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NUMERICAL SIMULATION AND PARAMETRIC STUDY OF SUPERSONIC WING FLUTTER

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Abstract

Wing flutter phenomenon is one of the important problems in supersonic aircraft design mainly when the airplane passing through the speed of sound. The main goal of this article is assessing wing flutter in supersonic regime and studying the effective parameters role in flutter event. Several wings with different sweepback angles and materials in a range of altitudes are considered. Supersonic flutter characteristics are calculated using K-method and validated by experimental data where a good agreement is observed. The results also show how the various parameters may change the supersonic wing flutter characteristics, i.e. speed and frequency.

1 Introduction

Aeroelasticity is the study of the effect of aerodynamic forces on elastic bodies. One of the interesting problems in aeroelasticity is the stability of structures such as airplanes in flight, when small disturbances of an incidental nature induce more or less violent oscillations. It is characterized by the interplay of aerodynamic, elastic, and inertia forces, and is called a problem of aeroelastic instability. Phenomena like divergence and flutter are the examples of static and dynamic instabilities. Calculation of flutter characteristics of aerospace vehicles is one of the main problems that aeroelasticians were faced during last decades and numerous methods have been developed and successfully employed for this purpose [1-2]. Due to such developments, flutter calculations can be performed now via the commercial packages which are based on advanced computational techniques. One of these softwares is ZAERO [3] that recently has been used by many designers and researchers. For instance, Lee and Weisshaar [4] have investigated the flutter characteristics of folding wings using ZAERO and MSC. Nastran. They showed that the flutter dynamic pressure increases with the folding angle. Canfield [5] has implemented ZAERO for flutter speed and frequency calculations to determine the most critical natural modes of vibration for F-16 ventral fin and design piezoelectric actuators for reducing buffet-induced vibrations. In his work, a finite element model (FEM) of the fin was developed, tuned and optimized to closely match published modal frequencies. In another research, ZAERO was used for validation of the results of a program that was developed by Henshaw et al [6]. The program has provided a set of methods to allow realistic consideration of non-linearity in the aeroelastic design and qualification of aircraft.

There are different effective parameters in the wing's dynamic instability, such as, wind speed, flow regime, altitude, center of mass position, materials, geometry and effect of body presence. In the present work, flutter characteristics of supersonic wings will be investigated and the effects of material type, altitude and sweepback angle will be studied. The flutter characteristics will be calculated by ZAERO software package, K-method is used for the calculation of flutter characteristics and lifting surface theory is used for unsteady supersonic flow modeling. The results will be compared with the experimental data and the ZAERO software's capabilities will be demonstrated and discussed by means of comparisons. Finally, some concluding remarks will be presented.

2 Governing Aeroelastic Equations

Aeroelastic response of flight vehicle is a result of the mutual interaction of inertial and elastic structural forces, aerodynamic forces induced by the static or dynamic deformation of the structure, and external disturbance forces. The equation of motion for the aeroelastic system in terms of discrete system can be derived based on the equilibrium condition of these forces:

$$\bar{M}\ddot{x}(t) + \bar{K}x(t) = F(t) \qquad (1)$$

where $\bar{M}$ and $\bar{K}$ are the mass and stiffness matrices generated by the structural finite element method, and **x**(t) is the structural deformation. **F**(t) can be generally divide into two parts; the aerodynamic forces induced by the structural deformation and external forces:

$$F(t) = F_{aero}(x) + F_e(t) \qquad (2)$$

Figure 1 presents a functional diagram that illustrates the aeroelastic interaction of these structural and aerodynamic forces.

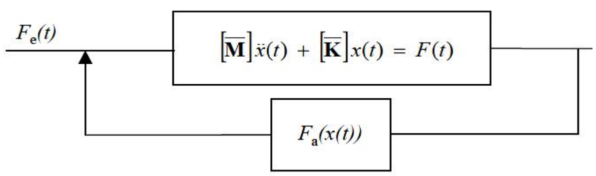

Figure 1. Aeroelastic functional diagram

The flutter matrix equation can be readily transformed into the Laplace domain and results in an eigenvalue problem in terms of s:

$$\left[s^2\bar{M} + \bar{K} - q_\infty \bar{H}\left(\frac{sl}{V}\right)\right]x(s) = 0 \qquad (3)$$

where H represents the aerodynamic transfer function and l is the reference length. Since the finite element model of aircraft structure normally contains a large amount of degrees of freedom, the size of the mass and stiffness matrices are usually very large. Hence, solving the eigenvalue problem of equation (3) directly would be computationally costly. To circumvent this problem, one introduces the "*modal approach*" which can be expressed as:

$$x = \varphi q$$

where φ is the modal matrix whose columns contain the lower order natural modes. Normally, less than ten numbers of the lowest natural modes are sufficient for the flutter analysis of a wing structure. For the whole aircraft structure, fifty natural modes are usually sufficient [3]. **q** is the generalized coordinates which are the eigenvectors to be determined. The equation is presented in terms of the generalized mass matrix **M**, the generalized stiffness matrix **K**, the generalized aerodynamic force matrix **Q**, and the generalized coordinate **q** and can be rewritten as:

$$\left[s^2 M + K - q_\infty Q\left(\frac{sl}{v}\right)\right]q = 0 \qquad (4)$$

where

$$M = \varphi^T \bar{M} \varphi \qquad (5)$$

$$K = \varphi^T \bar{K} \varphi \qquad (6)$$

$$Q\left(\frac{sL}{V}\right) = \varphi^T \bar{H}\left(\frac{sL}{V}\right) \qquad (7)$$

Introducing a non-dimensional Laplace parameter **p** as below:

$$p = \frac{sl}{v} = (\gamma k + ik) \qquad (8)$$

where k is the reduced frequency $k = \omega l/V$ and ω is the harmonic oscillatory frequency, l is the reference length, and V is the velocity of undisturbed flow. Then equation (4) becomes:

$$\left[\left(\frac{V}{L}\right)^2 M^2 p^2 + K - q_\infty Q(p)\right]q = 0 \qquad (9)$$

The basic equation for flutter analysis by the K-method can be written as following:

$$\left[-\omega^2 M + (1 + ig_s)K - q_\infty Q(ik)\right]q = 0 \qquad (10)$$

Eq. (10) is obtained by replacing **p** by ik in Eq. (9), where ig_s is the added artificial complex structural damping that is proportional to the stiffness. K-method equation can be obtained by substituting dynamic pressure equation into Eq. (10) and dividing the resultant equation by ω^2:

$$\left[M + \frac{\rho}{2}\left(\frac{L}{k}\right)^2 Q(ik) - \lambda K\right]q = 0 \qquad (11)$$

where $\lambda = (1 + g_s)/\omega^2$.

If rigid body modes exist, Eq. (11) cannot be solved directly since it contains some trivial solutions. Therefore, it is necessary to eliminate these trivial solutions by partitioning Eq. (11) into rigid body and elastic modes:

$$\begin{bmatrix} M_{rr} & 0 \\ 0 & M_{ll} \end{bmatrix} \begin{Bmatrix} q_r \\ q_l \end{Bmatrix} + \frac{\rho}{2}\left(\frac{L}{K}\right)^2 \begin{bmatrix} Q_{rr} & Q_{rl} \\ Q_{lr} & Q_{ll} \end{bmatrix} \begin{Bmatrix} q_r \\ q_l \end{Bmatrix} - \lambda \begin{bmatrix} 0 & 0 \\ 0 & K_{ll} \end{bmatrix} \begin{Bmatrix} q_r \\ q_l \end{Bmatrix} = 0 \quad (12)$$

where the subscripts r and l denote the rigid body modes and elastic modes, respectively. Since:

$$\{q_r\} = -\bar{M}_{rr}^{-1}\bar{M}_{rl}\{q_l\} \quad (13)$$

Equation (12) can be reduced to:

$$\left[\left[-\bar{M}_{lr}\bar{M}_{rr}^{-1}\bar{M}_{rl} + \bar{M}_{ll}\right] - \lambda[K_{ll}]\right]\{q_l\} = 0 \quad (14)$$

$$\bar{M}_{lr} = \frac{\rho}{2}\left(\frac{L}{k}\right)^2 Q_{lr} \quad (15)$$

$$\bar{M}_{rr} = M_{rr} + \frac{\rho}{2}\left(\frac{L}{k}\right)^2 Q_{rr} \quad (16)$$

$$\bar{M}_{rl} = \frac{\rho}{2}\left(\frac{L}{k}\right)^2 Q_{rl} \quad (17)$$

$$\bar{M}_{ll} = M_{ll} + \frac{\rho}{2}\left(\frac{L}{k}\right)^2 Q_{ll} \quad (18)$$

To solve for the eigenvalue λ, it is required to perform unsteady aerodynamic computations at several given reduced frequencies. For n structural modes, there are n eigenvalues corresponding to n modes at each reduced frequency. The flutter frequency ω_f, the airspeed V_f, and the artificial damping g_s are given as:

$$\omega_f = \frac{1}{\sqrt{Re(\lambda)}}, \; V_f = \frac{\omega_f L}{k}, \quad g_s = \omega_f^2 \; Im(\lambda) \quad (19)$$

3 Results and Discussion

First, for validation of the numerical method, flutter speed and frequency of wings with different sweepback angle are calculated and compared with experimental data. A schematic wing configuration is shown in Figure 2.

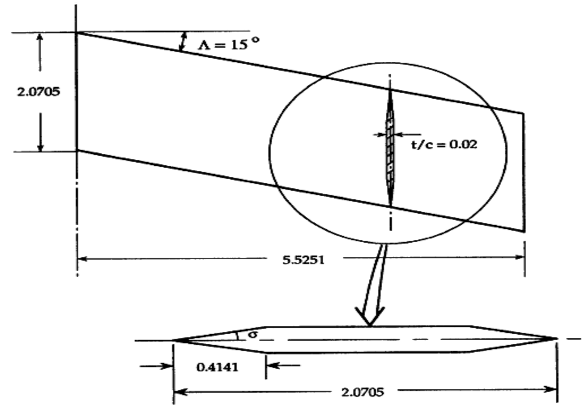

Figure 1. Wing Configuration

As mentioned above, wing's natural frequency should be calculated before aeroelastic analysis. Result of different wings with different sweep angles are shown in Table1 and compared with experimental data [7]. Modal analysis is performed by MSC.NASTRAN commercial software. About 8000 nodes are used in natural frequency calculation.

Table 1. Natural frequency of sweptback wings

Sweepback angle (deg)	Material	1st natural frequency (HZ)	1st natural frequency (Experiment)	Error %	2nd natural frequency (HZ)	2nd natural frequency (Experiment)	Error%
15	Mg	37	35.2	4.8	2.18	209.2	4.5
	Al	36	34.8	3.3	210	201.8	3.9
45	Mg	45	43.2	4	220	212.6	3.6
	Al	42	40.02	4.7	210	199.3	4.9
60	Mg	47	45	4.2	200	190.9	4.5
	Al	50	48.4	5.2	214	202.68	5.2

For flow analysis using lifting surface theory, wing should be paneled. In this study, 100 panels used in chordwise direction and 200 panels in spanwise of wing. As shown in Tables 2 and 3, aeroelastic analyses performed in different sweepback angles and air density are in good agreement with experimental data [7].

Table 2. Flutter speed of sweptback wings

Sweepback angle (deg)	Material	Air density (Kg/m^3)	V_f (m/s) [experiment]	V_f (m/s) [Numerical]	Error %
15	Mg	0.497	619.15	630.5	1.8
	Al	0.252	390.40	399.8	2.4
45	Mg	0.314	619.15	624.7	0.8
	Al	0.324	390.40	396.4	1.5
60	Mg	0.355	619.15	624.9	0.9
	Al	0.669	390.40	396.2	1.5

Table 3. Flutter frequency of sweptback wings

Sweepback angle (deg)	Material	Air density (Kg/m^3)	ω_f (Hz) [experiment]	ω_f (Hz) [Numerical]	Error %
15	Mg	0.497	146	148.1	1.4
	Al	0.252	102	107.4	4.9
45	Mg	0.314	170	166.8	1.8
	Al	0.324	180	191.7	6.5
60	Mg	0.355	180	177.5	1.3
	Al	0.669	174	178.6	3

Maximum errors are related to transonic speeds that are because of nonlinear behavior of flow that cannot be captured by lifting surface theory. Now for investigating the effect of different parameters like sweep angle and air density, wings with sweep angles 15 and 60 degree at three different altitudes are considered. Aeroelastic analysis results are given in Table 4. As shown in Table 4, increasing air density will reduce flutter speed and will increase flutter frequency. Aerodynamic force and moment increment cause this result. As we know the flutter phenomena results from fluid and solid energy exchanging. So by increasing air density and air kinetic energy, flutter may accrue in lower speeds. Also by decreasing sweep angle, flutter speed and its frequency will reduce. These results are presented and compared in Figures (2a-2d) too.

Table 4. Effect of different parameters on flutter characteristics

Sweepback angle (deg)	Material	Air density (Kg/m^3)	Numerical data V_f (m/s)	Numerical data ω_f (Hz)
15	Mg	0.257	788.1	144
		0.497	630.5	148.14
		0.669	474.1	156.6
	Al	0.252	399.8	117.4
		0.464	316.8	118
		0.669	264.1	119.8
60	Mg	0.355	624.9	177.5
		0.464	564.8	203.5
		0.669	477.2	207.7
	Al	0.355	-	-
		0.464	489.3	174
		0.669	396.2	178.66

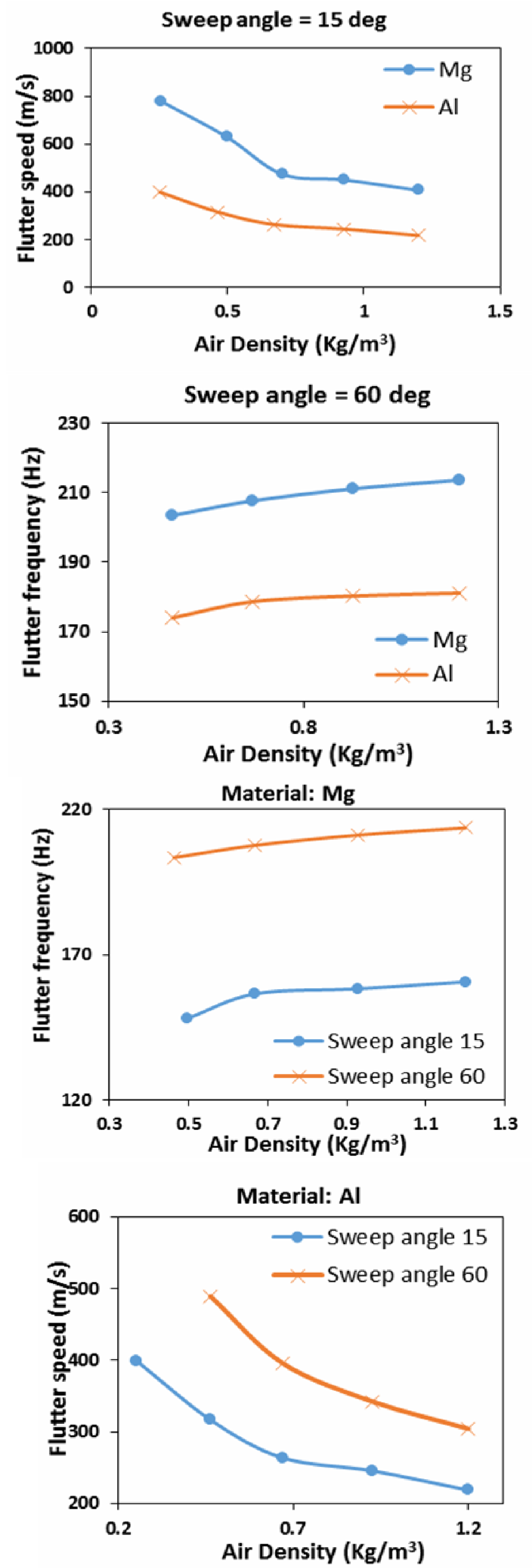

Figure 2. Effect of air density, material and sweep angle on flutter characteristics of supersonic wing

4 Conclusions

In this paper, the capability of ZAERO software for flutter analysis is verified and validated by experimental data. The obtained results have shown that this software is reliable for the supersonic wing flutter calculations. Also the effects of parameters such as air density, material and sweepback angle on the flutter characteristics of the wing are investigated. It is concluded that, reducing sweepback angle will result in flutter speed and frequency reduction and increasing air density will decrease flutter speed and increase flutter frequency of the supersonic wings.

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