ROBUSTIFICATION OF AN $\mathcal{H}_2$ AUTOPILOT FOR FLEXIBLE AIRCRAFT BY SELF–SCHEDULING BASED ON MULTI–MODEL EIGENSTRUCTURE ASSIGNMENT

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Abstract

This paper describes the synthesis of a self–scheduled controller robust with respect to parametric uncertainties. Traditionally, scheduling is done a posteriori by interpolation of gains computed for several points within the parameter space. The method proposed here, named (Mu-μ)–iteration, is based on worst–case analysis and multi–model eigenstructure assignment. It permits the designer to synthesize a robust low order controller by considering a priori its dependence of some scheduling parameters. The method is then applied to robustify an initial $\mathcal{H}_2$ autopilot for flexible aircraft satisfying all design specifications for a model with given mass and airspeed. It will be shown that the final controller satisfies all design criteria even when mass and airspeed vary significantly.

1 Introduction

The evolution in the aeronautical industry leads to high capacity aircraft development. The optimization of the design of these large aircraft, which includes high aspect ratio and new materials like composites, makes them become more flexible. This evolution increases the interaction between aeroelastic dynamics and control laws, known as aeroservoelasticity.

Classic static eigenstructure assignment techniques with output feedback [6] offer the possibility to decouple input/output transfers in that way that for example an airspeed command does not affect the vertical speed and vice versa. However, it may be unsuitable for the direct computation of flight control laws for modern aircraft because of the upcoming interaction of rigid and flexible modes [7].

$\mathcal{H}_2/\mathcal{H}_\infty$ control techniques are very powerful in satisfying frequency domain constraints via weighting functions. Controller activity can be minimized in some frequency intervals while passenger comfort is maximized. They present furthermore good robustness characteristics against dynamic uncertainties as neglected high order dynamics [1]. Unfortunately, their robustness with respect to parameter uncertainties, as airspeed, mass, centre of gravity location, altitude and so on, is poor [14, 2]. Synthesis methods dedicated to parameter robustness [14] are often too conservative and expensive in computational effort.

Modern modal control techniques keep the simplicity of eigenstructure assignment approaches while adding more degrees of freedom (d.o.f.) for synthesis by introduction of dynamics or self–scheduling. Dynamics are used to fix roll–off criteria as well as to structure the controller matrix, *i.e.* to avoid the injection of fast acceleration measurements in slow command signals like the engine command [3]. Dynamics are also used to assign several models simultaneously and hence to robustify an initial controller (static modal or dynamic $\mathcal{H}_2/\mathcal{H}_\infty$ controller) designed at one model with respect to unmeasurable parameter variations [4, 2] while preserving its nominal good time/frequency domain behaviour. Static self–scheduling allows to use measures (for example mass and airspeed) to robustify a controller with respect to those parameters [5] while reducing controller activity.

This paper will show that the use of dynamic self–scheduling permits to combine all these features. It is divided into two parts :

- The first part will present the modal multi–model techniques and their combination.

- In the second part the combination is successfully applied to an initial $\mathcal{H}_2$ autopilot for large flexible transport aircraft described in [1].

2 Notations

System: Consider a Linear Time Invariant (LTI) system with n states, m inputs and p outputs written in state space form

$$\begin{aligned} \dot{x} &= A(\Delta)x + B(\Delta)u \\ y &= C(\Delta)x + D(\Delta)u \end{aligned} \tag{1}$$

where x is the state vector, y the output vector and u the input vector. $A(\Delta) \in \mathrm{R}^{n\times n}$, $B(\Delta) \in \mathrm{R}^{n\times m}$, $C(\Delta) \in \mathrm{R}^{p\times n}$ and $D(\Delta) \in \mathrm{R}^{p\times m}$ are the systems state space matrices depending on the uncertainty matrix Δ. This matrix can be complex or real, structured or unstructured following the type of uncertainties considered during the synthesis. The input/output transfer of the system (1) can also be considered under the standard form (Linear Fractional Transformation LFT) of the upper part of Fig. 1.

Controller: The control feedback is defined by:

$$u(s) = K_{sched}(s, \Delta')\, y(s) \tag{2}$$

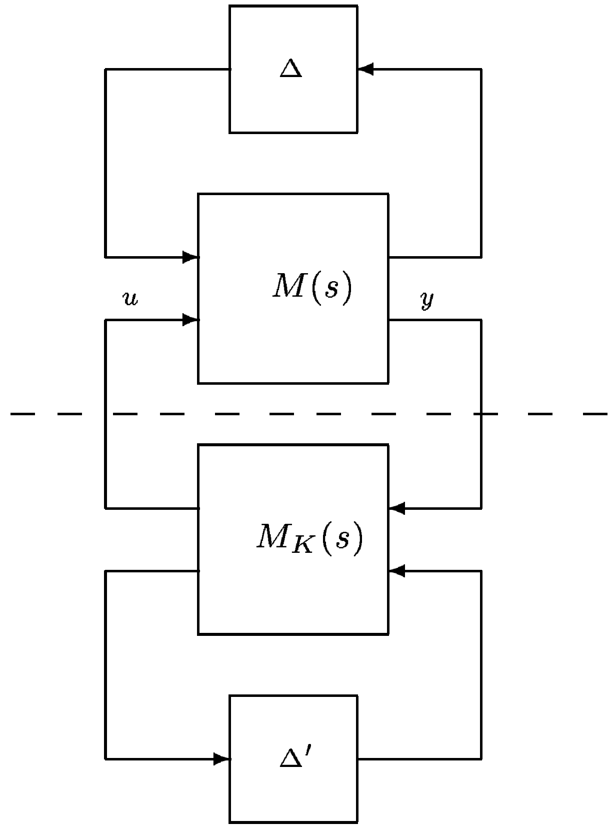

Fig. 1 LFT–formulation of the system controlled by a dynamic self–scheduled output feedback

where $K_{sched}(s, \Delta')$ is a dynamic scheduled gain. Δ' is the matrix of constant or slowly varying real parameters which are measured or at least observable and which will be used for scheduling. The elements of Δ' form a subspace of Δ.

3 Design procedure

3.1 Single–model modal control

In the non–scheduled case $K_{dyn}(s) = K_{sched}(s, \Delta')$ Proposition 3.1 from [9] generalizes the traditional eigenstructure assignment of [13] for the use of dynamic controllers.

Proposition 3.1 *The triple $\mathcal{T}_i = (\lambda_i, v_i, w_i)$ satisfying*

$$\begin{bmatrix} A(\Delta) - \lambda_i I & B(\Delta) \end{bmatrix} \begin{pmatrix} v_i \\ w_i \end{pmatrix} = 0 \tag{3}$$

is assigned by the dynamic gain $K_{dyn}(s)$ if and *only if*

$$K_{dyn}(\lambda_i)\underbrace{\left[C(\Delta)\,v_i + D(\Delta)\,w_i\right]}_{E(\Delta)} = w_i \qquad (4)$$

The input directions w_i and right eigenvectors v_i associated to the closed loop eigenvalue λ_i also depend on the perturbation matrix Δ, however this dependence is omitted in the formulae in order to simplify notation. They can be fixed by various methods. The method respecting mostly the system's natural behaviour is the *orthogonal projection* of the eigenvector $v_{i,ol}$ associated to the open loop eigenvalue $\lambda_{i,ol}$ on a vector v_i belonging to λ_i

$$v_i = V(\lambda_i)\,\left[V(\lambda_i)^T\,V(\lambda_i)\right]^{-1} V(\lambda_i)^T\,v_{i,ol} \qquad (5)$$

where $V(\lambda_i)$ and $W(\lambda_i)$ are two matrices with $\mathrm{rank}\,[V(\lambda_i)] = m$ such that

$$\left[\begin{array}{cc} A(\Delta)-\lambda_i I & B(\Delta)\end{array}\right]\left(\begin{array}{c} V(\lambda_i)\\ W(\lambda_i)\end{array}\right) = 0$$

See [12] for the theoretical background and further details. For aeronautical applications refer to [9, 10]. This projection is used in the following.

3.2 Multi–model modal control

Multi–model eigenstructure assignment [12] is done by simultaneously assigning triples $\mathcal{T}_i$ for several models which reduces to solve a set of equality constraints of type (4) for the transfer matrix $K_{dyn}(s)$ where its entries are

$$K_{ij,dyn}(s) = \frac{\mathcal{N}_{ij}(s)}{\mathcal{D}_{ij}(s)} = \frac{b_{ij,k}\,s^k + \cdots + b_{ij,1}\,s + b_{ij,0}}{a_{ij,k}\,s^k + \cdots + a_{ij,1}\,s + a_{ij,0}} \qquad (6)$$

The choice of the models to treat with and the triples to assign is determined by an analysis of the stability and/or performance robustness (see Steps B.1 and B.2). Common or different denominators $\mathcal{D}_{ij}(s)$ of the matrix $K_{dyn}(s)$ are fixed *a priori* with a sufficiently high order to realize the desired 'roll–off'. Furthermore, its degree must be chosen in order to offer enough numerator coefficients $b_{ij,k}$, the tuning parameters (degrees of freedom). Generally, this choice offers too much degrees of freedom for the resolution of the equality constraints so that the problem is solved by minimizing a criteria of type $||K_{dyn}(j\omega_i) - K_0(j\omega_i)||_F$ over a certain frequency interval ω_i where $K_0(s)$ is a reference controller (often a simple gain) synthesized for an initial model M_0. This reduces to minimize a quadratic criteria under linear constraints. The procedure used in the following will be called (Mu-μ)–iteration. See also Fig. 2.

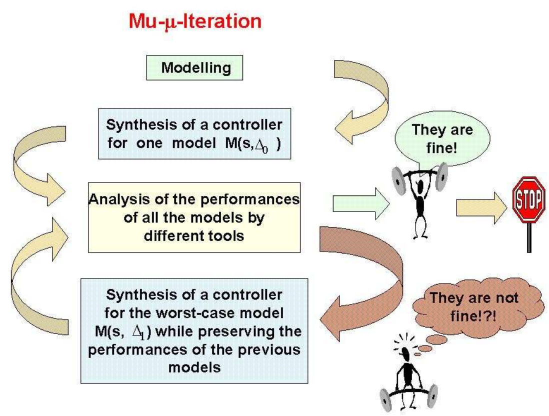

Fig. 2 The multi–model (Mu-μ)–Iteration

Procedure: (Mu-μ)–iteration

Step A.1 — Elaborate a first initial design on a nominal model M_0 with Δ_0. All kinds of synthesis methods can be applied at this step ($\mathcal{H}_\infty$ control, LQG optimal control, μ–synthesis, *etc.*..). In the case of initial non–modal approaches, look for an eigenstructure assignment having the same characteristics as the initial controller, see section 3.4.

Step B.1 — Proceed to a multi–model analysis of the pole map or a real μ–analysis like proposed in [11]. If the design is satisfactory for all values of Δ, then **stop**. Otherwise identify the *worst–case* model M_{wc}, determine its critical triple $\mathcal{T}_i$ and continue with **Step B.2**.

Step B.2 — Improve the behaviour of the *worst–case* model M_{wc} by replacing the $\mathcal{T}_i$ by $\mathcal{T}_i^*$ respecting the specifications while preserving the properties of all the models M_0, ..., M_{wc-1} treated before. Return to **Step B.1**.

Remark: See [8] for some general rules on multi–model eigenstructure assignment. For example to avoid incompatible assignments, models being as 'far' as possible from each other in the considered parameter space should be treated and/or some constraints on models treated before should be relaxed. Incompatible assignments will be found by a non–invertible matrix $E(\Delta)$ in Eq. (4).

In [9] the (Mu-μ)–iteration procedure is successfully applied to the design of a multi–input/multi–output longitudinal aircraft autopilot being robust with respect to parameter variations in mass m, centre of gravity x_{cg}, control signal time delay τ and airspeed V. The necessary degrees of freedom for this multi–model eigenstructure assignment have been achieved by the use of a second order dynamic controller $K_{dyn}(s)$.

3.3 Multi–model modal self–scheduling control

The controller $K_{sched}(s,\Delta')$ of the lower part of Fig. 1 depends on the varying parameters contained in Δ'. These parameters are measurable so that the interpolation formula can be calculated in real time. In classical interpolation schemes different controllers are first designed for several models and *a posteriori* interpolated. Non–interpolability can occur. A big advantage of the proposed approach is that the controller structure (*i.e.* the interpolation formula) is chosen *a priori* and hence taken in account during the design process. This choice is often set by physical constraints, can be supported by an open–loop analysis and is, in most cases, not restrictive.

Take for example a scheduling with respect to parameter δ_1 and an interpolation formula

$$K_{sched}(s,\delta_1) = K_0(s) + \delta_1 K_{\delta_1}(s) + \delta_1^2 K_{\delta_1^2}(s) \quad (7)$$

Then the synthesis of this controller can be done following Proposition 3.2 from [10].

Proposition 3.2 *The determination of such a self–scheduled controller is equivalent to the synthesis of a multi–model modal controller*

$$K_{dyn,eq}(s) = \begin{bmatrix} K_0(s) & K_{\delta_1}(s) & K_{\delta_1^2}(s) \end{bmatrix} \quad (8)$$

with respect to system

$$\left[A(\Delta), B(\Delta), \begin{pmatrix} C(\Delta) \\ \delta_1 C(\Delta) \\ \delta_1^2 C(\Delta) \end{pmatrix}, \begin{pmatrix} D(\Delta) \\ \delta_1 D(\Delta) \\ \delta_1^2 D(\Delta) \end{pmatrix} \right] \quad (9)$$

which is nothing else than increasing the number of outputs p of system (1), i.e. *here, the output number is finally $3p$.*

Hence, it is sufficient to apply our design procedure (Mu-μ)–iteration on the augmented system (9) for the controller $K_{dyn,eq}(s)$ and to extract from it the matrices $K_0(s)$, $K_{\delta_1}(s)$ and $K_{\delta_1^2}(s)$ for the realization of $K_{sched}(s,\delta_1)$.

The augmentation of the output number offers so the additional degrees of freedom necessary for the simultaneous resolution of some linear constraints of type (4), the same degrees of freedom formerly obtained by applying a dynamic feedback in Proposition 3.1. Therefore, it is possible to use static components K_i for $K_{dyn,eq}$. For example, the problem of [9] could have been treated by a self–scheduled controller of the following type

$$\begin{aligned} K_{sched}(\delta m,\, \delta x_{cg},\, \delta V,\, \delta\tau) = K_0 \\ + K_{\delta m}\,\delta m + K_{\delta m^2}\,(\delta m)^2 + \cdots \\ + K_{\delta x_{cg}}\,\delta x_{cg} + K_{\delta x_{cg}^2}\,(\delta x_{cg})^2 + \cdots \\ + K_{\delta V}\,\delta V + K_{\delta V^2}\,(\delta V)^2 + \cdots \\ + K_{\delta\tau}\,\delta\tau + K_{\delta\tau^2}\,(\delta\tau)^2 + \cdots \end{aligned}$$

supposing that all parameter variations are measurable.

The combination of both, augmentation of output number and application of dynamic feedbacks, will offer some more degrees of freedom so that the designer can still add more linear constraints of type (4) and/or ask for some bandwidth objectives or frequency response constraints. This is what is done for the application to the highly flexible aircraft in section 4.

Or, use self–scheduling for parameter variations which are easily measurable and dynamics to account for parameter variations which are not measurable. Assume that in the above example of [9] the time delay $\delta\tau$ is not measurable (which is indeed the case) then a controller with the following structure can be used

$$
\begin{aligned}
K_{sched}(s,\delta m,\delta x_{cg},\delta V) = K_0(s) \\
+K_{\delta m}(s)\,\delta m + K_{\delta m^2}(s)\,(\delta m)^2+\cdots \\
+K_{\delta x_{cg}}(s)\,\delta x_{cg} + K_{\delta x_{cg}^2}(s)\,(\delta x_{cg})^2+\cdots \\
+K_{\delta V}(s)\,\delta V + K_{\delta V^2}(s)\,(\delta V)^2+\cdots
\end{aligned}
$$

In all cases, the (Mu-μ)–iteration design procedure can be applied as introduced above with one minor adaptation. During the analysis step (**Step B.1**) the expression of the controller $K_{sched}(s,\Delta')$ has to be rewritten in an LFT (standard form) manner in order to extract from the overall system a generalized $(M-\Delta)$ form as described in Fig. 1.

3.4 Some precisions on the Step A.1 : an equivalent modal dynamic controller

If the initial feedback design stems from non–modal approaches, it is necessary to find an equivalent modal dynamic controller. A modal analysis of the closed loop system is performed in order to identify the dominant modes and associated eigenvectors following a systematic procedure :

1 — Plot the impulse response with respect to all modes,

2 — Calculate modal matrix Λ of the closed loop system,

3 — Perform a modal decomposition of the impulse response by Λ,

4 — Plot the contribution of all modes to the impulse response by histogram or time plots,

5 Choose the most contributing modes.

Let us recall that the time response of a linear system $\dot{x}=Ax+Bu$, $y=Cx$ is equal to

$$
y(t)=\sum_i C v_i \int_0^t e^{\lambda_i(t-\tau)}\,u_i B u(\tau)\,d\tau
$$

where λ_i and v_i are respectively the already defined eigenvalue and right eigenvectors, u_i are the so–called left eigenvectors. In order to detect the leading modes relative to a given input, it suffices to simulate separately each term

$$
C v_i \int_0^t e^{\lambda_i(t-\tau)}\,u_i B u(\tau)\,d\tau \qquad (10)
$$

Just the modes with non–negligible effect are taken into account.

4 Application to the design of a lateral flight control law for highly flexible aircraft

4.1 Model description and specifications

The models used in this study are linearized models of the lateral motion of a flexible aircraft around equilibrium points. The system is a large carrier aircraft in which flexibility was intentionally degraded in order to evaluate the relevance of control law synthesis techniques in highly critical cases. The models [1] are 60^{th}–order state–space representations with 2 control inputs (aileron deflection dp and rudder deflection dr) and 44 measurements: 4 measurements (lateral acceleration n_{y_i}, roll rate p_i, yaw rate r_i and roll angle ϕ_i) at 11 measurement points regularly spaced along the fuselage $(i=1,\ldots,11)$. The state vector x contains:

- 4 rigid states (yaw angle β, roll rate p, yaw rate r, roll angle ϕ),
- 36 states corresponding to 18 flexible modes modeled between 8 and $80\,rd/s$ (generalized coordinates q_j and $\dot{q}_j$, $(j=1,\ldots,18)$),
- 20 secondary states that represent the dynamics of the servo–control surfaces and aerodynamic lags.

The models are available for 3 different flight conditions (corresponding to 3 different airspeeds $V_1=250\,kts$, $V_2=320\,kts$ and $V_3=350\,kts$) and 6 different loading cases (corresponding to 3 mass cases with $m_1=277\,t$, $m_2=410\,t$ and $m_3=505\,t$ and the trim tank being empty $\Delta m=0$ or full $\Delta m=20\,t$).

The following list summarizes the various specifications:

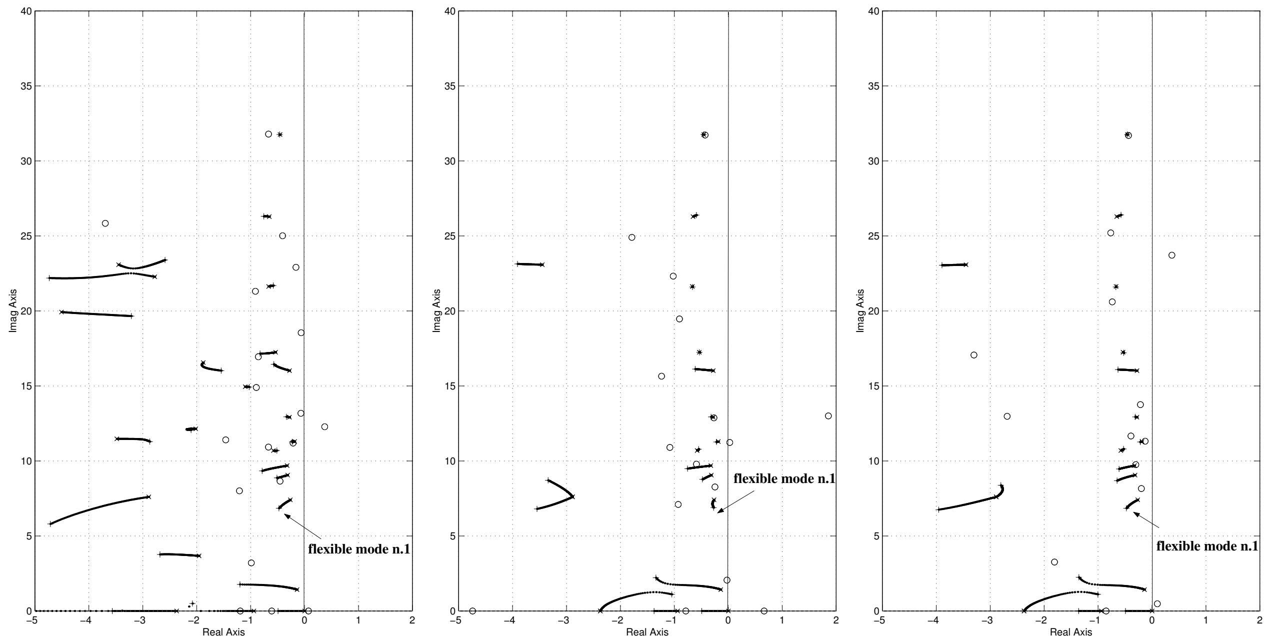

Fig. 3 Root loci at m_1 for the initial $\mathcal{H}_2$ controller (on the left), the intermediate (in the middle) and the final modal dynamic controller $K_0(s)$ (on the right)

- **S1**: Dutch roll damping ratio > 0.5,
- **S2**: templates on the step responses w.r.t. β and p,
- **S3**: roll/yaw channel decoupling,
- **S4**: no degradation of the damping ratios of flexible modes, or furthermore, an increase of the damping ratios of low frequency flexible modes in order to improve comfort during turbulence, especially, the accelerations n_y at all measurement points of the closed loop system have to be smaller than half the maximum value $n_{y,max}$ in open loop when the aircraft enters turbulences
- **S5**: the previous performances must be robust with respect to the various loading cases,
- **S6**: the previous performances must be robust with respect to the various flight conditions.

4.2 Initial $\mathcal{H}_2$ controller

The design method and the initial design of the $\mathcal{H}_2$ controller is described in [1]. It has been assigned to satisfy the design criteria **S1 to S5**, *i.e.* satisfying all performance criteria for the flight condition low speed V_1 on high mass model m_6 and most of the performance criteria for the other loading cases $m_1 - m_5$ at V_1. It is a 22^{th} order controller using the 4 measurements of measurement point 6.

4.3 Equivalent modal dynamic controller $K_0(s)$

At **Step A.1** of the (Mu-μ)–iteration, first the structure Eq. (6) of the equivalent modal dynamic controller $K_0(s)$ has to be chosen. Using 4 measurements and the 2 inputs defines a controller $K_0(s)$ matrix of 2×4. The dominant modes of the initial controller have to be identified in order to fix the denominators $\mathcal{D}_{ij}$. A modal decomposition using Eq. (10) of the controller's time responses reveals that the eigenvalues $\lambda_{c,1,2} = -17.64 \pm 17.59\,i$, $\lambda_{c,3,4} = -1.83 \pm 3.22\,i$ and $\lambda_{c,5,6} = -1.55 \pm 8.67\,i$ should be maintained. The $a_{ij,k}$ of $\mathcal{D}_{ij}$ are hence fixed with $k = 1,\ldots,6$. For simplicity, all $\mathcal{D}_{ij}$ are equal. To eliminate a direct transfer, the $\mathcal{N}_{ij}$ order is set to 5 which offers $5\,(2\times 4)$ design parameters $b_{ij,k}$.

The modal analysis of the system's behaviour when fedback with the initial $\mathcal{H}_2$ controller — using again Eq. (10) — delivers the eigenvalues which have to be assigned in the closed loop by $K_0(s)$. For the model M_0 with m_6 it is sufficient to place the rigid body modes as

- the dutch roll mode with $\lambda_{1,2} = -0.9670 \pm 1.7670\,i$
- the spiral and roll mode with $\lambda_{3,4} = -0.8796 \pm 0.0542\,i$

and the corresponding vectors v_i and w_i assigned by the initial controller. $K_0(s)$ is computed using 2 equations of type (4). A multi–model analysis (**Step B.1**) via root loci for all mass cases shows that the first flexible mode at about $9.5\,rad/s$ has been destabilized in the low mass configuration m_1. See the root locus in the middle of Fig. 3. The real part of its eigenvalues $\lambda_{flex,1}$ is shifted to the right (from -0.8 to -0.5). A worst–case model $M_{wc,1}$ is detected. In order to improve its behaviour, $\lambda_{flex,1}$ should be shifted back to its initial value for model m_1 while projecting the open loop eigenvector $v_{flex,1,ol}$ using Eq. (5). In **Step B.2** the worst–case behaviour is improved while still assigning the rigid body modes for model m_6 using now 3 equations of type (4), 2 for model m_6, 1 for model m_1.

On the right root locus of Fig. 3, it can be observed that the first flexible mode is indeed shifted back to the left as the initial controller does. The resulting 6^{th} order $K_0(s)$ is equivalent to the initial 20^{th} order $\mathcal{H}_2$ controller as additionnally shown on Fig. 4. The frequency responses of lateral acceleration n_y corresponding to passenger comfort in wind turbulences at three different measurement points are depicted. On the right hand, the responses for the model m_6 are given, on the left hand side those for model m_1. The dash–dotted lines correspond to the initial $\mathcal{H}_2$ controller, the solid lines to the equivalent modal dynamic controller $K_0(s)$, the dotted lines correspond to the open–loop behaviour. For model m_6, the two responses are almost equal. The peaks are indeed smaller than half the open–loop peak response (see criteria **S4**). For model m_1, controller $K_0(s)$ fits very well with the initial one, at frequencies of about $11\,rad/s$ it is even better, *i.e.* the peak is at least not higher than the open–loop one. These results have also been confirmed by time plots.

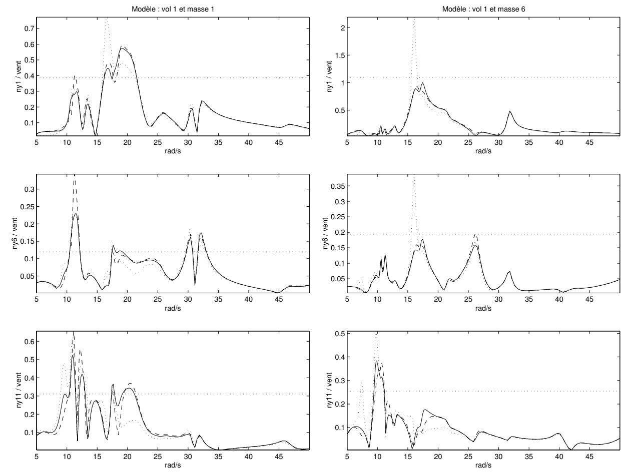

Fig. 4 Frequency domain responses of lateral acceleration $n_{y,i}$ of the open–loop (dotted lines), the initial $\mathcal{H}_2$ controller (dash–dotted lines) and the equivalent modal dynamic controller $K_0(s)$ (solid lines) for the two models m_1 (left) and m_6 (right)

4.4 Extension of the initial controller to flight conditions V_2 and V_3

The initial controller has been assigned for a good compromise for the different loading cases at flight condition V_1. The frequency domain responses of $n_{y,i}$ on Fig. 5 show that the initial $\mathcal{H}_2$ controller (dashed line) does not perform well at high speed V_3, especially at mass m_1.

To improve its behaviour for speed variations without deteriorating its initial performances, a dynamic self–scheduled controller of type (7) using airspeed variation δV as scheduling variable is designed. The interpolation formula

$$K(s,\delta V) = K_0(s) + K_V(s)\,\delta V + K_{V^2}(s)\,\delta V^2 \quad (11)$$

is chosen which is inspired by the fact that all aeronautical forces and moments depend on the dynamic pressure $\frac{\rho}{2}V^2$. In order to offer enough design parameters $b_{ij,k}$, $K_V(s)$ and $K_{V^2}(s)$ are dynamic matrices of size 2×4. For the missile example treated in [5], it was sufficient to use static terms K_i. For simplicity, their denominators $\mathcal{D}_{ij}$ are fixed in the same way.

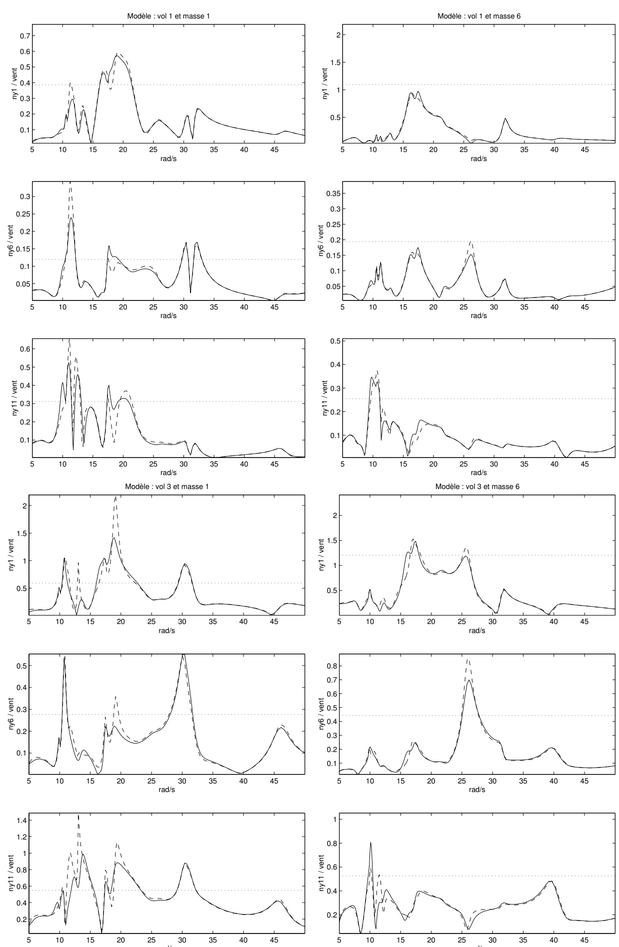

Fig. 5 Frequency domain responses of lateral acceleration $n_{y,i}$ of the initial $\mathcal{H}_2$ controller (dashed line) and the dynamic modal self–scheduled controller $K_{sched}(s, \delta V)$ (solid line) for the two models m_1 and m_6 at speeds V_1 and V_3

On root loci corresponding to the system fed back by the initial controller, it can be observed that a cluster of flexible modes between 10 and $20\,rad/s$ shifts to the right, when the airspeed passes from V_1 to V_3. This shift can be avoided by placing the $K_V(s)$ and $K_{V^2}(s)$ poles at $-0.9 \pm 10i$ and $-2 \pm 17i$. As before, we start the multi–model design procedure of section 3.2 with model M_0 at high mass m_6 and low airspeed V_1. The rigid modes corresponding to dutch roll and roll and spiral mode are fixed as in section 4.3. Applying Prop. 3.2 and Prop. 3.1 a first controller $K_{sched}(s, \delta V)$ is computed. A multi–model analysis for all mass and speed cases (**Step B.1**) highlights that for the model $M_{wc,1}$ at low mass m_1 and high airspeed V_3 a flexible mode at about $17\,rad/s$ shifts to the right. To improve this behaviour, at **Step B.2** the rigid modes are placed for model M_0 and simultaneously, the flexible mode $\lambda_{flex,m_1,V_3} = -0.2628 \pm 17.4015\,i$ for model $M_{wc,1}$ applying 3 equations of type (4), 2 for model M_0 and 1 for model $M_{wc,1}$. A second multi–model analysis (**Step B.1**) reveals that a second worst–case appear : it is the model for high mass m_6 and high speed V_3. The destabilized flexible mode is assigned at $\lambda_{flex,m_6,V_3} = -0.7776 \pm 9.2734\,i$ for model $M_{wc,2}$ while maintaining the constraints for $M_{wc,1}$ and M_0 applying 4 equations of type (4 at **Step B.2**. A last multi–model analysis proofs that the final controller $K_{sched}(s, \delta V)$ of order 14 satisfies the design criteria for all flight configurations in mass and speed. See the solid line on Fig. 5. Especially the high frequency response peak at about $20\,rad/s$ in $n_{y,1}$ could have been divided by two as well as the peak at $14\,rad/s$ in $n_{y,11}$ could have been reduced to 75% of its initial height without deteriorating significantly the initial frequency responses. This improved behaviour can also be checked via time responses and root loci.

4.5 Additional dynamic self–scheduling with respect to mass variations δm

In order to refine the performances in the different loading cases m_i, an additional term $K_m(s)\,\delta m$ is added in the interpolation formula (11) for a new self–scheduled controller $K_{sched}(s, \delta V, \delta m)$ with respect to speed and mass variations. A dynamic 2×4 controller matrix $K_m(s)$ is chosen with four controller poles fixed to $-2.8 \pm 12.8\,i$ and $-3.2 \pm 22.5\,i$ following the same approach has for K_V and K_{V^2} in section 4.4. Two additional flexible modes at $10\,rad/s$ and $30\,rad/s$ can be assigned for the model $M_{pc,1}$ at low mass m_1 and high speed V_3 while maintaining the assignments of section 4.4. The system's behaviour could have been still improved with a 18^{th}–order controller (which is still smaller than the 22^{th}–order initial controller), especially for model $M_{pc,1}$ and also the intermediate models in mass m_2 – m_5 which is checked on the frequency response plots as well as time plots and root loci.

5 Conclusions

In this paper, first the classical and then the multi–model modal control design methods like dynamic and self–scheduled approaches are presented. Their different application fields are pointed out reaching from a simple controller synthesis to the retuning of already existing controllers. Retuning means improvement of global controller performances including order reduction and restructuring. These methods have been successfully applied to the retuning of an initial 22^{th}–order $\mathcal{H}_2$ controller for an highly flexible transport aircraft. Its excellent performances for high mass at low speed could have been extended to all flight conditions, *i.e.* low and high mass at low and high speed. The controller order could have been reduced to 18 at the same time.

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