# FROM BASEL TO ST. PETERSBURG AERONAUTICAL AND SCIENTIFIC MILESTONES: DANIEL BERNOULLI AND LEONHARD EULER A RUSSIAN-SWISS HERITAGE

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#### **Abstract**

The paper adduces some facts about the history of aeronautical sciences. The purpose is to explore and compare the activity of various generations of scientists, who made valuable contributions to the development of aeronautical sciences during XVIII-XX centuries.

The paper aims to explore how the scope of scientific activity of the outstanding researchers changed over time and which factors influenced their output.

## 1 International Scientific Lives – Globalization in the Past

The names "Bernoulli" and "Euler" are widely known in aeronautics and fluid mechanics theory.

Both scientists have given their names to fundamental equations, which are used today to demonstrate the pressure (energy) conservation in fluids and model the inviscid fluid flow. Both Bernoulli and Euler are of Swiss origin, from Basel. However, they have realized their scientific achievements in the scientific community of St. Petersburg.

Their scientific life must be seen in the context of the time: the XVIII century laid down the basis of the intense and broad scientific development, which was succeeded in the XIX century and beyond by an increasing number of scientists.

The early XVIII century presented famous scientists like, for example, Russian Mikhail Lomonosov and Swiss Horace-Bénédict de Saussure.

Bernoulli and Euler must be seen in the context of their tradition. It is a characteristic of the scientific and technical communities that there has always been an intense exchange of information and continuous contacts between the individuals. In our aerospace community such contacts are particularly important and fruitful. ICAS is an example of lived practice.

# 2 Aeronautical Sciences – Way to a Broad Field of Activity

Following the traditions of Bernoulli and Euler, L. Prandtl and N. I. Zhukovsky created modern aeronautical science.

Their successors were engineers and scientists as well: the professors Chaplygin and Ackeret had made important contributions to aerospace science, development of transonic aerodynamics and management of aeronautical centers such as the large TsAGI and the small Institute of Aerodynamics at the ETH in Zurich. According to the size of the countries one was large and the other one was small, but both people were dedicated to progress of aeronautics.

Chaplygin and Ackeret were active in theory as well as with experiments. Ackeret was internationally active whereas Chaplygin worked only in Russia. The similarities are striking, nevertheless.

Nature and extension of the sciences evolved enormously at all times. Lomonosov and de Saussure were active in an extremely wide field, which included many sciences as well as culture at the same time.

They were called "Universal-Gelehrte" or "universal scholars". Some specialization can already be noticed with Bernoulli and Euler as will be shown later on. Finally, Chaplygin and Ackeret have dedicated their efforts mostly to aerospace, which was still a relatively small field of activity at that time.

Today, aeronautical science is extremely wide and the specialization is progressing as shown by the number of topics at the ICAS Congresses and the authors involved.

Another interesting question is "Why human flying started only after 150 years passed since the creation of the first and fundamental laws of fluid mechanics?". It seems obvious that early flight was much more the undertaking of courageous and practical people than of scientists. Also the early ideas of human flight (originating from Leonardo da Vinci) did not inspire to establish a bridge in between.

# 3 "Founding fathers" of aeronautical sciences.

# 3.1 Mikhail-Vasilyevich Lomonosov (1711-1765)



Fig. 1: Mikhail Lomonosov

Lomonosov (shown in Fig. 1) was born in Denisovka village (Arkhangelsk province) in 1711. He studied mineralogy and metallurgy in Germany in Marburg and Freiburg. In 1745 Lomonosov has become the Professor of chemistry at St. Petersburg. This is the period when he devotes a lot of attention to the experimental activities.

In 1748 Lomonosov formulated the law of "conservation of matter", which is regarded as one of his most famous postulates and reads as follows: "All changes in nature are such that inasmuch is taken from one object insomuch is added to another. So, if the amount of matter decreases in one place, it increases elsewhere. This universal law of nature embraces laws of motion as well, for an object moving others by its own force in fact imparts to another object the force it loses."

In 1750s Lomonosov devoted a lot of time to physics while developing wave theory and kinetic theory of gases.

In 1760 he also became engaged a lot with astronomy and, in particular, conducted the observations of Venus' atmosphere.

Lomonosov was famous for a multitude of his talents, which included artistic traits besides scientific expertise. He was famous both as a chemist, physicist, astronomer, geographer, geologist, mosaicist and poet.

# 3.2 Horace-Bénédict de Saussure (1740-1799)

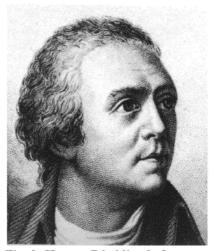


Fig. 2: Horace-Bénédict de Saussure

De Saussure (shown in Fig. 2) was born in Conches (Switzerland) in 1740. He defended his dissertation "Sur la Nature du Feu" at the Académie in 1759. De Saussure published the book, which described his experience of

climbing the mountains, titled *Les Voyages dans les Alpes* in 1779. In 1783 De Saussure conducted the research on hygrometry and measurement devices. In 1784 he met de Montgolfier and Pilâtre de Rozier in Lyon. In 1787 he conducted barometric measurements at the peak of Mont Blanc.

De Saussure also carried out further and lightning conductor geology studies experiments. He developed such instruments as the anemometer. magnetometer and cyanometer. De Saussure extended his mountaineering activities and published even more accounts of his journeys. He also constructed the first solar oven.

## 3.3 Daniel Bernoulli (1700-1782)



Fig. 3: Daniel Bernoulli

Bernoulli (shown in Fig. 3) was born in Groningen as a son in a family of physicists and mathematicians in 1700, soon after his birth the family went to Basel. Bernoulli studied medicine at the University of Basel, Heidelberg and Strasbourg. He earned a PhD in anatomy and botany in 1721. In 1724 Bernoulli was honored by the Académie des sciences in Paris. Being a close friend of Euler, Bernoulli decided to go to St. Petersburg in 1725 as a Professor of mathematics. In Russia he works a lot on his main scientific work titled Hydrodynamica (Hydrodynamics). After having some health problems, Bernoulli returns to Basel in 1733 where he successfully worked as a Professor of anatomy, botany and physics at the University of Basel until his death in 1782.

Bernoulli's main scientific postulates include the concept of conservation of energy within the flow and relation between the variations of pressure and velocity, leading to the well known Bernoulli equation (6).

## 3.4 Leonhard Euler (1707-1783)



Fig. 4: Leonhard Euler

Euler (shown in Fig. 4) was born in Basel in 1707. He studied at the University of Basel and had separate lessons in mathematics from Johann Bernoulli (the father of Daniel Bernoulli). By the offer of Daniel Bernoulli (after the death of his brother Nicolas), Euler moved to St. Petersburg in 1727 and took up a position at the Mathematics Department of the Imperial Russian Academy of Sciences. In 1733 he succeeded Daniel Bernoulli as the head of mathematics department.

In 1741 Euler moved to Berlin Academy. Euler published a lot of works, of which *Mecanica (Mechanics)* is the most important one for aerospace sciences.

In 1766 Euler decided to move back to St. Petersburg where he lived until his death in 1783.

Besides formulating the very famous Bernoulli equation, Euler was also the first scientist to describe the equations of mechanics of fluids without friction.

The other notable publications of Euler include the following: *Principles of Motion of Fluids* (1752), *General Principles of State of Equilibrium of Fluids* (1753) and *General Principles of Motion of Fluids* (1755).

Euler considered pressure as a point property that could vary from point to point

throughout a fluid flow. He derived the differential equation, which describes the connection between pressure and velocity for inviscid flow. In particular Euler equations describe the conservation of mass (1), impulse (2) and energy (4):

$$\frac{\partial \rho}{\partial t} + \nabla (\rho V) = 0 \tag{1}$$

where:

 $\rho$  is the density of the fluid;

*t* is the time, which is used to describe the changes in the fluid flow;

 $\nabla$  is the Hamiltonian operator for the three-dimensional space;

V is the speed of the fluid flow.

$$\begin{cases} \rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} \\ \rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} \end{cases}$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z}$$
(2)

where:

p is the pressure in the fluid flow;

x,y,z are the coordinates along the axes, which form a three-dimensional coordinate system used for the description of changes of properties within the fluid flow;

u, v, w are the components of the fluid flow velocity vector  $\overline{V}$  (3):

$$\overline{V} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{3}$$

$$\rho \frac{D(e+V^2/2)}{Dt} = -\nabla(pV) \tag{4}$$

where:

e is the energy of the fluid flow.

These equations are based on second Newton's law (5):

$$F = M \frac{d^2 x_m}{dt^2} \tag{5}$$

where:

*F* is the force, which is applied to the body; *M* is the mass of the body;

 $x_m$  is the coordinate of the axis, which coincides with the direction of the body movement.

Euler integrated the differential equations (1), (2) and (4) yielding the first derivation of what we call Bernoulli's equation. Therefore the credit for Bernoulli's equation (6) should be shared by Euler.

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{6}$$

where:

 $p_1$ ,  $p_2$  are the values of pressure in points 1 and 2 along the fluid flow;

 $V_1$ ,  $V_2$  are the values of fluid flow speed in points 1 and 2.

Further application of Bernoulli's equation was made first by Joseph Lagrange, who first published his fundamental work on analytical mechanics in 1788. Pierre-Simon Laplace made simplifications and approximations of the mentioned above equation and published his works in 1789. And, of course, Claude-Louis-Marie-Henri Navier and George Stokes were the ones to adduce the full equation of motion of fluid substances with taking into account the friction and thus extending Euler's equations (around 1840).

## 4. Successors

#### 4.1 Sergey Chaplygin (1869 – 1942)

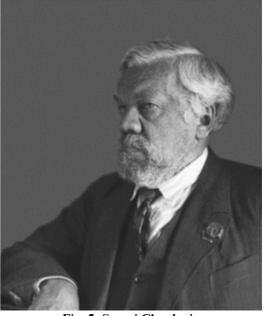


Fig. 5: Sergei Chaplygin

Chaplygin (shown in Fig. 5) was born in Ranenburg (Ryazan province) in 1869. He graduated from Moscow University in 1890.

The Doctor thesis of Chaplygin, which he defended 1902, *About gas streams* has illustrated the possibility of reducing the general problem of stationary two-dimensional isentropic movement of compressible gas to the solution of one linear partial differential equation (7). This publication was one of the founding works of gas dynamics.

$$\frac{\partial^2 \Phi}{\partial \Theta^2} + \frac{\upsilon^2}{1 - \frac{\upsilon^2}{\sigma^2}} \frac{\partial^2 \Phi}{\partial \upsilon^2} + \upsilon \frac{\partial \Phi}{\partial \upsilon} = 0 \tag{7}$$

where:

- $\Theta$  is the angle between the fluid particle velocity and *x*-axis;
- $\nu$  is the fluid particle speed;
- c is the speed of sound;
- $\Phi$  is the function, which is defined by equation (8):

$$\Phi = -\varphi + x\nu_{x} + y\nu_{y} \tag{8}$$

where:

 $\varphi$  is the velocity potential, which is defined by equation (9):

$$\varphi = -\Phi - \upsilon \frac{\partial \Phi}{\partial \upsilon} \tag{9}$$

x, y are the coordinates of the fluid particle, which are calculated according to the system of equations (10):

$$\begin{cases} x = \frac{\partial \Phi}{\partial v_x} = \cos \Theta \frac{\partial \Phi}{\partial v} - \frac{\sin \Theta}{v} \frac{\partial \Phi}{\partial \Theta} \\ y = \frac{\partial \Phi}{\partial v_y} = \sin \Theta \end{cases}$$
 (10)

 $v_x, v_y$  are the components of fluid particle velocity vector, which are calculated according to equations (11) and (12):

$$v_{r} = v \cos \Theta \tag{11}$$

$$\upsilon_{\nu} = \upsilon \sin \Theta \tag{12}$$

From 1893 Chaplygin taught mechanical engineering and applied mathematics at several educational institutions including Moscow State University.

In 1918-1919 Chaplygin was the first Rector of the 2<sup>nd</sup> Moscow State University. This

University was afterwards divided into a number of large independent Universities, which still exist today.

In 1918 Chaplygin was invited by Zhukovsky to start work on the creation of TsAGI (**Ts**entralniy **A**erogidrodinamicheskiy **I**nstitut).

In 1924 he was elected as a corresponding member of the Russian Academy of Sciences and got full member status in 1929.

In 1928-1931 Chaplygin was the Director of TsAGI.

In 1931-1941 Chaplygin was in charge of the creation of TsAGI aerodynamics laboratories.

In 1910 Chaplygin provided an analytical derivation for the Zhukovsky law formula (13):

$$\overline{F}_{lift} = -\rho_{gas} \overline{V}_{gas\infty} \times \overline{\Gamma} l \tag{13}$$

where:

 $\bar{F}_{lift}$  is the lifting force;

 $\rho_{gas}$  is the density of the gas;

 $\overline{V}_{\rm gas\infty}$  is the velocity of undisturbed uniform gas flow:

 $\overline{\Gamma}$  is the circulation of gas flow velocity;

is the length of the wing section, which is perpendicular to the plane that contains the profile cross-section.

Simultaneously with Zhukovsky himself Chaplygin proved that circulation  $\Gamma$  in the Zhukovsky law is uniquely determined by the finiteness of the speed at the trailing edge of the wing profile (this statement is now known as Zhukovsky – Chaplygin postulate).

The postulate goes as follows: "Of all possible kinds of streamlining of a sharp trailing edge wing the only ones, which are implemented in nature, are those that provide finite values of the trailing edge speed."

In 1913 Chaplygin published one of the world's first works in the finite span wing theory. He made the emphasis on taking the tip vortices into the account.

Chaplygin also attempted to derive the lifting force and induced drag formulas. The results of his research were not confirmed by Zhukovsky experiments, but were afterwards used in "Prandtl's inductive theory".

In 1919 Chaplygin offered a method of approximate integration of differential equations while proving an original inequality theorem ("Chaplygin theorem").

Chaplygin headed the group of scientists from TsAGI, who founded SibNIA in 1941, and died in Novosibirsk in 1942.

## 4.2 Jakob Ackeret (1898 – 1981)



Fig.6: Jakob Ackeret

Ackeret (shown in Fig. 6) was born in Zurich, Switzerland, in 1898. He received his diploma in mechanical engineering and a doctorate from ETH Zurich (Swiss Federal Institute of Technology) in 1920 where he worked as an assistant of Professor Stodola. Since 1921 Ackeret worked on aerodynamic research and became a department head in Göttingen under the supervision of Professor L. Prandtl.

Returning to Switzerland in 1927 Ackeret became chief engineer at the Escher-Wyss company and in 1930 he created the Institute of Aerodynamics at the ETH and habilitated as a Professor there.

Ackeret was one of the pioneers in the theoretical and experimental investigation of supersonic flow about airfoils and in wind tunnels. In 1925 he published the small perturbation theory of supersonic flow of a perfect gas over a thin airfoil, which is often described as "Ackeret Theory". Fig. 7 illustrates the experiments, which were carried out by Ackeret in this field.

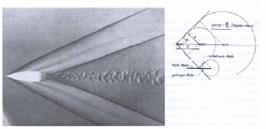


Fig. 7: Visualization of supersonic stream similar to the experiments which were carried out by Ernst Mach, leading Ackeret to introduce the Mach-number for the relationship between speed of the projectile and the speed of sound [3]

The simplification of the non-linear Euler equations for inviscid flow finally led to the simple linear equations (14) and (15), which can be used for calculation of lift and drag coefficients in a supersonic flow for thin airfoils at small angles of attack:

$$C_l = \frac{4\alpha}{\left(V_{gas\infty}^2 - 1\right)^{1/2}} \tag{14}$$

where:

 $C_l$  is the lift coefficient;

 $\alpha$  is the angle of attack;

 $V_{gas\infty}^2$  is the speed of undisturbed uniform flow.

$$C_d = \frac{f(\alpha, t)}{\left(V_{gas\infty}^2 - 1\right)^{1/2}} \tag{15}$$

where:

 $C_d$  is the drag coefficient;

 $f(\alpha,t)$  is some function of the angle of attack and body shape/thickness (the values of this function and their distribution depend on the shape of the body)

At the ETH Ackeret designed, built and operated the world's first supersonic closed-circuit wind tunnel (shown in Fig. 8), which was driven by one of the first multi-stage axial compressors ever created. Such compressor permitted independent variation of the values of Reynolds and Mach numbers. Soon afterwards a similar wind tunnel was built according to Ackeret's design in Guidonia, Italy. This tunnel was capable of attaining Mach 4. Ackeret introduced the term "Mach number" at his inaugural lecture at the ETH in 1929, which was first published in the SBZ (Ref. 3).

By the beginning of WW II Ackeret was the leading European authority on supersonic flows.

Ackeret died in Küsnacht, Switzerland in 1981.

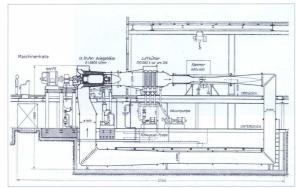


Fig. 8: The first closed-cycle supersonic wind tunnel at the ETH in Zurich, 1934.

#### **5** Conclusions

Advances in research and development are often made in parallel. This was shown above on the examples of some great scientists. These scientists have sometimes worked independently and sometimes collaborated in close contact while attaining these results.

The early scientists were active in an extremely broad field: they worked and were famous as scientists, artists, writers, poets.

With time progressing, the scientists became more specialized in one area of physics (e.g. fluid mechanics). They also very often worked as engineers (Ackeret) and stimulated designs or managed institutes directly (Chaplygin).

Fundamentally, scientists needed an excellent environment (St. Petersburg) and continuous exchange of information and experience (Lomonosov, Bernoulli, Euler, Ackeret) to produce the appropriate results.

ICAS always was and still remains in the tradition of such information exchange experience and environment.

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