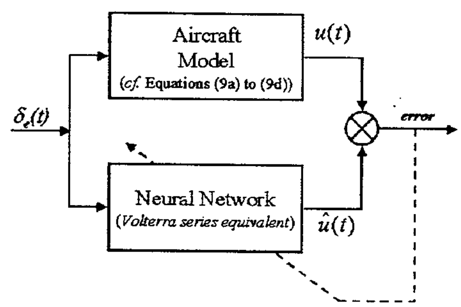

Figure 3 – Identification scheme.

Although the employment of random input forms to aircraft identification schemes may not be practical, here these types of inputs are chosen because of their mathematical value. As a first justification, one can observe that the adopted aircraft equations of motion (*cf.* Equations (9a) to (9b)) have been assumed with linearised aerodynamic loads. Since the only non-linear effect is due to the coupling of rigid body velocity variables and neglecting complex non-linear transient aerodynamic responses, the aircraft dynamics can, in principle, be treated as any other dynamic system with weakly non-linear behaviour. The second justification is that by presenting random inputs, a broader range of frequencies and amplitudes can be associated to the system during the identification process.

To perform the identification process, band-limited white noise is assumed to compose the input motion. The maximum absolute amplitude for the elevator angle is limited to a range of 1.5° to 2°. Larger angles have been avoided because, even being the aircraft[14] linearly stable, non-linear marginal instability could jeopardise the identification process. In addition, aircraft long period motion response is also assumed. The training pattern has a sample interval of 1.0 *second* to facilitate adequate representation of the motion-induced responses.

In order to assist the identification process, the input/output time-series pattern for the network training is subdivided into two parts. One part is used for the training itself (*training set*) while the other one is used to perform measurements to the resulting network output (*test set*). The training set is taken in a random (may be not sequential) from the half of the total input/output time-series pattern and the remaining set is the test one. Time windows depending on the assumed number of time-delays, *N*, are presented to the network and the resulting error measure is used in the back-propagation algorithm. After back-propagating the accumulated errors from feed-forwarding the training set inputs (batch mode), the test set is presented to the network and the total error (sum of squared errors) is obtained. After the end of this step, an *epoch* of the training is completed. If this error measurement is lower then a user-defined value (0.01 in this case), the identification process is stopped. An another stop criterion is based on the number epochs.

To compose the input training pattern a noise power value of 0.0005 and sample time of 2π are adopted. These parameters have been taken from observing both the input limitations described above and the system output behaviour.

The neural network training parameters are: number of time delays (*N*) in the input series is 10; number of hidden neurons, (*M*) is 50; starting learning rate of 0.01; and starting momentum constant of 0.85. The training has been carried out in 10,000 epochs.

Figure 4 presents a comparison between the aircraft non-linear response obtained by simulations of the Equations (9a) to (9b) and the respective Volterra series representation after completion of the neural network training process.

The resulting neural network equivalent to a Volterra series representation of the non-linear dynamic system can also be used to extract the kernels. Using the methodology exposed in the previous section, the first- and second-order Volterra kernel approximations of the identified aircraft non-linear dynamics model can be calculated. Figures 5 and 6 depict the two kernels, respectively. Higher-order kernels could also be extracted, but the extensive number of variables would turn their illustration very difficult.

To test the robustness of the identified model a set of arbitrary characteristic motion-induced horizontal scalar velocity histories are presented to the neural network. Figures 7 to 12 show the comparison of the desired simulated outputs of the network equivalent to the Volterra series representation.

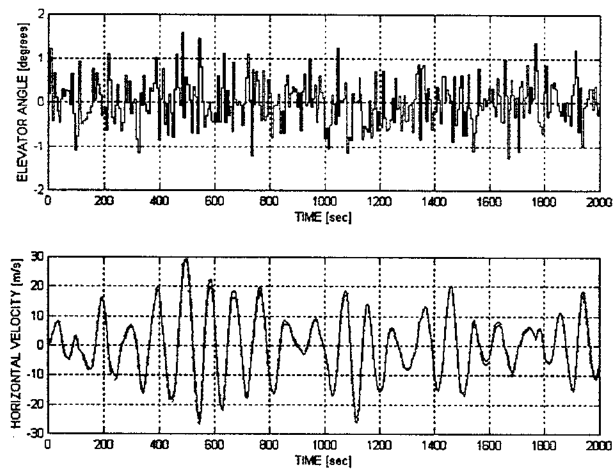

Figure 4 – Horizontal scalar velocity response due to elevator angle motion - training results (solid: aircraft model simulation; dashed: neural network output).

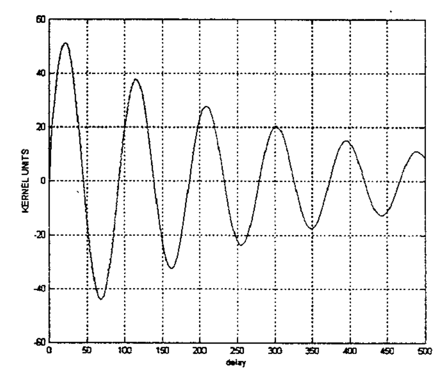

Figure 5 – First-order kernel of the Volterra series representation of the horizontal scalar velocity response to variations of the elevator angle.

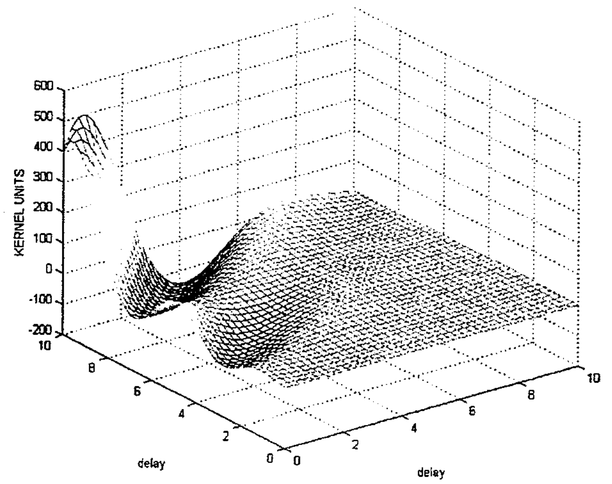

Figure 6 – Second-order kernel of the Volterra series representation of the horizontal scalar velocity response to variations of the elevator angle.

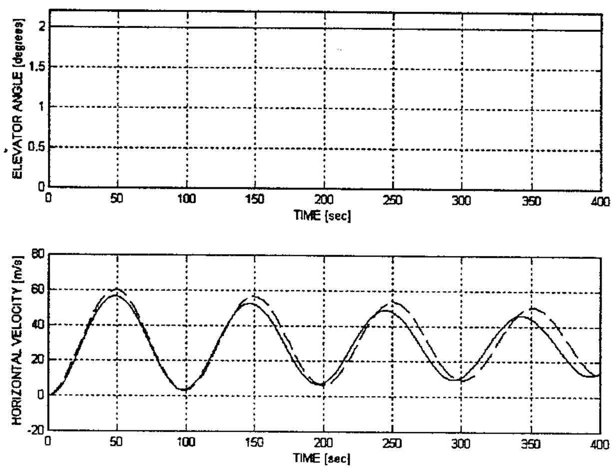

Figure 7 – Horizontal velocity response to a 2° amplitude elevator angle step input (solid: aircraft model simulation; dashed: neural network output).

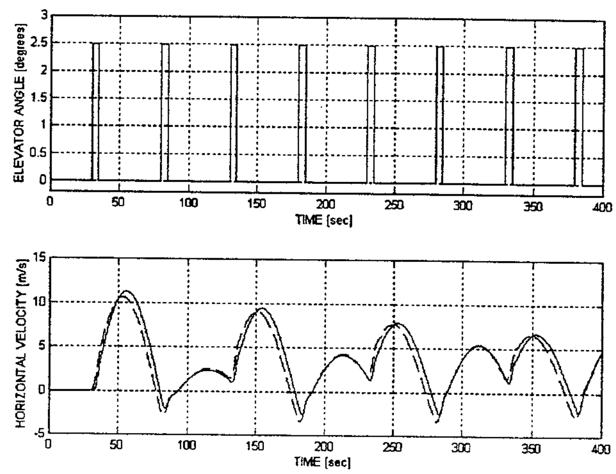

Figure 8 – Horizontal velocity response to a 2.5° amplitude, 5*s* at max. amplitude, 50*s* period, elevator angle squared pulses input (solid: aircraft model simulation; dashed: neural network output).

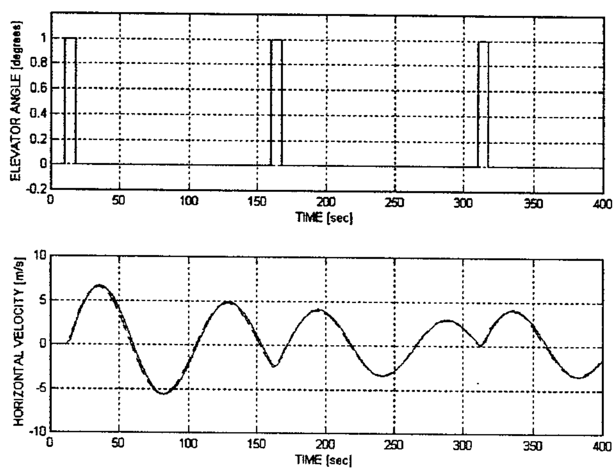

Figure 9 – Horizontal velocity response to a 1° amplitude, 10*s* at max. amplitude, 150*s* period, elevator angle squared pulses (solid: aircraft model simulation; dashed: neural network output).

Discussions

The ability of the neural network model to capture the essential features of the non-linear aircraft longitudinal dynamics can be observed in emulations for a broad range of motion-induced history used during the training process. In contrast to the time demanded by the training process to identify the model, the final network evaluations are fast enough to allow real-time predictions of non-linear dynamic system responses, justifying applications in analysis and control design. Moreover, the Volterra series approach allows subsequent bilinear representation.

For the identified neural network model the non-linear behaviour of the horizontal scalar velocity response is adequately captured. Few discrepancies can be observed in the network model output after training. These are mainly associated to imperfections in the training algorithm (e.g., local minima problem) and to the non-linear characteristics of the horizontal velocity response. In addition, the statistical features of the random input series used for training may have induced to these imperfections.

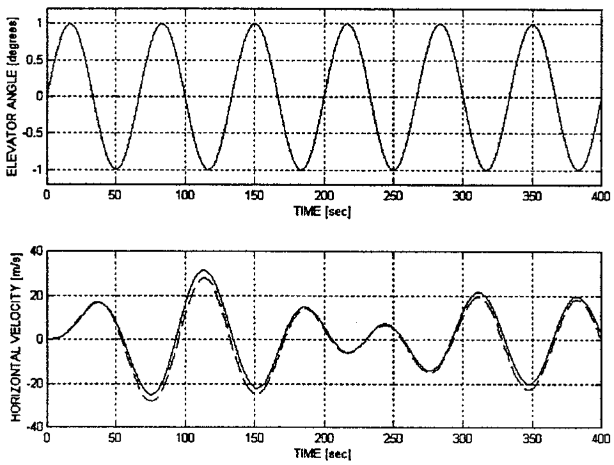

Figure 10 – Horizontal velocity response to a 1° amplitude, 0.015 *Hz* frequency, elevator angle sinusoidal input (solid: aircraft model simulation; dashed: neural network output).

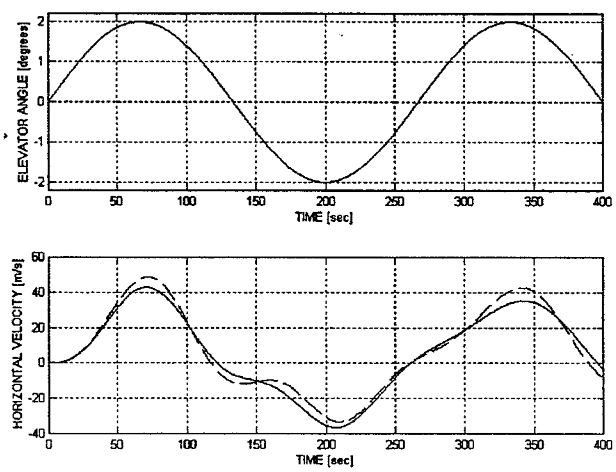

Figure 11 – Horizontal velocity response to a 2° amplitude, 0.00375 *Hz* frequency, elevator angle sinusoidal input (solid: aircraft model simulation; dashed: neural network output).

Generally, the predictive capabilities of the identified network model are shown to be satisfactory for the majority of the generalisation test cases within the training limits. This also confirms the underlying capability of Volterra functional series for the aircraft non-linear dynamics representation. Examining the first two Volterra kernel approximations, one can observe how the series is composed to represent the degrees of non-linearity corresponding to this dynamic system. The first-order kernel (*cf.* Figure 5) represents the linear component of the Volterra series and can be associated to the unit impulse response to the system. The second-order kernel gives an idea of how the non-linear effects are distributed.

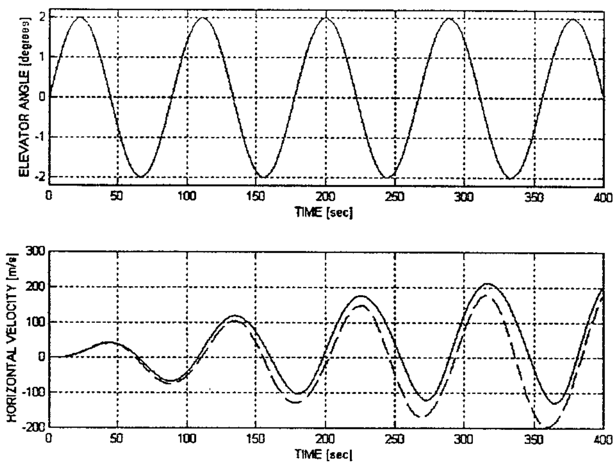

Figure 12 – Horizontal velocity response to a 2° amplitude, 0.015 *Hz* frequency, elevator angle sinusoidal input (solid: aircraft model simulation; dashed: neural network output).

When tested with a 2° amplitude step-input elevator motion (*cf.* Figure 7) which leads to a typical long period phugoid response, the identified model reveals reasonable approximation of the horizontal velocity response. The delay encountered between the desired and predicted responses can be related to a loss of frequency description in the training pattern, or the need for more time-delays (N) in the composition of the inputs to the neural network. Another aspect related to the delay could be the motion amplitude that is over the maximum value used to limit the training pattern (*cf.* Figure 4).

Figures 8 and 9 present generalisation tests for squared pulse types of elevator motion. The case in Figure 8 corresponds to the horizontal velocity ($u(t)$) response to 2.5° amplitude squared pulses, starting at 30 *s*, 5 *s* duration in maximum amplitude, and period of 50 *s*. The identified model reveals a time lead-lag with respect to the desired response, although the main features of the horizontal velocity response history has been captured. A possible reason for this deficiency could also be associated to the maximum amplitude of the input motion that is higher than the adopted in the training pattern. In the case of Figure 9, however, the identified network model satisfactorily predicted all the features of the horizontal velocity response history within the training limits. The input of 1° amplitude squared pulses, starting at 30 *s*, 10 *s* duration in maximum amplitude, and period of 150 *s*, comprises the case.

The behaviour of the identified model is also tested for characteristic sinusoidal inputs of the elevator angle with different amplitudes and frequencies. Figures 10 to 12 shows the prediction results to sinusoidal motion-induced histories. Despite the differences and the prediction disparities, the identified network model maintains the main features of the horizontal velocity response history. It can be observed that frequency and amplitude features of the input motion history significantly influence the prediction capabilities of the identified model. As far as amplitude is concerned, the same kinds of discrepancies observed in the previous generalisation tests have been associated to the sinusoidal-type of motion-induced horizontal velocity responses. Specifically, the cases shown in Figures 11 and 12 present this dissonance.

The frequency effects to the predicting horizontal velocity responses have shown that the training process failed to provide complete modelling. However, the main features of the motion-induced responses have been all captured. Some large errors can be observed in cases with certain extreme frequency and amplitude values (*cf.* Figure 11). Particularly, in Figure 12 the horizontal scalar velocity unstable response reveals that in these circumstances, the model comprehensively fails to predict the combined frequency and amplitude features of the system response. This case comprises a 2° amplitude sinusoidal input motion with frequency of 0.015 *Hz*. Interestingly, the identified network model has been capable of predicting the instability, from what can be inferred that the resulting model provides reasonable representation of the system non-linear dynamic characteristics.

Conclusions

Volterra functional series, obtained by an equivalent neural network model, provides a suitable representation of an aircraft non-linear longitudinal dynamics. Supported by the rigorous concepts underlying the Volterra functional series approach to non-linear systems identification, the successful application of neural networks to determine an equivalent representation ensures both reliability and accurate mathematical description for the resulting model.

Comparing this approach to other non-linear identification methods, the neural network technique has provided a much more powerful tool for a systematic assessment of equivalent Volterra functional series of non-linear dynamic systems representation.

The principal advantages of the approach are associated to the premise that an exact description for a continuous non-linear, causal, time-invariant system can be furnished by a Volterra functional series. Moreover, the application of neural networks for the systematic assessment of all Volterra kernels, facilitates the production of parametric models allowing fast evaluation of the system response.

Generalisation tests have shown good predictive capabilities of the identified model within the training limits. Delays in the response predictions demonstrated by the test cases have been the only significant imperfections related to the identified model. Such errors can be related to training deficiencies or poor training pattern. The identified model has also presented adequate performance in a test case where the input motion induced to unstable condition of the aircraft response. For the aims of this investigation these results can be considered satisfactory.

Extensions for the production of multiple-input, multiple-output system representations, application of the models in the analysis and control design of aircraft non-linear dynamics, and improvements to the neural network training algorithms will be the further steps of this research work.

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