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WAKE VORTEX ENCOUNTER LOADS. FLIGHT TESTS AND NUMERICAL SIMULATIONS

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Abstract

"A flying aircraft generates a turbulent wake as a direct consequence of its aerodynamic lift generation. This wake consists of a high amplitude of swirling air flow velocities concentrated in a region of relatively small spatial extent trailing behind the generator aircraft. Another aircraft entering into this wake may be significantly impacted by the vortex flow". This is the leading text in the web page of the European Commission Research & Initiative Proposal WakeNet3 and it is a good introduction to the problem addressed in this paper.

The following chapters will present the wake vortex encounter (WVE) problem from a dynamic loads standpoint. The numerical simulation methodology and assumptions for loads calculation will be explained, as well as the models and tools used for that purpose. An extensive Flight Test campaign has been dedicated to study the phenomenon and validate the overall loads calculation process. Some parametric studies, devoted to a better understanding of the phenomenon and its sensitivities to some geometrical parameters will also be presented. Finally, a stochastic approach to the loads calculation has been performed in order to link the loads levels with their probabilities. This kind of approach will permit to assess the definition of wake vortex scenarios for future regulations.

1. Introduction: Wake Vortex Encounter relevance

Wake Vortex Encounter (WVE) has always been identified as a potential hazardous scenario for the aircraft that crosses the wake. Many reports and papers in the past have been devoted to study this type of incidents, although most of them focused either on the handling qualities (HQ) and subsequent piloting techniques of the crossing aircraft or on the aerodynamic characterization of the wake.

- From the handling qualities standpoint the pilots receive instructions on how to operate when encountering a wake of an aircraft ahead of them. In the 1970s, "The International Civil Aviation Organization (ICAO) defined 'Minimum Wake Turbulence Separations' for worldwide application. These separations are based on three dedicated aircraft classes (Light, Medium and Heavy) depending on aircraft maximum take-off weight." [1]

- From the aerodynamic characterization point of view there have been many attempts to measure the wake aerodynamic properties and subsequent simplified aerodynamic models have been derived from these measurements [2], [3].

Nevertheless, the amount of papers devoted to address the specific problem of "dynamic loads" [4] experienced by an aircraft during a wake crossing has been significantly more reduced. There is no reference to WVE load cases in the regulations no matter civil or military. However, test experience shows that a WVE is an event that may generate large dynamic loads on the crossing aircraft.

This is the motivation of the present paper supported by the forecast that wake vortex encounters may become more common in the future:

- In the civil field because *"more and more airports are operating at their capacity limit during peak hours (...)Traffic density is increasing in general, leading to more and more aircraft operating in close vicinity to each other and thus potentially increasing the risk from wake encounters"* [1].

- In the military field, wake vortex encounters may be a consequence of the aircraft operation (an aircraft crossing its own wake during a 360º turn in a surveillance mission or crossing the wake of a preceding aircraft while training dog fighting maneuvers, etc.).

The next chapters of this paper will show the whole process of the wake vortex encounter (WVE) loads simulation:

- WVE loads calculation. Methodology tools and models.
- Flight Test campaign. Validation of loads simulation process and A400M wake characterization.
- Parametric studies.
- Stochastic approach.

This paper compiles in a single text all the information widespread in other recent publications [5][6][7]. In addition, the stochastic approach results are herein presented for the first time.

2. WVE loads calculation: methodology, tools and models

2.1. Similarities and differences with discrete gust

Conceptually, a wake vortex encounter may resemble a "special" kind of gust excitation that the aircraft may eventually suffer. Even so, there are some specific characteristics in this excitation that makes it much more complex than the "*1–cos*" book-case gusts described in the airworthiness regulations. These special characteristics had made necessary to adapt some of the tools used for the gust loads calculation and even to develop a new one to generate such a complex excitation.

The "classical" models and tools used in our work for both gust and wake simulations are as follows:

- A NASTRAN Finite Element Method (FEM) condensed model (see Figure 1) representing the aircraft (A/C) structure stiffness. The mass is accounted for by means of lumped masses discretely attached to the structure. The Doublet Lattice Method (DLM) model of Figure 1 is used to account for the unsteady aerodynamic of the lifting and control surfaces [8]. A modal reduction method is used to project the grids degrees of freedom over a truncated modal base, which reduces the problem from several thousand degrees of freedom to only a few hundreds.

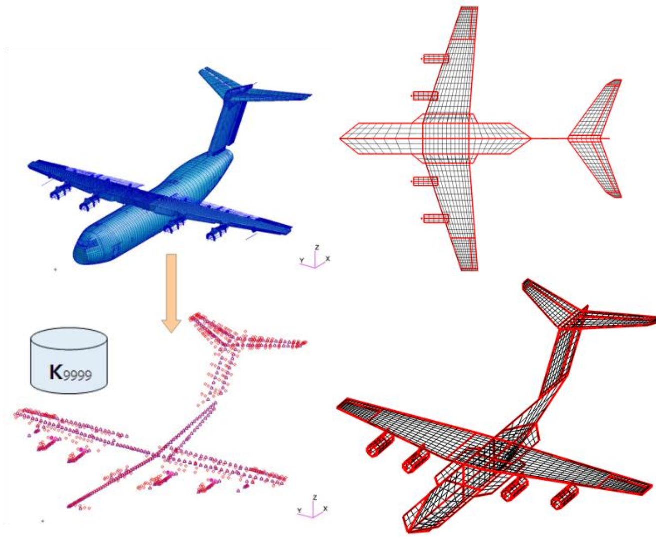

Figure 1: A400M structural dynamic FEM model (left) and unsteady aerodynamics DLM model (right)

- The DYNRESP solver [9], is used to calculate the modal response for subsequent loads analysis. The code combines FFT based linear solution in the frequency domain with time domain solution for introducing nonlinear effects. The linear application of this work includes linear control loops with sensor/force coupling terms to account for gyroscopic and 1p engine loads. The aerodynamic force model associated with the 1p propeller loads includes a unsteady delay in their development.

- The DYNLOAD in-house software is used for the loads recovery. It is based on the summation of forces method to integrate inertial, aerodynamic and other external forces along the A/C components. These will be added to steady 1g loads, calculated externally with specific flexible quasi-steady models.

The main differences between the "traditional" gust excitation and the wake encounter one are described here:

- The discrete gust excitations regarded in the regulations have (1-cos) smooth velocity profiles, with gust lengths ranging from 30 to 350 ft, whereas wake velocity profiles are sharp, typically with two positive peaks and two negative ones. The sharpness of the wake velocity profile entails higher frequency contents than discrete gust excitations, as showed in Figure 2.

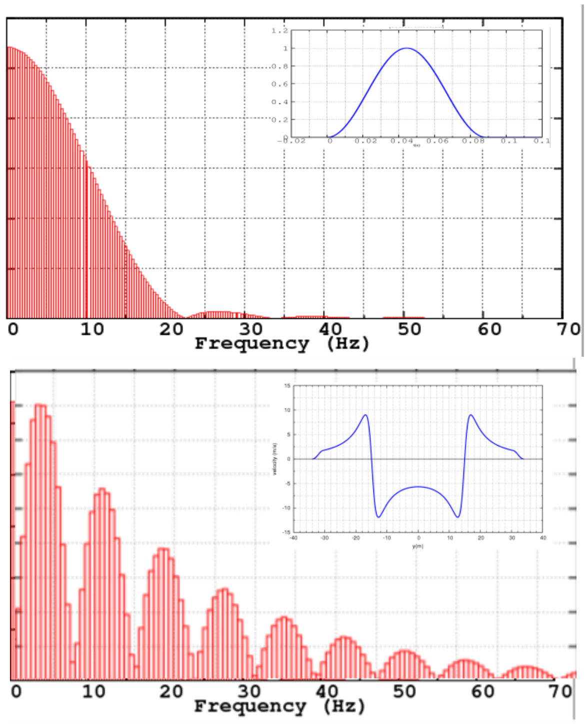

Figure 2: discrete (1-cos) gust (top) and wake (bottom) typical time-velocity profiles and frequency contents

- In the classical discrete gust problem, the velocity field is uni-dimensional (along the aircraft trajectory) and unidirectional (pure vertical or lateral). This means that all the panels of the aerodynamic model undergo the same velocity profile, regardless its *x*, *y* and *z* coordinates, with a delay that depends only on the longitudinal coordinate *x* of each panel with respect to a given reference point at which the uniform gust profile is defined. The gust front is always perpendicular to the aircraft trajectory.

The encountered wake profiles for the crossing aircraft depends, on the other hand, also on the height and span position of the DLM boxes. The velocity field induced by the wake varies with the distance to the vortices plane (while the gust is uniform) and besides that, it can be crossed at any angle, not only perpendicularly. Therefore, as shown in Figure 3, one of the wings generally "meets" the wake before the other one and for an aircraft with T-tail configuration the velocity profiles at the HTP are significantly different from the ones at the wing. Furthermore, the direction of the velocity vector varies along the flight path, being purely vertical only at the vortices plane and at their symmetry plane. This causes the aircraft to be excited in both the vertical and lateral directions (see Figure 3).

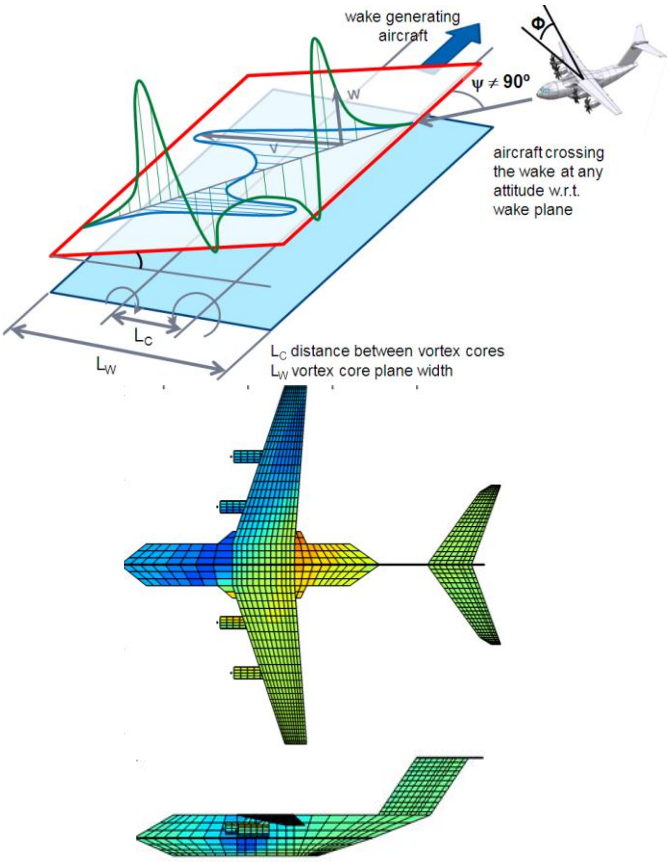

Figure 3: Induced velocities on the aircraft in a generic wake crossing

2.2. Wake vortex aerodynamic model

Prandtl slender wing theory states that a finite lifting surface embedded in a potential airflow yields one vortex at each tip as a result of the conservation of vorticity. Depending on the planform of the wing, the vortices may be concentrated at the tips or distributed along the trailing edge. Propellers and high lift devices also affects the wake vortex distribution. It is assumed in this work that the wake is regarded as a couple of counter-rotating vorticity tubes rather than as a vortex sheet (Figure 4). It consists of two infinite and horizontal vorticity tubes, and condensed into two single parameters:

- separation between vortices cores L_C
- vortex core radius r_c, whose physical meaning is explained later.

This simplified approach is more accurate the longer is the streamwise distance travelled from the generating aircraft. For most operations, encounters of the near field of the wake (less than a few wing spans) are considered highly unlikely and undesirable. The present work is focused only on encounters that happen in the far wake, where the assumption of a pair of counter-rotating vorticity tubes is valid [10].

The vortices tubes may be considered of infinite length when compared with the wing span. The induced velocity at a point P is then given by the Biot-Savart law. The velocity will be confined in the plane $Y_W Z_W$ that is perpendicular to the vortices and contains the point P (Figure 5).

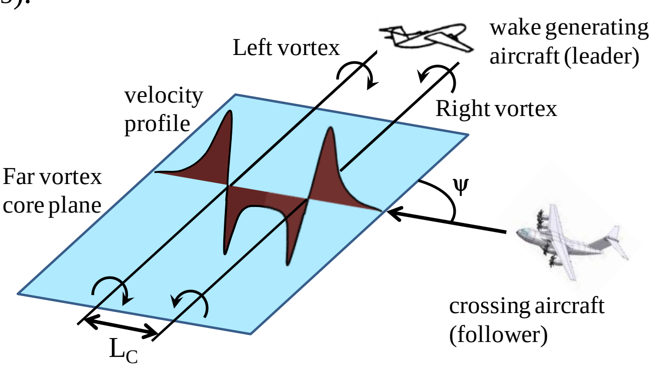

Figure 4: Far wake model

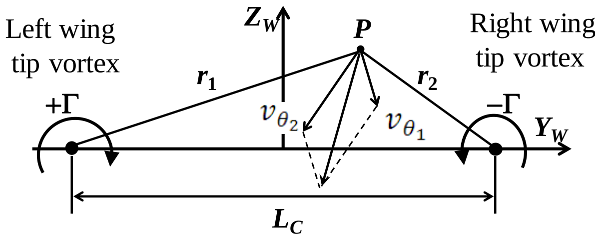

Figure 5: Velocity induced by far wake

The resultant velocity at point P is the combination of the azimuthal velocities v_{θ_1} and v_{θ_2} induced by the right and left vortex tubes. To avoid the singularity of the classical potential theory at the vortex core, some expressions have been developed to represent the actual fact that the velocity is finite at the vortex centre. Hallock-Burnham [2] and the Oseem-Lamb [3] presented two of the most popular formulations. For this work, the azimuthal velocities are given by the Hallock–Burnham model:

$$v_{\theta_1} = \frac{\Gamma}{2\pi r_1}\left(\frac{{r_1}^2}{{r_1}^2 + {r_c}^2}\right) \qquad v_{\theta_2} = \frac{-\Gamma}{2\pi r_2}\left(\frac{{r_2}^2}{{r_2}^2 + {r_c}^2}\right)$$

where r_1 and r_2 are the distances from point P to the vortex cores, r_c is the vortex core radius and Γ is the circulation. This expressions resemble the potential theory, but the denominator has been modified with the addition of a new term, the vortex core radius squared. The Hallock-Burnham model conjectures that each vortex has two regions:

- a core region, where the tangential velocity grows from zero at the vortex centre up to a maximum value reached at the vortex core radius.
- an external region where the tangential velocity drops asymptotically towards zero as the distance grows, as predicted by potential theory.

The circulation of the vortices Γ is approximated by a simplified expression of the Kutta-Joukovsky formula:

$$\Gamma \cong \frac{W_{ac} \cdot g \cdot n_z}{\rho_\infty U_\infty s_w b}$$

where ρ_∞ is the air density, U_∞ is the flight speed, W_{ac} is the aircraft mass and n_z is the load factor (all parameters referred to the generating aircraft).

In order to take into consideration the fact that the wake vorticity difuses after a certain time, a wake aging model has been implemented. The wake aging is a complex phenomenon that depends on many atmospheric parameters as well as on the wake generation characteristics and, of course, on the elapsed time (wake age). There are many ways to model it, and the one adopted for this work has been the Sarpkaya decay model [11].
It considers some parameters, like the aircraft weight, the load factor and some atmospheric characteristics conservatively estimated, together with the elapsed time, to yield a reduction factor to be applied to the vortices circulation. This means that the whole velocity field is scaled from the initial one.

$$\Gamma_{aged} = \Gamma_{nominal} * (\text{decay factor})$$

The dependency of the decay factor with the elapsed time for a sample flight condition is showed in the plot below (Figure 6). Mind the abrupt diminishing of the circulation after two minutes since the wake is generated. It completely disappears one minute later.

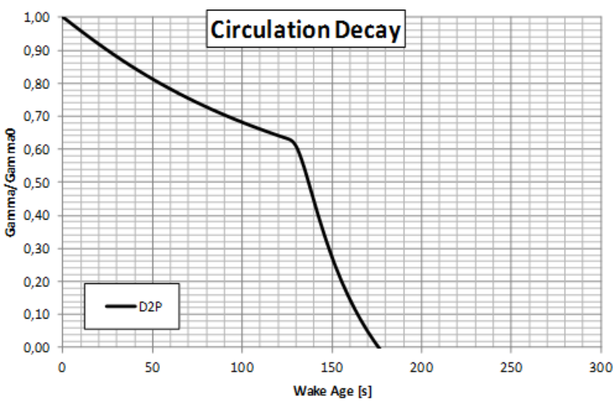

Figure 6: Typical wake vortex decay

2.3. Wake vortex excitation on the DLM boxes

In order to generate the excitation forces that the vortex velocity field yields over the DLM boxes of the crossing aircraft, an in-house tool named WESDE has been developed. Besides the aerodynamic wake model, already described, it has to account for the geometrical parameterization of the crossing.
For this purpose a simplifying assumption has been considered: that the crossing aircraft maintains a straight trajectory and constant attitude and velocity with respect to the exciting field, not affected by the excitation from the flight mechanics standpoint. On the other hand, the presence of the crossing aircraft obviously affects the vortex velocity field. However, in order to be consistent with the DLM formulation, the excitation forces are calculated with the angles of attack induced on the DLM boxes assuming that the aircraft is a "ghost" that does not affect the excitation field it is crossing.

The geometric parameters that define the crossing of the wake are shown in Figure 7 and consist of the following:
- two heights of a reference point of the aircraft over the vortices (H_1 and H_2) and one angle, yaw (ψ) to fix the aircraft trajectory.
- another two angles, roll (ϕ) and angle of attack (α), to set the aircraft attitude.

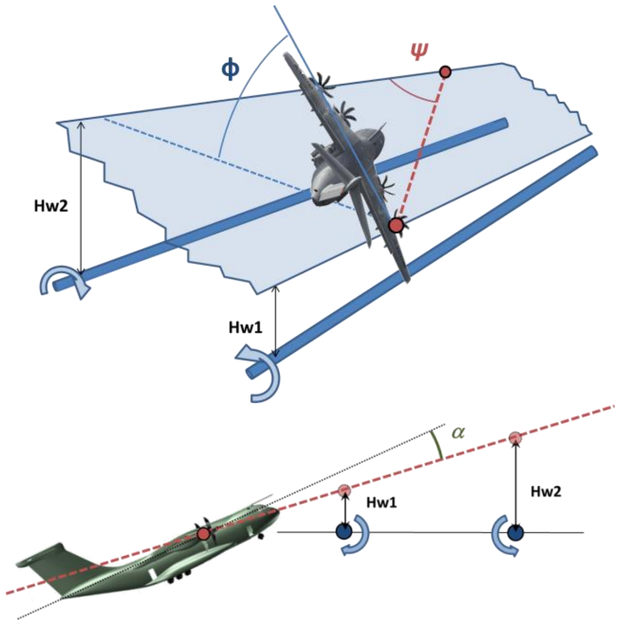

Figure 7: geometric parameterization of the wake crossing

Given the characteristics of the velocity field and the geometric versatility of the crossing, in the most general case, each box of the aerodynamic model will be subjected to a different excitation time history (see Figure 8).

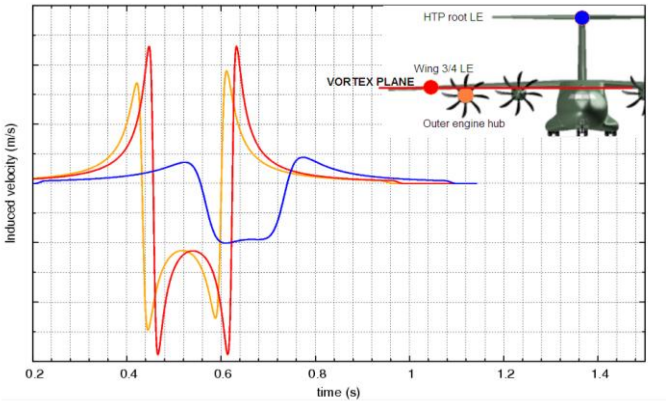

Figure 8: example of different velocity profiles at three locations along the aircraft

This is exactly the output of WESDE, a matrix with the normal velocity time histories of each DLM aerodynamic box.

The solver and postprocessor tools (DYNRESP and DYNLOAD) have had to be adapted to deal with this important increase of the excitation complexity: from a unique velocity profile for all the aerodynamic boxes in the classical gust excitation, to one profile per aerodynamic box in the wake problem.
The flowchart of the whole loads calculation process is shown in Figure 9:

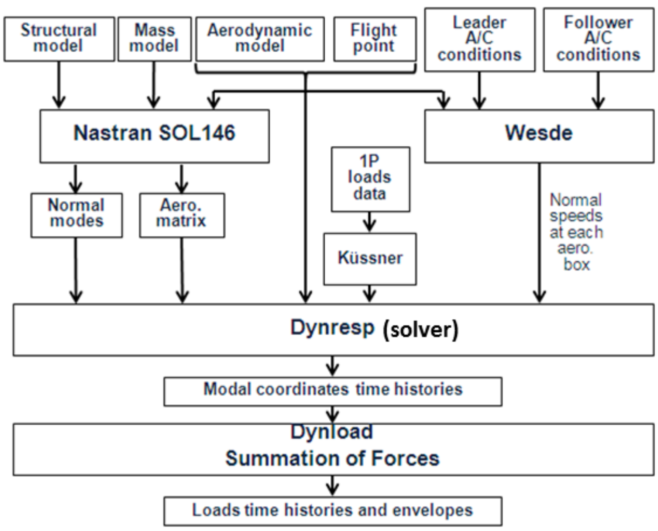

Figure 9: Wake vortex encounter calculation schema

3. Fight Tests campaign: validation of loads calculation and A400M wake characterization

In order to validate the described loads calculation methodology, an extensive flight test campaign with the A400M was performed in 2012, split in two separate parts:

- Part 1(140 runs): A400M crossing the wake of a jet airliner. It allowed assessing the response of the A400M to an already known wake and validating the simulation methodology and tools.
- Part 2 (109 runs): A400M crossing the wake of another A400M (a way to replicate for instance the aircraft crossing its own wake in a wind up turn). It was also used to characterize the wake of the A400M.

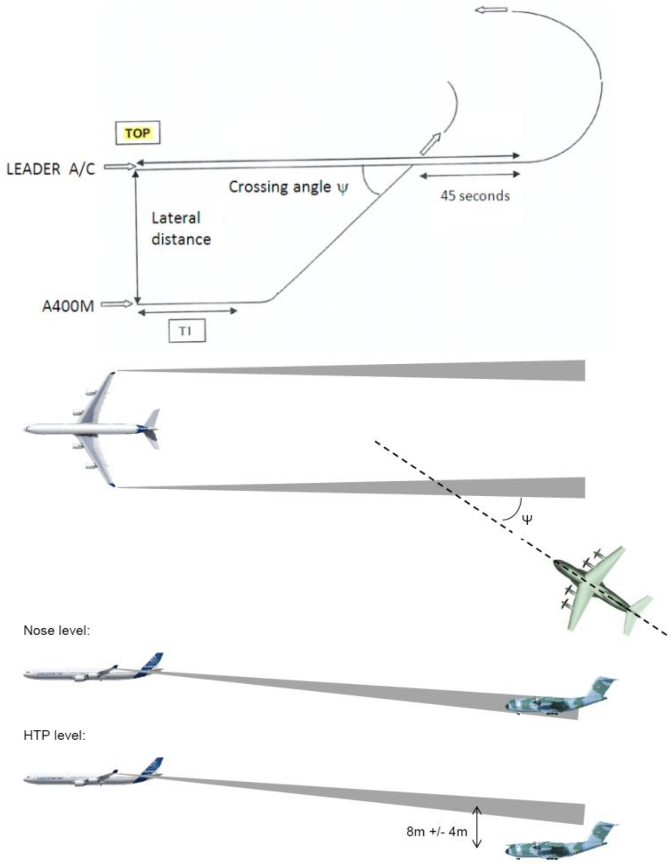

Figure 10: Intentionally WV encounter Flight Test procedure

Dynamic loads and accelerations while crossing the wake have been monitored using strain gauges and accelerometers distributed over the entire aircraft (Figure 11and Table 1)

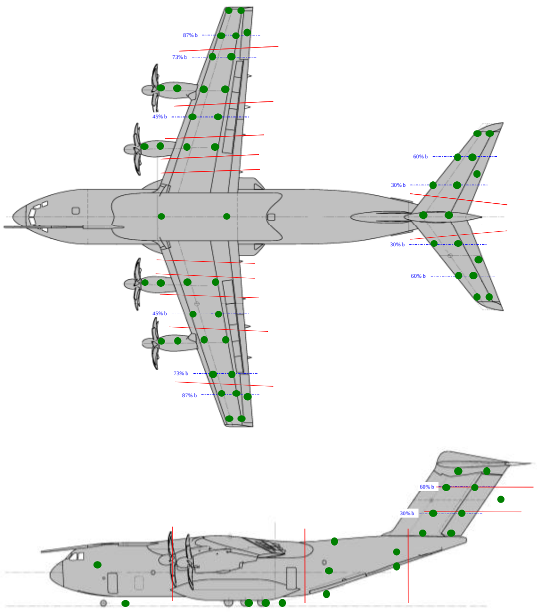

Figure 11: Accelerometers (green dots) and monitoring stations for loads (red lines) used in A400M WVE FT campaign

	NX	NY	NZ	TOTAL
Wing	12	-	28	40
Fuselage	6	6	6	18
HTP	6	-	16	22
VTP	4	9	-	13
LG	7	7	7	21
				114

Table 1: Accelerometers summary

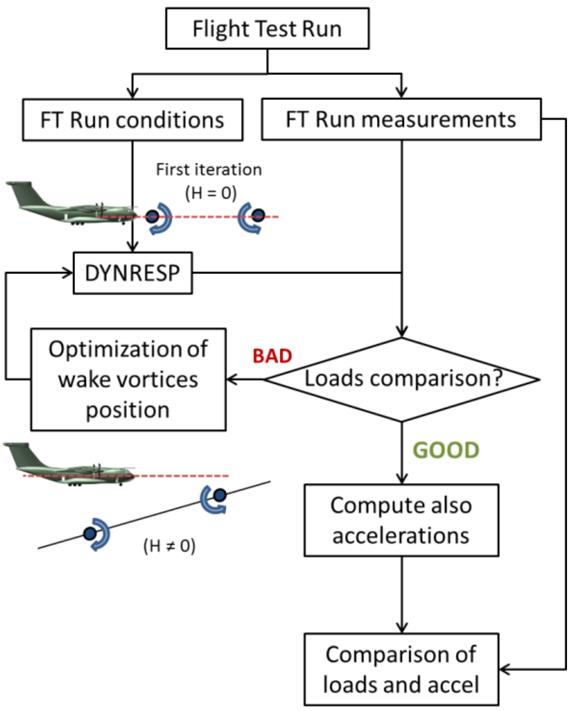

Figure 12: Numerical simulations of flight test wake encounters

The strategy to fit the numerical simulations to flight test results (Figure 12) involves an iterative process to find out the actual distance of the crossing aircraft with respect to the vortices.

Other uncertainties, like the vortex decay or the vortices separation, can be determined as part of this iterative process. Predictive models of circulation decay due to vortex aging can be also included in the analysis. Sample comparisons of flight test results with numerical simulations are shown in Figure 13, exhibiting excellent agreement.

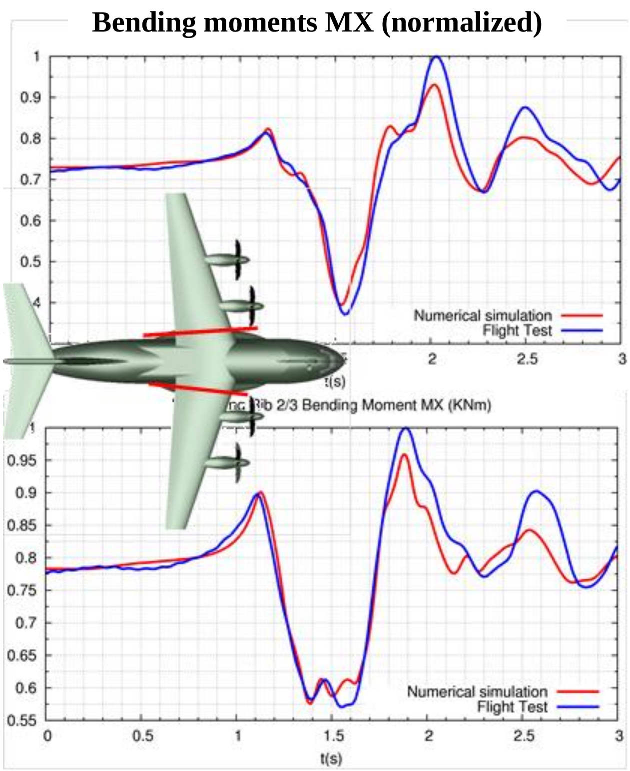

Figure 13: WVE numerical simulations of A400M wing root bending moment compared with flight test results (encounter at an angle of ψ=40º).

4. Sensitivity studies to vortex height and crossing speed/yaw angle

Before launching a complete loads loop, sensitivity studies have been done in order to better understand the influence of three critical parameters on the WVE loads: the distance to the vortices, the crossing speed and the yaw crossing angle.

- Overflying distance of the lifting surfaces (specially HTP) over the wake vortices.

These plots (Figure 14) show that the H_{crit} for several tail loads has a strong correlation with the angle of attack:

- at $\alpha = 0$, $H_{crit} = 0$. In this attitude the whole HTP DLM plane hits the vortices plane.
- at $\alpha \neq 0$, There is a H_{crit} .that maximizes the effect of the wake on the HTP plane

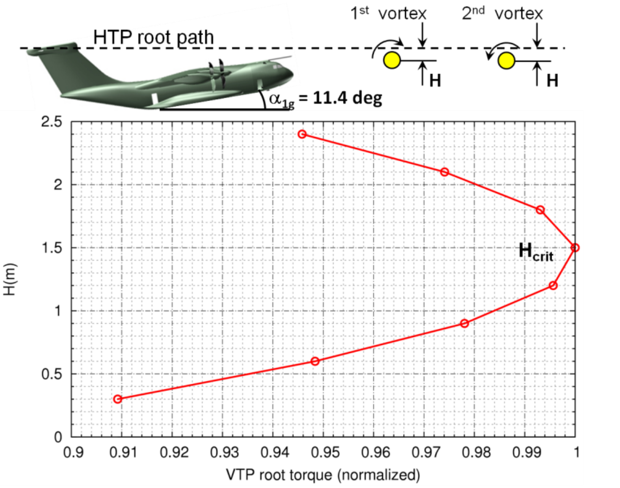

Figure 14: Sensitivity of VTP root torsion moment to wake vortices height H

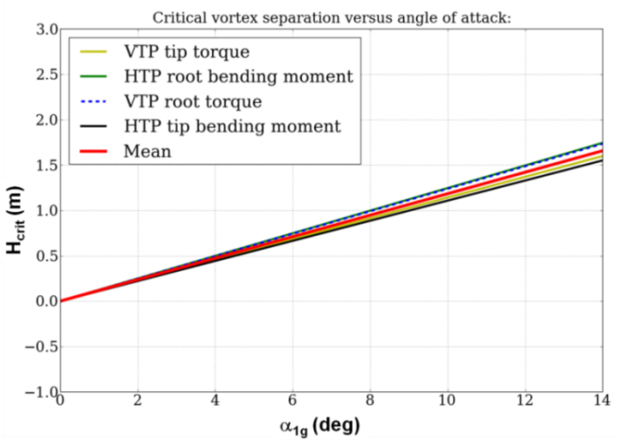

Figure 15: Effect of angle of attack on critical H

- Crossing speed/yaw angle

Figure 16 shows the variation of the VTP root torsion moment, which is a magnitude dominated by a single mode, the VTP torsion. In this kind of loads, that dominating mode can be excited tuning the crossing of the two vortices with the mode frequency. This can be achieved with several combinations of flight speed and crossing angle that makes the time lapse between the two vortices velocity peaks match the frequency of the loads driving mode. Those combinations follow a law of constant U • sin ψ, which gets the same crossing time for different speeds and angles. This can be easily visualized in the coloured plot (Figure 16).

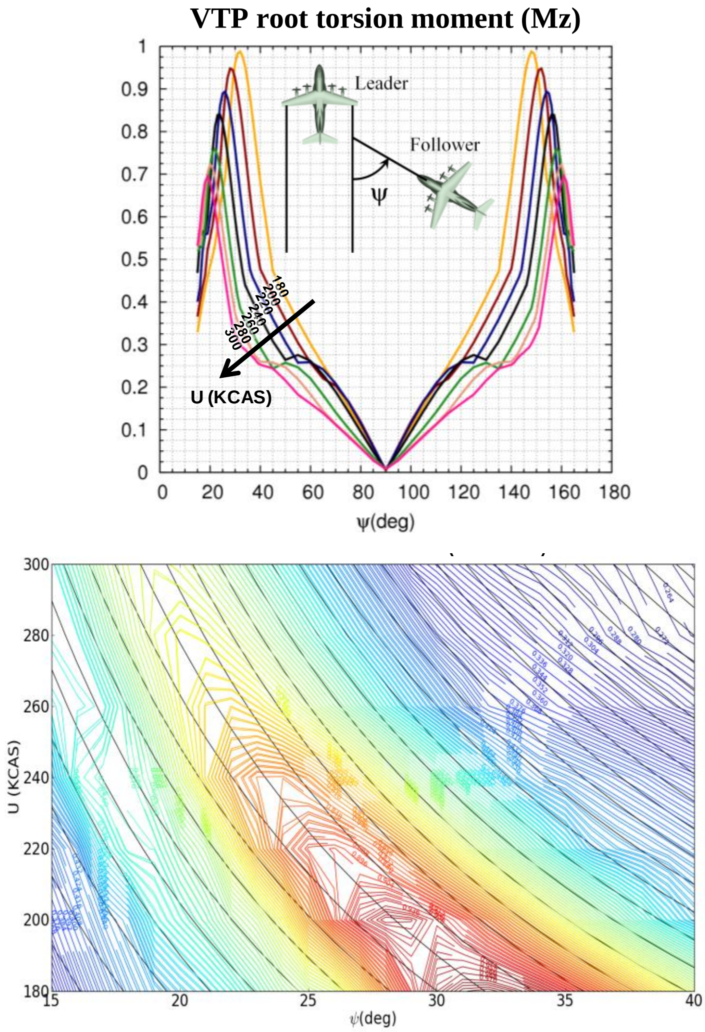

Figure 16: Variation of VTP root torsion moment MZ with ψ and flight speed U (bottom: iso U·sinψ).

On the other hand, there are other load magnitudes dominated by several modes, like the HTP bending moment. For these ones, there is not a clear relationship between the flight speed and the crossing angle that yields the maximum loads (see Figure 17)

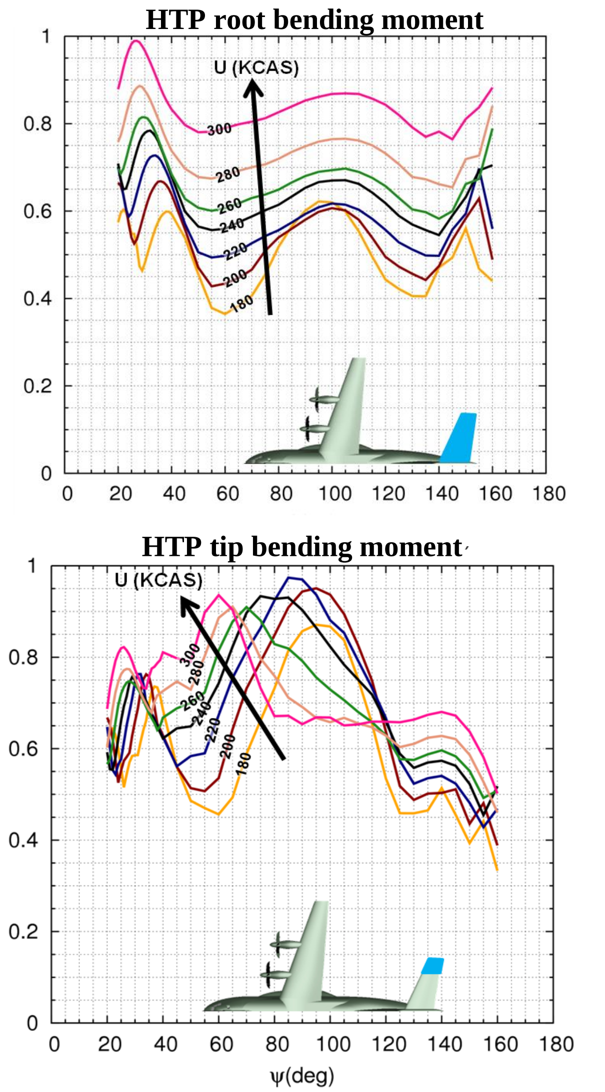

Figure 17: Variation of HTP root (top) and external HTP (bottom) bending moment MX with ψ and flight speed U.

5. WVE Stochastic approach.

The good matching of WVE simulations with FT measures, together with the robustness of the method and tools developed and the relative short run time of each simulation (60 min, aprox), allows the use of the presented methodology to afford an “industrial” loads loop calculation to assess the WVE event from dynamic loads standpoint.

Nevertheless, the great amount of parameters needed to define a wake crossing simulation, their wide ranges of variation and the complex dependence of the loads with them (showed in the parametric analyses), justifies the use of a statistical approach to the problem. Furthermore, the stochastic technique might be used to determine an adequate level of loads and its probability in order to cover the lack of proper WVE scenarios definition in the current Airworthiness Regulations.

5.1. WVE Considered scenario. Assumptions

The stochastic study is focused on self-wake crossing events during turning up maneuvers. This automatically link some of the variables involved in the definition of each event:

- The bank angle of the aircraft when generating the wake is directly related to the load factor of the maneuver. Therefore, the relative position (height) of both tubes of vortices is also linked to the load factor.
- The time considered for the wake aging in the decay model is also related with the load factor, in the assumption of a perfect circular turn at constant velocity:

$$\Delta t = \frac{2\pi \cdot V}{g\sqrt{n_z^{\,2}-1}}$$

$$n_z = \frac{1}{\cos\phi}$$

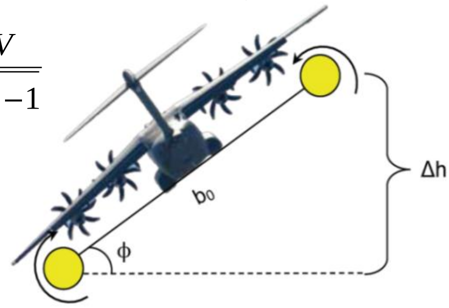

Figure 18: vortices generated in a turning maneuver, load factor and time to a self-crossing

5.2. WVE Stochastic variables and probability distributions

Given the described scenario, the set of parameters that defines the wake crossing are selected according to the probability distributions of certain stochastic variables:

- The aircraft weight and flight point parameters: Altitude, A/C speed, A/C mass, flaps configuration, etc… are obtained from A400M fatigue missions, which are divided in flight segments of different duration.

  After removing the cruise segment, as no self-wake encounter events are foreseen in this condition, the rest of the segments of each mission are sorted in order to better fit their duration to a Gaussian distribution.

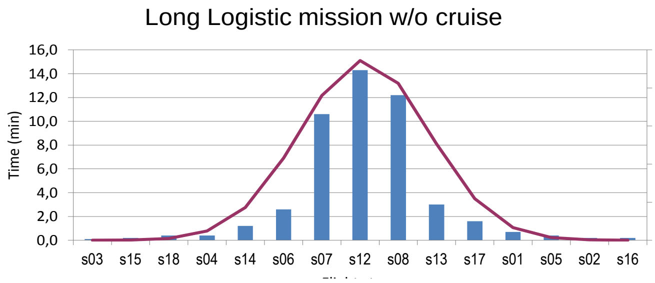

Figure 19: segments duration sorted to fit a normal distribution

- Load factor of the aircraft (Nz) during a turn up manoeuvre: The distribution of probability of the different values of Nz is also obtained from the fatigue missions and follows an exponential law:

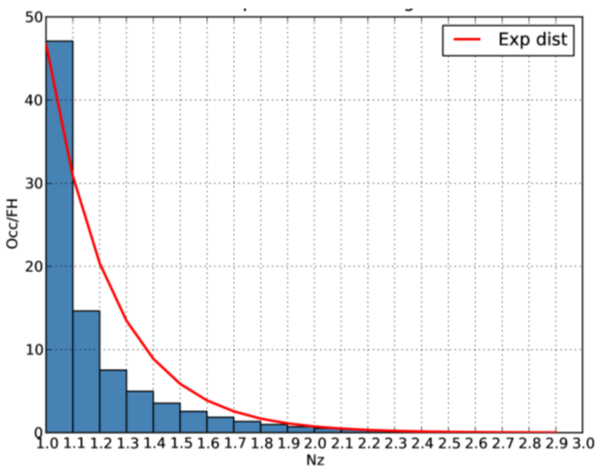

Figure 20: histogram (occurrence per flight hour) of load factors at turning manoeuvres

- Bank angle of the A/C when it crosses the wake: this parameter is assumed to follow an equal-probable distribution ranging from zero to the value corresponding to the load factor of the manoeuver that generates the vortices. Only angles from zero to 5deg are calculated, as these are the attitudes that lead to higher loads. The rest of angles are considered statistically but they are not explicitly calculated.
- Overflying distance (H_1) of the reference-point (HTP Root for tail impacts) over the first vortex encountered. All the heights in the range of interest [-20m, 20m], are assumed to have the same probability.
- Yaw angle: crossing angle at which the following aircraft enters the wake. $\Psi<90°$ means left wing first while $\Psi>90°$ means right wing first. All the angles in the studied range [20deg, 160deg], are assumed to be equally probable.

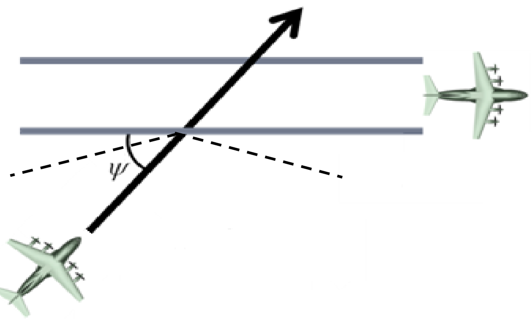

Figure 21: crossing yaw angles range

The values of all the variables out of the ranges considered in the analyses are assumed to lead to very low loads level (they don't worth to be simulated) but their probability of occurrence is taken into account when evaluating the probabilities of the different loads levels per hour of flight.

5.3. WVE stochastic methodology

The stochastic analysis has been divided in ten blocks of one thousand "relevant cases" each. For each "shot" the procedure is the following:

- The parameters that define the case are randomly chosen according to the distributions described above.
- If that combination of parameters is considered relevant from the loads standpoint (based on previous sensitivity analyses), the simulation is calculated. Otherwise, the loads are automatically assigned zero value without calculating anything. The case is however taken into account statistically without increasing the count of relevant cases of the block.
- the output variables (loads results) are stored for further statistical treatment when all the cases of each block have been calculated.

Each block of 1000 shots is assumed to envelope all the possible events occurrences in a whole aircraft life (10^{+5} flight hours).

The management of the whole process is performed by a program named ST-ORM. It controls the run of each of the "shots", as described above, and performs the statistical analysis of the results for each block.

5.4. WVE stochastic results

The following plots show (Figure 22) the typical output of the statistical analysis of the results: the variation of a relevant load with some of the stochastic variables. The trends observed in the clouds of shots are consistent with the ones concluded from the previous sensitivities analyses.

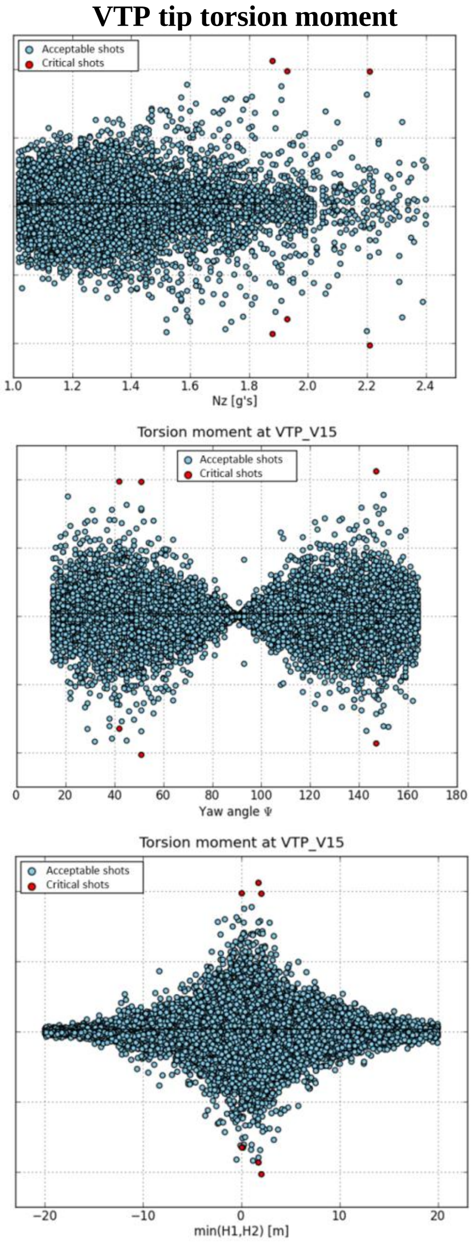

Figure 22: ant-hill plots. VTP tip torsion moment (Mz) vs Nz, Ψ and H_1

The following 2D envelopes show the correlated pairs of load magnitudes at certain relevant locations of the tail (HTP and VTP roots). The proximity to the reference load envelope confirms the relevance of the WVE events from the loads standpoint. The points of the envelopes closer to the reference loads are also highlighted in red in the ant-hill plots of Figure 22.

Although the envelopes of the different blocks of shots are not identical, their similarity shows the consistence of the method and gives confidence on the loads level obtained.

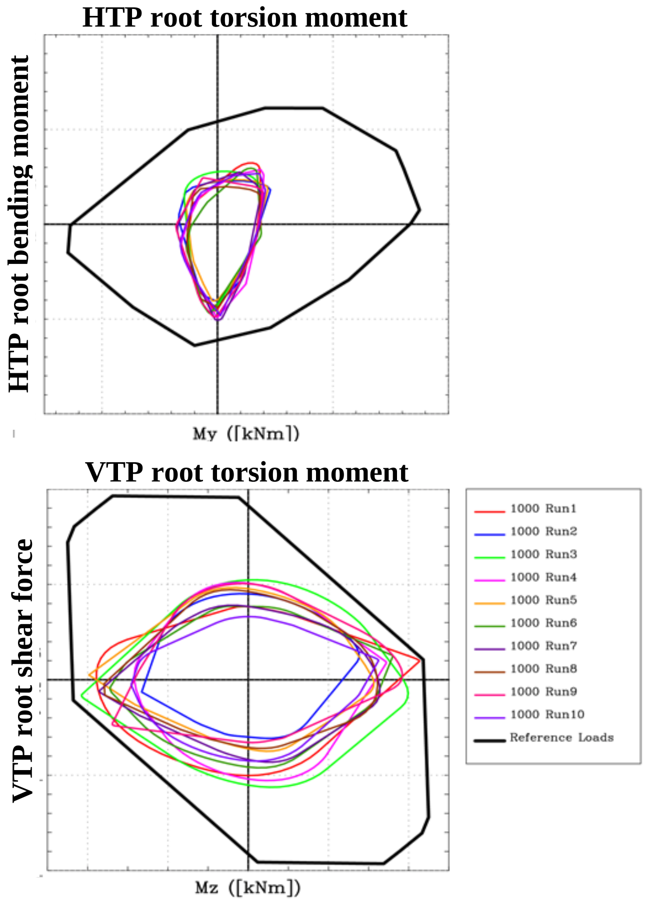

Figure 23: 2D tail loads. Stochastic blocks envelopes vs reference loads

6. Conclusions

The wake vortex encounter scenario has been introduced, highlighting its relevance from the dynamic loads standpoint (especially for T-tail heavy transport aircraft).

The paper has described a numerical methodology to compute WVE dynamic loads using models and tools that resemble (1-cos) gust analysis although the process exhibits important differences from the classical gust approach.

A comprehensive and dedicated flight test campaign was used to validate the numerical methodology. Comparisons between test results and computations have shown an excellent agreement.

The final step has been to determine suitable load envelopes that could be representative of this WVE scenario. For this final step, a stochastic approach has been followed and 10 sets of 1000 shots obtained (each one assumed equivalent to 10^{+5} flight hours and therefore linked to what could be regarded as limit loads). Although the load envelopes of each one of these 10 sets have not been fully identical, they have been very close to each other, thus giving confidence that these levels could be a good representation of the WVE scenario on the entire life of the aircraft.

References

[1] Web page of the European Commission Research & Innovation Proposal (Transport): WakeNet3-Europe http://ec.europa.eu/research/transport/projects/items/wakenet3_europe_en.htm (accessed 8-April-2013).

[2] Burnham, D.C. and Hallock, J.N.: Chicago Monostatic Acoustic Vortex Sensing System, Volume IV: Wake Vortex Decay, Technical report DOT/FAA/RD-79-103 IV, July 1982, 208 pages.

[3] Devenport, W.J., Rife, M.C., Liapis, S.I. and Follin G.J. "The Structure and Development of a Wing-Tip Vortex". Journal of Fluid Mechanics, 1996, vol. 312, pp. 67-106.

[4] Bisplinghoff, R. L., H. Ashley, and R. L. Halfman. "Aeroelasticity". Dover Publications, Inc., Mineola, NY,

[5] Karpel, M., Shousterman, A. Reyes, M. and Climent, H. "Dynamic Response to Wake Encounter". 54th AIAA/ASME/ASCE /AHS/ASC Structures, Structural Dynamics, Materials Conference and Co-located Events, Boston, Massachusetts, 8-11 April 2013.

[6] Claverías, S., Karpel, K., Cerezo, J., Torralba, M.A., Reyes, M. and Climent, H. "Wake Vortex Encounter Loads Numerical Simulation". Proceedings of the International Forum of Aeroelasticity and Structural Dynamics IFASD 2013. Bristol, 24-27 June 2013.

[7] Climent, H., Lindenau, O., Claverias, S., Viana, J.T., Oliver, M., Benitez, L., Pfeifer, D., and Jenaro-Rabadan, G. "Flight Test Validation of Wake Vortex Encounter Loads." Proceedings of the International Forum of Aeroelasticity and Structural Dynamics IFASD 2013. Bristol, 24-27 June 2013.

[8] Rodden, W.P. (Ed.), "MSC/NASTRAN Handbook for Aeroelastic Analysis". The MacNeal Schwendler Corp. Rpt. MSR-57, November 1987.

[9] Karpel, M., Shousterman, Climent, H. and Maderuelo, C. "Dynamic Aeroservoelastic Response with Non-linear Structural Elements". 54th AIAA/ASME/ASCE /AHS/ASC Structures, Structural Dynamics, Materials Conference and Co-located Events, Boston, Massachusetts, 8-11 April 2013.

[10] Ginevsky, A.S., and Zhelannikov, A.I., "Vortex wake of aircrafts", Springer 2009.

[11] Sarpkaya, T., "New Model for Vortex Decay in the Atmosphere," Journal of Aircraft, Vol. 37, No. 1, 2000, pp. 53-61

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