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MULTI OBJECTIVE OPTIMIZATION OF TRANSONIC AIRFOIL USING CST METHODOLOGY WITH GENERAL AND EVOLVED SUPERCRITICAL CLASS FUNCTION

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Abstract

CST method is a powerful parameterization method because of its simplicity, robustness, and its ability to be generalized into various possible shapes of aerodynamic bodies. The geometry from CST itself is mainly determined by the formula of class and shape function. Application of CST to transonic airfoil optimization is still rare and more studies are needed. This work studies the application of CST and modified CST to the transonic airfoil multi-objective optimization problem. Various scheme for optimization such as free lower and upper bound and reuse of the information from existing airfoil are performed in this work. An Euler solver with NSGA-II algorithm is employed to find the Pareto front with maximizing lift and minimizing drag coefficient as the objectives. Furthermore, robust optimization with the L/D ratio as the objective is also performed. Results show that reusing the information of existing airfoil could help to reduce the possible shape of airfoil, and thus makes the optimization scheme more effective. Moreover, the evolved class function also perform fairly well compared to the standard class function. It is also found that the mean and standard deviation of L/D ratio are two highly conflicting objectives.

1 Introduction

Contribution to the aerodynamic performance of an aircraft mainly comes from the wing as its main aerodynamic forces generator. The design and selection of an airfoil, as a cross sectional part of the wing, shares a large and important portion in the wing design itself. Optimization plays another role where the performance can be optimized to obtain benefits such as lower full consumption if the drag is minimized or high endurance if the L/D ratio is maximized. To optimize an airfoil, typically the shape of an airfoil is altered during the optimization process to obtain a specific goal. However, to use all set of coordinates for the design variables is almost impossible. To greatly reduce the design variables from almost infinity to a finite set, airfoil parameterization method is employed to represents an existing or totally new airfoil.

Example of such airfoil parameterization methods are Hicks Henne function, PARSEC, B-spline parametrization, and the newly introduced Kulfan parameters, or CST method [1]. The latter is introduced by Kulfan and there are already some researches tried to explore the capabilities of CST method. CST method is a powerful parameterization method because of its simplicity, robustness, and its ability to be generalized into various possible shapes of aerodynamic bodies. CST with low order polynomial is also suitable for airfoil preliminary design and optimization purpose since it only needs few parameters to gives a specific shape of airfoil.

Application of CST to the optimization of transonic airfoil is still rare and more studies are needed. This work studies the application of CST and modified CST to the multi-objective optimization problem of transonic airfoil. Study is performed on how lower and upper bound affects the optimization search with CST and non-standard class function is also tried. The goal of this paper is to do a preliminary study of how CST behaves in multi objective optimization of transonic airfoil case. Beside of that, preliminary experiment of robust optimization with CST is also performed with stochastic expansion as the uncertainty quantification method. Robust optimization is important to ensure that the optimum solution is robust to the uncertainties. Another goal of this paper is to stand as a first step into the study of CST parameterization coupled with robust optimization methodology for more complex cases of transonic airfoil optimization.

2 CST for Airfoil Parameterization

CST is a relatively new airfoil parameterization that uses Bernstein Polynomial as its building block to generate aerodynamic shape. Generally speaking, the original CST is developed to generate various aerodynamic shapes and is not limited to the airfoil shape only. For example, CST could generates the shape of wing, fuselage, supersonic aircraft, and else. The specificness of the shape is depends on the class function of CST. For an airfoil shape, the class function is defined with fixed parameters inside the class function itself. Modification of CST is also done with the addition of trailing edge ordinate location and its thickness which are not exists in the original CST. The good thing about Bernstein polynomial, and CST itself, is that the smoothness of its curve is beneficial for optimization purpose. Because of the number of parameters can be tuned easily by altering the order of Bernstein polynomial, CST is suitable for either conceptual or preliminary design of airfoil. The expression for CST is given on the next paragraph.

The upper surface and lower surface of CST where C and S are the class and shape function, respectively, are expressed as:

$$\left(\frac{z}{c}\right)_{upper} = C_{N2}^{N1}\left(\frac{x}{c}\right) S_U\left(\frac{x}{c}\right) + \frac{x}{c}\frac{z_{te}}{2c} + + \frac{x}{c}\frac{dz_{te}}{2c} \tag{1}$$

$$\left(\frac{z}{c}\right)_{lower} = C_{N2}^{N1}\left(\frac{x}{c}\right) S_L\left(\frac{x}{c}\right) + \frac{x}{c}\frac{z_{te}}{2c} + + \frac{x}{c}\frac{dz_{te}}{2c} \tag{2}$$

Where z_{te} and dz_{te} and are trailing edge ordinate location and trailing edge thickness, respectively, addition of z_{te} into the equation results in modified CST while standard CST omits this term. N1 and N2 are constant with values of 0.5 and 1, respectively. The class function C_{N2}^{N1} is defined as:

$$C_{N2}^{N1}\left(\frac{x}{c}\right) = \left(\frac{x}{c}\right)^{N1}\left(-\frac{x}{c}\right)^{N2} \tag{3}$$

The general shape function S_U (for upper surface) and S_L (for lower surface are defined as:

$$S_U\left(\frac{x}{c}\right) = \sum_{i=0}^{N_U} A_U(i) S\left(\frac{x}{c}, i\right) \tag{4}$$

$$S_L\left(\frac{x}{c}\right) = \sum_{i=0}^{N_L} A_L(i) S\left(\frac{x}{c}, i\right) \tag{5}$$

Where A_U and A_L are upper and lower surface weights, respectively. N_U and N_L are the CST order of upper and lower surface, respectively. The component shape function, which could be different for upper and lower surface depending on CST order N, is expressed as:

$$S\left(\frac{x}{c}, i\right) = K_i^N \left(\frac{x}{c}\right)^i \left(-\frac{x}{c}\right)^{N-i} \tag{6}$$

With K is a binomial coefficient and defined as follows:

$$K_i^N = \frac{N!}{i!\,(N-i)!} \tag{7}$$

Figure 1 shows an example of airfoil with and without additional trailing edge ordinate location using the same CST weights for both upper and lower surface.

Powell modify the existing CST for better representation of transonic airfoil [2]. The modification is done by using Genetic Programming to find new class function that "optimized" the shape-fitting for transonic airfoil. For general SC airfoil, the class function for upper surface is:

$$x(1-x)(1.265x - 1.4x^{1.28x-0.55}) \tag{8}$$

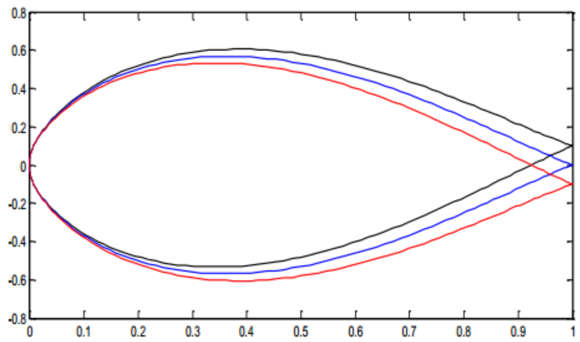


Fig. 1 : Standard CST (blue line), CST with $z_{te} = 0.1$ (black line), and CST with $z_{te} = -0.1$ (red line).

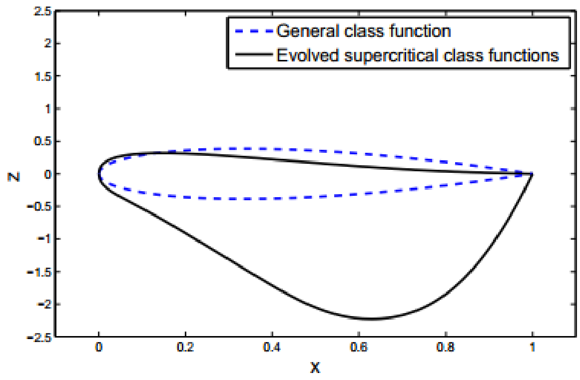


Fig. 2 : Comparison of general and evolved class function with same weights

The class function for lower surface is:

$$x(1-x)(-1.4x^{1.28x-0.55} - 11.335x + 0.0399\sin(18.8x+1.3) - 1.2) \tag{9}$$

Example of the output from this evolved class function is shown in Figure 2 which is adopted from Powell [2].

In the original paper, this evolved class function is used on single-objective optimization of transonic airfoil to reduce the drag. The extension and study of this new class function for multi-objective optimization of transonic optimization is performed in this paper. Global optimization is more difficult for this evolved class function because of the high sensitivity of the geometric shape to the evolved shape parameters. Moreover, the relation of CST parameters to the real geometry is not intuitive. Clearly, there should be a way to reduce this difficulties and we tried to address this on the next chapter.

3 Deterministic ransonic airfoil multi-objective optimization design with CST

In this paper we perform a preliminary study on the implementation of CST parameterization to the airfoil multi-objective optimization case. One of the difficulties of CST is that its parameter value is not straightforward and closely related to the real airfoil parameter such as PARSEC. This could become a difficulty if one want to perform global optimization using CST as its airfoil parameterization since it is difficult to find suitable CST parameter value. Here in this paper we propose a simple approach to find suitable CST parameter for global optimization by using the information from well-known airfoils. In this paper we use a set of well-known supercritical airfoils to generate the lower and upper bound for the optimization scheme. The variable bounds are found by analytically fitting some well-known supercritical airfoil using CST with general and evolved supercritical class function and then use the maximum and minimum values of these CST parameters. The objective in this work is to maximize C_l (equivalent to minimize $-C_l$) and minimize C_d, which is performed to address all aspects of aerodynamic design [3]

The mathematical model for the optimization is:

$$\text{minimize:} \ -C_l, C_d \tag{10}$$

Before we proceed further, we tried to investigate the effect of additional order of CST to be implemented into the optimization algorithm (solver, optimization, and mesh generation are explained in chapter 4). It is clear that the additional parameter is an advantage, because the degree of freedom of the geometry is higher. However, it comes with a cost of additional complexity for the optimization. The optimization is done using general class function and the value of upper and lower bound are obtained using the existing airfoil. The cases tried are standard CST without (Standard) and with additional trailing edge term (modified) and with 5 and 6 variables, denoted by CST 5 Std, CST 5 Mod, CST 6 Std, and CST 6 Mod. The result that is depicted on

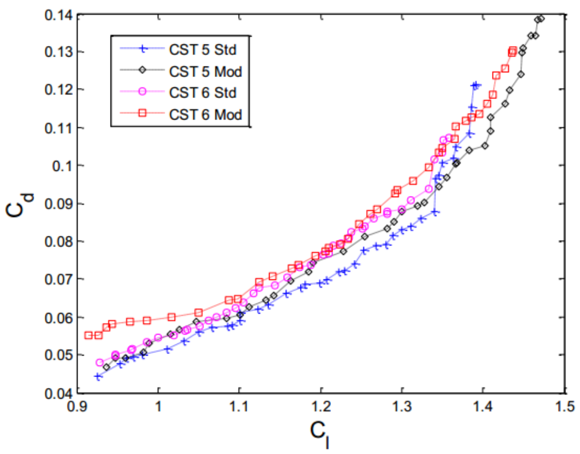


Fig. 3 : Pareto front of transonic airfoil preliminary optimization

Table 1: Free bounds

Variables	Lower Bound	Upper Bound
A_l^0	-0.5000	-0.0500
A_l^1	-0.5000	0.1000
A_l^2	-0.5000	0.1000
A_l^3	-0.5000	0.1000
A_l^4	-0.5000	0.1000
A_u^0	0.0500	0.5000
A_u^1	0.0500	0.5000
A_u^2	0.0500	0.5000
A_u^3	0.0500	0.5000
A_u^4	0.0500	0.5000
z_{te}	-0.02	0.02
dz_{te}	0	0.01

Figure 3 shows that the additional complexity dominates the advantage as it could be seen that the additional CST parameter only decrease the quality of the pareto optimum. Additional trailing edge term is also beneficial to widen the range of pareto front but is slightly decreased in term of converge. By seeing this preliminary result, further optimization process uses modified 4^{th} order CST with 5 variables for each surface.

This work uses CST with polynomial order of 4, means that there are 5 variables for each upper and lower surface. Together with trailing edge ordinate location and trailing edge thickness, there are 12 parameters that act as the optimization decision variables. The optimization in this research uses the following airfoils for finding the CST parameters: RAE-2822, SC(2)-0410, SC(2)-0610, SC(2)-0710,SC(2)-0412,SC(2)-0612, and SC(2)-0712. For free bounds, general class function, and Evolved Supercritical class function (ESCF) the lower and upper bound is shown in Table 1,2, and 3 respectively.

4 Flow Solver and Optimization Algorithm

To solve the flow field, an open source Euler Solver from SU2 [4] package is used to obtain the aerodynamic coefficients of airfoil generated by CST during the optimization process. Euler mesh is generated with O-grid farfield type and farfield distance of 20 chord from the airfoil with the example is shown in figure 4 and 5. The mesh is generated using the algorithm of Per-Olof Persson which provides a quick and high quality mesh generation [5]. A dedicated program is developed to easily interface the optimization algorithm (explained later) with mesh generation and flow solver. The input for mesh generation program is the coordinate of the airfoil, distance to the farfield, and mesh concentration near the airfoil.

NSGA-II [6] which is a multi-objective optimization algorithm based on the principle of natural evolution is used to optimize the shape of transonic airfoil. By using the principle of non-domination, NSGA-II continously update the solutions until it discover the pareto front or stopped on some pre-defined number of generations. The desired final solutions generated by NSGA-II is hopefully lies in or near the pareto front, which is a set of solutions that are non-dominating to each other.

The optimization algorithm, mesh generation, and flow solver is combined into a single script consists of MATLAB and Phyton script to perform the optimization continuously. Any non-converged solution is given a hard penalty so it could not survive in the recombination selection of evolutionary algorithm. The optimization for each scenario is performed with population size and maximum generation number of 50 and 50, respectively. The flight condition of Mach Num-

Table 2: General CST variable bounds

Variables	Lower Bound	Upper Bound
A_l^0	-0.2167	-0.1300
A_l^1	-0.1331	-0.0747
A_l^2	-0.2733	-0.1398
A_l^3	-0.2097	-0.0734
A_l^4	0.0378	0.3251
A_u^0	0.1275	0.2118
A_u^1	0.0974	0.1404
A_u^2	0.1603	0.2376
A_u^3	0.1556	0.2075
A_u^4	0.1981	0.3356
z_{te}	-0.0147	0.0005
dz_{te}	0	0.0060

Table 3: Evolved CST variable bounds

Variables	Lower Bound	Upper Bound
A_l^0	-0.1383	-0.0867
A_l^1	-0.0463	-0.0026
A_l^2	-0.0560	-0.0156
A_l^3	-0.0083	0.0065
A_l^4	0.0042	0.0200
A_u^0	0.0809	0.1434
A_u^1	0.1279	0.1899
A_u^2	0.2743	0.4226
A_u^3	0.1094	0.4595
A_u^4	1.5221	2.5328
z_{te}	-0.0147	0.0005
dz_{te}	0	0.0060

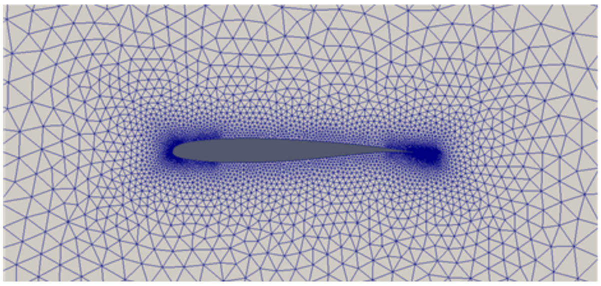


Fig. 4 : Sample of computational domain near airfoil

ber and angle of attack are 0.8 and 2^0, respectively.

5 Results

The optimization result in objective spaces is shown in figure 6. From the result it is visually clear that the result from Free bound dominates the result from general and evolved class function. To be noted is that there is no geometrical constraint because we want to see the maximum range that is attainable by CST parameterization on multi-objective transonic airfoil optimization. It is also clear to see that the evolved class function dominates the result from general class function but with small burden on the pareto front span.

Result shows that the general class function CST with free bound could find a wide range of airfoil by still generating feasible shape. The depiction of the flow field and airfoil shape from the optimization with maximum C_l and minimum C_d is shown in Figure 7 and 8, respectively. However, the shape of a set of airfoils itself is quite unrealistic and probably need constrains for remedy, which could add the complexity of optimiza-

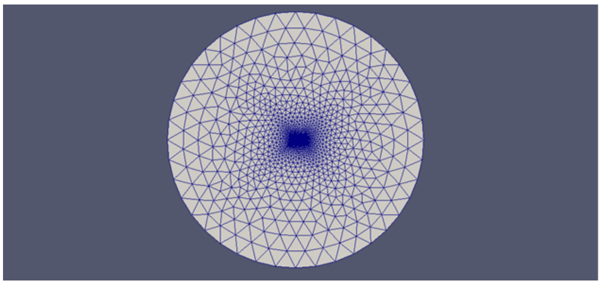


Fig. 5 : Sample of computational domain on the farfield

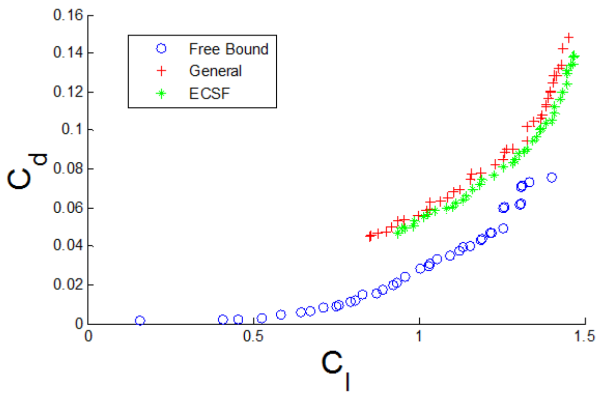


Fig. 6 : Pareto front from NSGA II

tion problem. However, it is interesting to be noticed that airfoils from free bounds generate very weak shock, thus a very low wave drag. To be noted is that the optimization using free bounds is performed to see the capabilities range of CST and is not mean to be used for real optimization because of the possibilities of unrealistic shape. Nonetheless, the result from free bound shows the extreme result very clearly where the airfoil with maximum lift coefficient has very high camber and very small thickness and vice versa for the airfoil with minimum drag. The result from this free bounds is possibly an unstable airfoil that is not robust to the uncertainty in the environmental condition.

Using maximum and minimum bound from existing airfoil could help this problem by generating realistic pareto optimum shape of airfoil. As it is already mentioned before, these bounds are generated using the maximum and minimum value of CST parameters derived from the existing airfoils. It could be seen on Figure 7 and 8 that the optimum airfoils still resembles the usual geometry supercritical airfoil which is a realistic shape. The shape is realistic for both airfoil with minimum drag and maximum lift coefficient and there is a clear trade-off between lift and drag coefficient as it is already well-known. The trend is also the same with free-bounds optimization where the minimum drag design doesn't generate strong shock waves while the maximum lift generates a strong and large negative pressure region with higher drag, the fact that was also mentioned in the paper of Oyama [8].

If the evolved class function is used, more superior pareto front could be obtained which is indicated by the major portion of the solutions that dominate the solution from CST with general class function. The objective function values look similar but it is clear to see the difference between both of the pareto optimum. Both pareto optimum are dominated by the result from free bounds, but the actual airfoil shapes are far more realistic.

There is no great difference on the optimum geometry by examining the shape generated by general class function and evolved supercritical class function. As it could be seen on Figure 9,

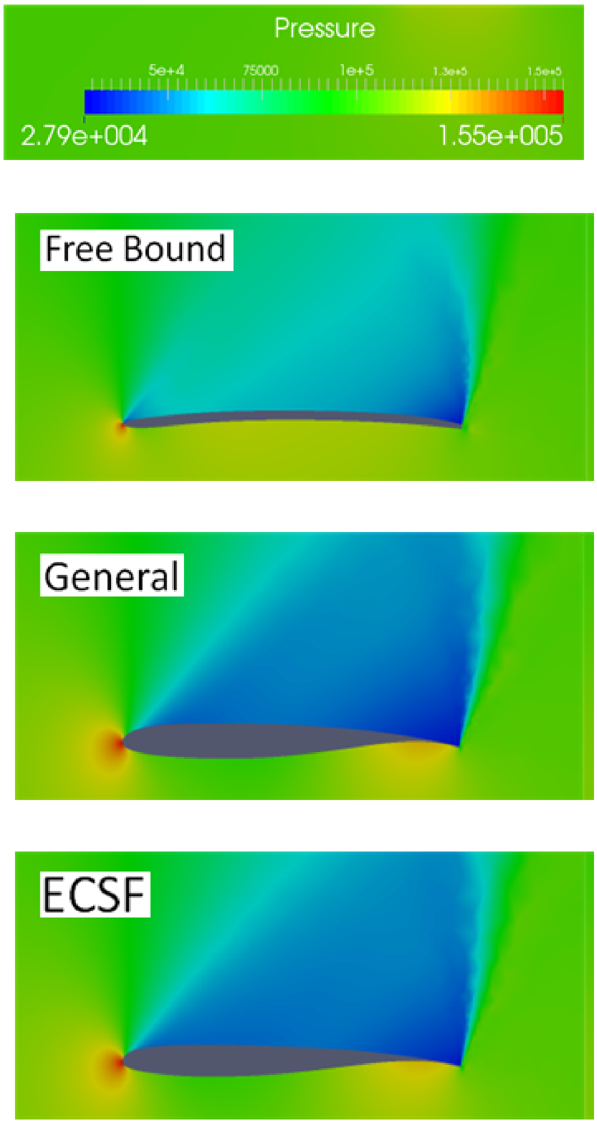


Fig. 7 : Airfoil with maximum C_l from all optimization

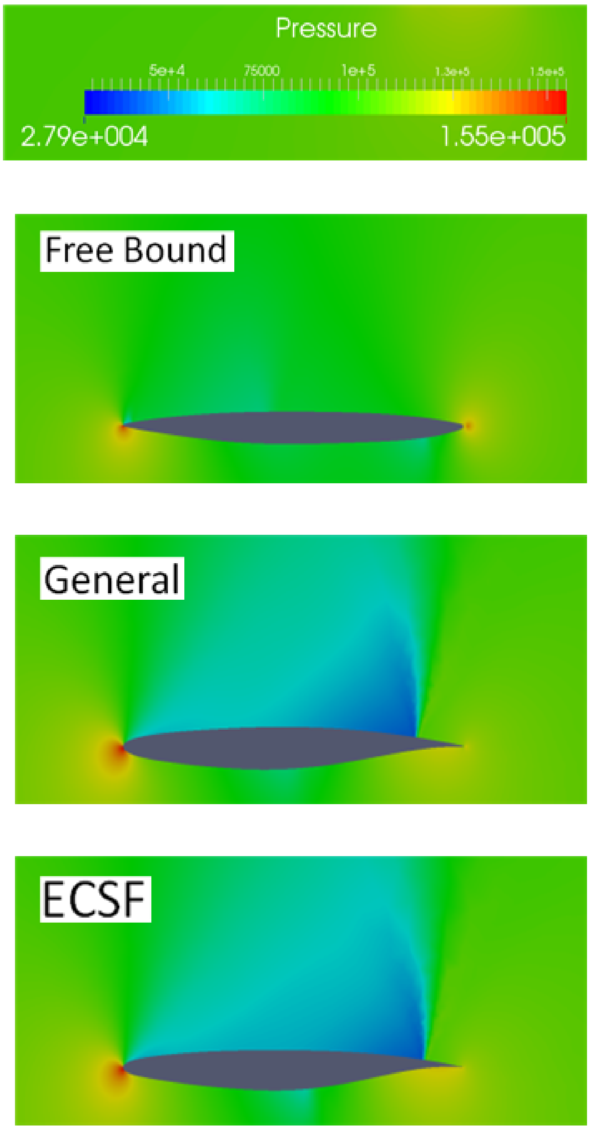


Fig. 8 : Airfoil with minimum C_d from all optimization

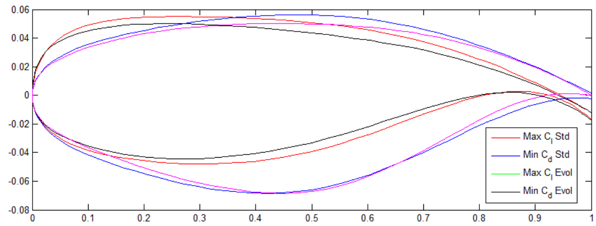


Fig. 9 : Extreme result from general and evolved class function

the extremum airfoil shape for both general and evolved class function roughly have the same feature. This is due in fact that both optimization scheme use the same existing airfoils to generate the lower bound and upper bound. Notice that the generated lower and upper bound are different for both scheme because of the different class function. The evolved class function has more control on the feature of the transonic airfoil because it is specially derived for the transonic airfoil shape. Meanwhile, even general class function can represent any shape of airfoil it find difficulties if there is a drastical change on the shape of airfoil such as in the lower surface of the supercritical airfoil. The airfoil with minimum drag coefficient is thicker than its maximum lift coefficient counterpart. This is because for high lift design the lower surface is moved upward which corresponds to the camber increase as it already pointed by Oyama [8]. The airfoil with maximum lift coefficient also takes the advantage of the freedom on trailing edge location where the trailing edge is now slightly bend downward. Please note that in this work the geometrical constraint are not imposed. It is seems that if the bounds are derived from existing airfoils, we could control the geometrical thickness without any additional constraint. However, it is recommended to still including the geometrical constraint on the optimization scheme.

The relation between CST parameters and actual shape is far than intuitive for evolved class function so careful consideration is needed to tune the bound of decision variables. Using the information from existing airfoils, any unrealistic shape could be prevented. Powell [2] also use similar approach where the starting point is based on the existing airfoil and slightly changes the base CST parameters. Examining the optimum result, the airfoil with maximum ratio of L/D coefficient try to balance these two conflicting objectives.

The next step after deterministic optimization is robust optimization. The evolved class function is chosen as the general class function because it shows good performance on deterministic optimization case.

6 Robust Optimization of L/D ratio

Robust optimization is an ongoing trend in field of aerodynamic optimization due to the realization that the deterministic optimization tends to find solutions that are fragile to the uncertainty. Examples of this uncertainty are the uncertainty in Mach number and angle of attack which are alleatoric that could be interpreted in probabilistic framework. There is also an epistemic (model form) uncertainty that is caused by our lack of knowledge to understand the true nature of the system. In airfoil transonic optimization literature, the robust optimization also could be defined as the optimization to minimize the expected drag coefficient in a range of Mach number. On the next step, we perform a multi objective robust optimization using CST with evolved class function. This is a preliminary robust optimization of CST with more complex robust cases is an ongoing research. Here, the goal is to maximize and minimize the mean and variance of L/D ratio, respectively.

$$\text{minimize: mean } L/D, \text{ std } L/D \tag{11}$$

The L/D ratio is chosen as the objective because it The uncertainty quantification technique used in this paper is the non-intrusive stochastic expansion method, where the underlying response is approximated by polynomials that are defined by the type of the probability distribution of the random variables. The non-intrusiveness of this method allow the use of code that can be treated as the the black box.

The concept of uncertainty quantification using stochastic expansion method is to approximate the functional form between the stochastic response output and each of its random inputs with the following chaos expansion:

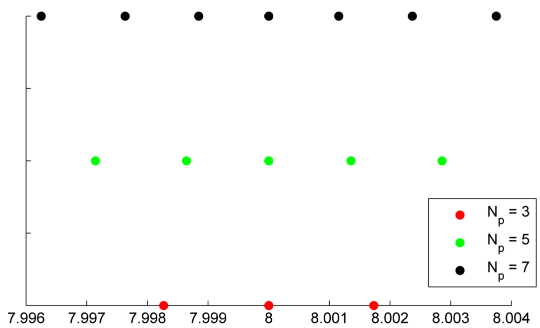


Fig. 10 : Collocation points for robust optimization case (x-axis is the Mach number

$$R = \sum_{j=0}^{\infty} \alpha_j \Psi_j(\xi) \tag{12}$$

The coefficient is estimated by performing spectral projection and using tensor product /sparse grid to expand the stochastic expansion and obtaining the collocation points:

$$\alpha_j = \frac{\langle R, \Psi_j \rangle}{\langle \Phi_j^2 \rangle} \tag{13}$$

For this preliminary robust optimization experiment, the random variable is the Mach number that normally distributed with mean of 0.8, and standard deviation of 0.001. To match the normal distribution, Hermite polynomial and quadrature are used to accurately approximate the response surface of the random variable where the depiction of Hermite polynomials could be seen of Figure 11 . The lower and upper bound of evolved CST is the same with the deterministic case as it is shown in table 3, except that the angle of attack is now set to zero. For this case the location of collocation points for 3,5, and 7 points are shown in Figure 10. In this work, we use the collocation point number of 5.

7 Robust optimization result

Result shows that the preliminary experiment of robust optimization of CST is able to find the trade-off between mean and the variance of the L/D ratio. As it could be seen on figure 12, mean and variance of L/D ratio are highly conflicting

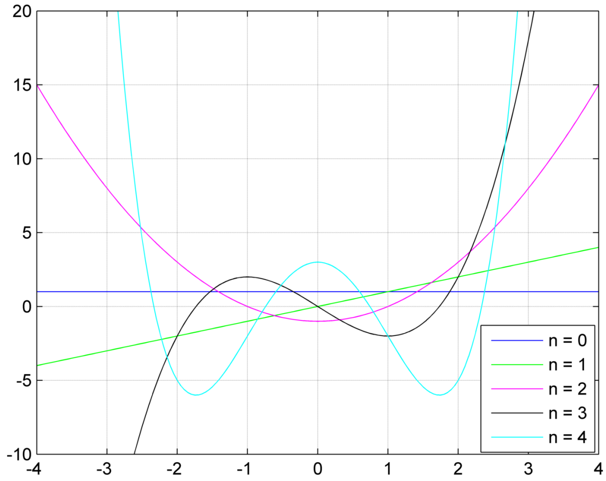


Fig. 11 : Hermite polynomials with order from 0 to 4

objectives. High lift to drag ratio only can be achieved with very high instability indicated by very high standard deviation. In other side, relatively low L/D ratio seems to possess the property of reasonable and low standard deviation of L/D ratio. The analysis is given on the following paragraphs.

Analysis is first done by seeing the C_l and C_d of the extreme airfoils (maximum mean and minimum standard deviation) that is listed on the table 4 where A and B are the airfoil with minimum mean and maximum standard deviation of L/D, respectively. The standard deviation of C_l and C_d of each airfoil is roughly similar, both airfoil seems to have the same variation trend of aerodynamic coefficient if uncertainty in Mach number is subjected. However, if the trend of C_l on the collocation points is analyzed, the airfoil with minimum mean of L/D ratio has a C_l trend that reduced if the Mach number is increased which is the opposite for the maximum standard deviation. Because the C_d trend for both airfoil is increasing together with the Mach number (caused by stronger shock), minimum mean airfoil has lower standard deviation of L/D ratio compares to the maximum standard deviation airfoil.

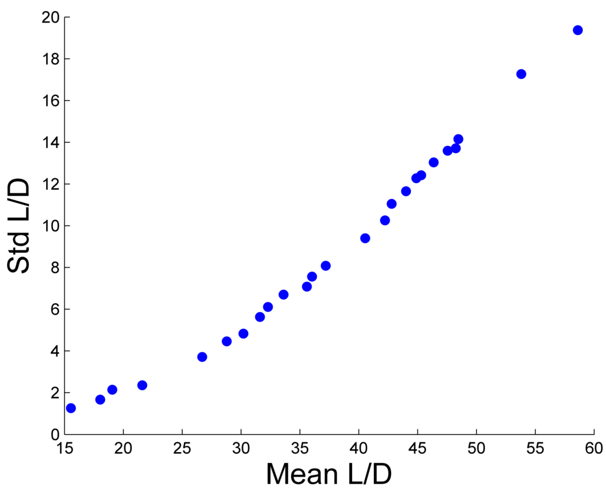


Fig. 12 : Pareto front of the robust optimization

Table 4: Aerodynamic coefficients at the collocation points

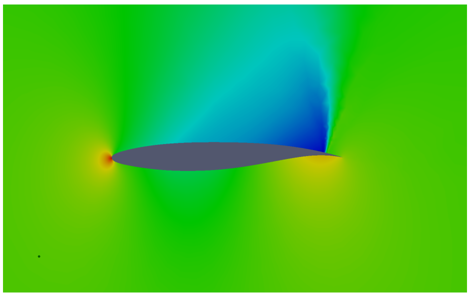
	C_l (A)	C_d (A)	C_l (B)	C_d (B)
M = 0.785	0.884	0.0447	0.576	0.004
M = 0.793	0.872	0.0503	0.600	0.007
M = 0.800	0.854	0.0552	0.624	0.011
M = 0.806	0.830	0.0596	0.644	0.017
M = 0.814	0.802	0.0635	0.660	0.023

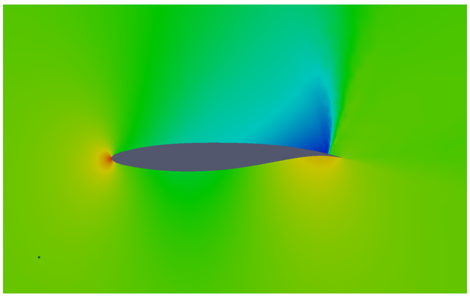
By analyzing the flow field of the extreme airfoils (Figure 13 and 14), both airfoil shows similar evolution of shock, even though the minimum mean airfoil shows stronger shock and thus resulting in stronger aerodynamic forces, and vice versa. This result in similar trend of the standard deviation of C_l and C_d with increasing Mach number.

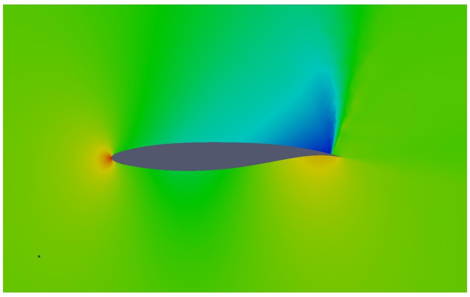
By seeing the geometry comparison as it depicted on Figure 15, there is no great difference between the geometries of airfoil with maximum mean and minimum variance of L/D ratio don't differ that much. This is because the lower and upper bound of the evolved class function only allow the geometry variation that resembles supercritical airfoil only. The airfoil with minimum standard deviation has higher camber on the trailing edge part compares to the other extreme airfoil but more analysis and data mining are needed to see if some certain feature of geometry really affect the mean and standard deviation of L/D ratio.

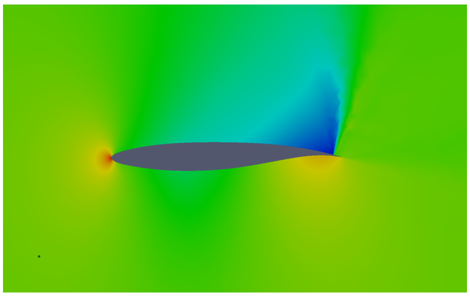
8 Conclusion

By comparing the three results from free bound, general, and evolved class function, some conclusions could be made. Optimization with free


(a) M = 0.785


(b) M = 0.793


(c) M = 0.800


(d) M = 0.806

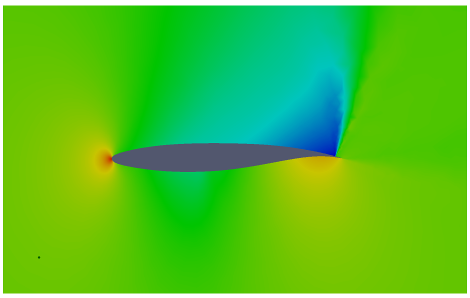
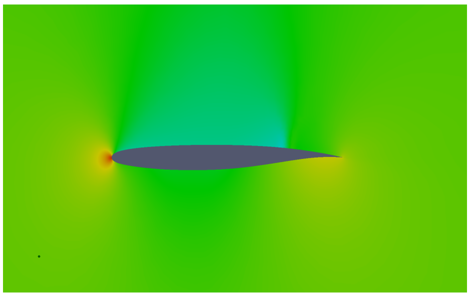
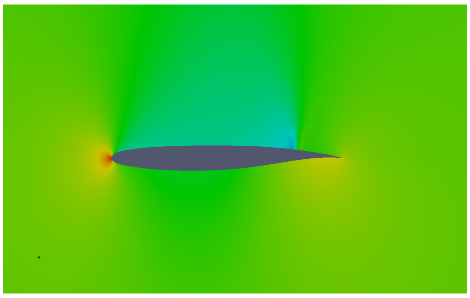
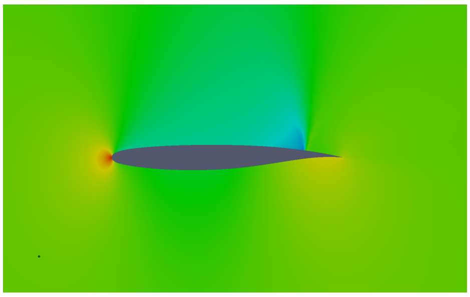
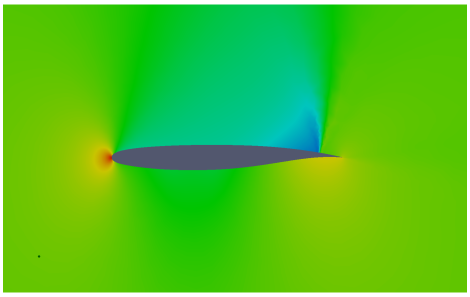

(e) M = 0.814

Fig. 13 : Flow field of airfoil with minimum mean of L/D and varying Mach number


(a) M = 0.785


(b) M = 0.793


(c) M = 0.800


(d) M = 0.806

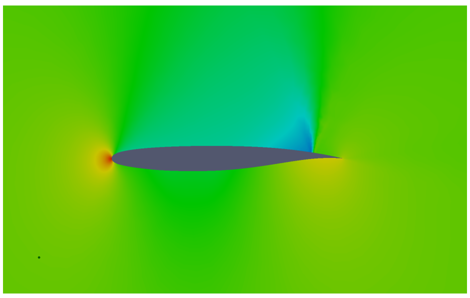

(e) M = 0.814

Fig. 14 : Flow field of airfoil with maximum std of L/D and varying Mach number

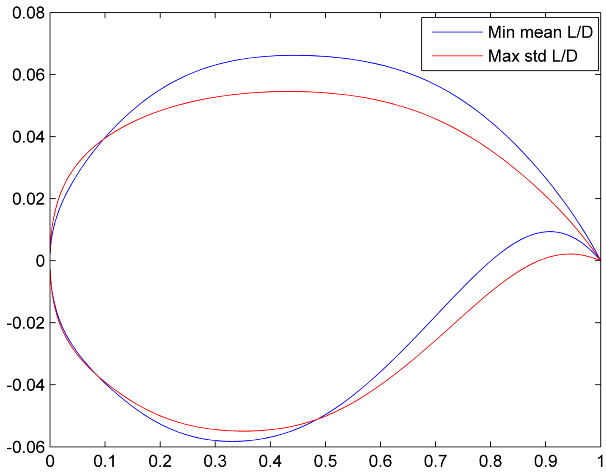


Fig. 15 : Geometry comparison of the extreme airfoils

bound is useful to see the range capability of CST but many additional consideration such as the possibility of far from realistic airfoil is generated. The additional thickness constraint could be imposed but because the design space is too wide it would becomes difficult since the ratio of infeasible to feasible design space is quite high. it is advantageous by limiting the bounds with the geometries of existing airfoil with either general or evolved supercritical class function. The advantage is located on the realistic shape given in the end of optimization procedure and more focused search. The evolved class function is superior if compared to the general class function which is indicated by its solutions that dominate the solutions from general class function. Therefore, for future optimization such as local optimization, evolved class function is preferable and it is attractive to be explored further. Some of works that specifically study CST parameterization can be found on [9], [10], [11], and [12].

Result from robust optimization also shows that the application of CST could find good representation of the tradeoff between mean and standard deviation of L/D ration, which are two highly conflicting objectives. Coupled with stochastic expansion, the optimization algorithm and CST provide an effective framework for robust optimization. However, more function evaluations are needed to see if anymore optimization is possible.

Included in our future research is to perform robust optimization with higher number of CST parameters, but since the optimization process could be very expensive, a combination of global and surrogate assisted local search is now our ongoing research. The use of local surrogate model has a purpose to accelerate and improve the quality of final pareto front in high dimension, motivated by the curse of dimensionality of global surrogate model in high dimension. The use of Navier stokes equation instead of Euler solver is also anticipated in the near future.

9 Future Work

We are working our way toward effective aerodynamic robust optimization with uncertainties and this work is a first step toward the airfoil robust optimization work. Future work will involves the robust optimization with CST and study the effectiveness of CST parameterization in airfoil robust optimization context.

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