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CONSIDERATION OF THE SERVICE LIFE TAKING INTO ACCOUNT A LOT OF ONE-TYPE STRESSED UNIT ELEMENTS

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Abstract

The probabilistic model of regular zone fatigue is proposed in this paper. The model with probability specified makes it possible to estimate the fatigue before the initiation of the first crack in regular zones (RZ) that are potentially dangerous as the multisite damages (MSD) can occur. This model demonstrates also the approach for RZ designing conditioned by the fatigue strength that is based on the optimal choice of geometric shapes and the mode of deformation with account not only the structural technological characteristics but also the quantity of one-type stress concentrators on different RZ.

1 History, problems and objectives

In the late 60s the operation intensity growth, the tendency to minimize weight by the application of the high-strength materials and the updating the static strength estimation resulted in the fact that the designing conditioned by static strength criteria did not provide the service life required. In the early 70s, the procedure was proposed [1] to estimate the wings of civil and transport aircraft as well as the recommendations to apply this procedure at initial design stages in order to select the allowable stresses level of the future structure conditioned by fatigue strength. The S-N curves (the panel curves) formed the basis of recommendations that are given in this paper. The panel curves were plotted based on test results of about 200 full-scaled civil aircraft wings panels. Which were tested under pulsating tension and had not less than two stringers been riveted to the skin and the structural irregularity (cuts, transversal joints, holes and so on). the Based on the results of comparing the analytical fatigue and the experimental one up to the initiation of cracks in those places where they emerged in these tests of the wing lower surface, in order to select the allowable stresses level in the most fatigue critical zones (rated sections) it was proposed to use the fatigue curves of sample with hole and for the rest of sections it was proposed to use the "panel" curves.

The approach to estimate the fatigue and the life time up to the present time is not changed and the general idea is that the fatigue distribution function (FDF) is assumed to be lognormal for the essential list of principal structural elements (PSE). The PSE averaged fatigue value is estimated by the results of full-scale test and in order to assess the life the normative coefficients are applied. During thirty years in the course of life tests the critical zones were detected and reinforced. This resulted into considerable growth of structural irregularities life, but while unriveting after the life tests completed it turned out that numerous cracks were detected caused by irreparable typical stress concentrators in panel joints, skin and stringers joints, spars joints that form the RZ.

Let the structure elementary segment that has only one typical stress concentrator be a unit element (UE). The quantity of RZ UE is estimated at thousands, *which based on level values that are equivalent to the typical flight stresses are divided into groups (segments) inside which when <u>highly accurately estimating</u> the fatigue the stress in each UE may be regarded as an equal one if the difference between the max value and the min one is less than 4.9MPa.*

Table 1. Total number of UE and equal stress segments as well as the range of UE and equal stress values at these segments for three types of A/C.

A/C	Number of UEs in wing lower surface RZ	Number of equal stress segments	RZ min stress, MPa	RZ max stress, MPa	Min number of UEs in segment	Max number of UEs in segment
Type 1	7866	14	58.6	150	42	1121
Type 2	15138	23	73.5	172	80	1626
Type 3	31456	13	86.7	150	110	8014

Each of UE is a critical zone where the crack may arise under operating time. The catastrophe may occur if the cracks arise in one transversal section of the wing and are not detected at the proper time. The crack that arises in fuselage RZ and that is not detected is more hazardous in terms of initiating the multisite damage (MSD). In order to indicate the problem the Table 1 shows the statistical data on UEs quantity per segment and on recalculated stresses under segmentation of wing lower surface into equal stress segments for three types of aircraft. Hereunder the word “segment” will be used instead of “equal stress segment”. Under such stresses levels and such a quantity of UEs on RZ zones the possibility can’t be excluded for cracks to arise in some aircraft RZ earlier than the design life expires. This was proved by the results of unriveting that followed the life tests. The UE averaged fatigue value is higher than the averaged fatigue value of the segment with unloaded hole, but the root-mean-square deviation (RMSD) in the first case is higher than in the second one, especially if the bolts with interference is used. And if the one-type UE quantity per a segment is taken in account then the selection of estimated stresses based on unloaded hole strip tests results may be incapable to provide the required enhancement of life characteristics.

Fig. 1 shows the mean UE segment fatigue (under normal logarithms distribution) to averaged value of minimal UE among the n family with the same parameters ratio under different RMSD values. Let the UE FDF true parameters of the 200 UE seam part be $a^{1}_{lg\,N} = 5.6$ и $\sigma^{1}_{lg\,N} = 0.25$ (with taking into account the $\bar{N} = 469845$ scattering. It is obviously that under $n = 200$ along the X axis and $\sigma^{1}_{\lg N} = 0.25$, the fatigue degradation coefficient along the У axis(Fig. 1) is equal to 5, i.e. the averaged fatigue before the first crack arises on the n UE part will be 469845/5=93969 cycles.

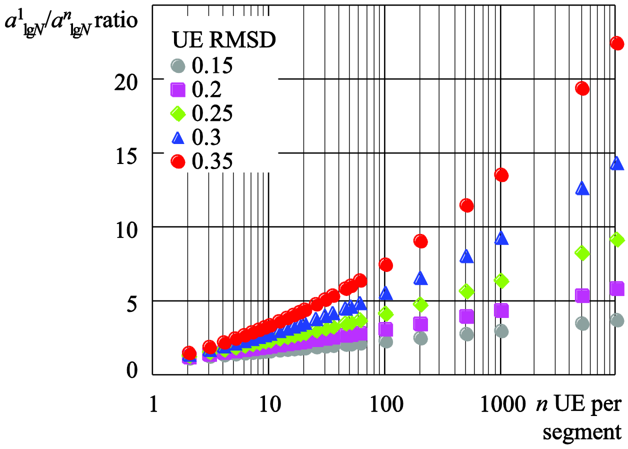

Fig. 1. The fatigue degradation coefficient vs. the RMSD and UE quantity per segment

To implement the fatigue prediction before the first crack arises in the RZ at the stage of design the following is required:

– The estimation procedure for maximal stresses (from the UE zero-to-tension stress cycle), which are equivalent by damageability to type flight mission profile. This procedure is to be performed with the acceptable accuracy and with taking into account the joints operational specificity;

– The RZ fatigue model that includes the FDF before the first crack arises in the RZ (Φ_e), the FDF of RZ part (F_y) and RZ i.e. the zone that includes several parts (F_{RZ});

– The methods to obtain the original data to calculate the UE FDF estimations with taking into account the technological and manufacturing factors;

– The statistical methods that allow obtaining the estimations of UE FDF parameters based on full-scale tests.

2 RZ fatigue estimation procedure

To calculate the mode of deformation in the late 80-s the methods were developed to estimate the maximal stress of zero-to-tension stress cycle that is equivalent to the typical flight. These methods were quite acceptable for preliminary estimation of the RZ stress state [2]. When the contact loads due to shear present in the zone, (landing gear hinge fitting, pylons, engines, flaps and ailerons and so on) one has learned to take into account their effects by use of K_{sm} coefficient [3]. As far as the results are being obtained at the design engineering stage, the initial estimations are possible to be corrected/updated.

Let the *assumptions* adopted to develop the probabilistic model be stated. If the PSE each taken separately determines the aircraft life then the RZ fatigue model must take into consideration all the UE family as a whole. The Russian experience in verification of specimens fatigue tests results confirms, often and with high probability, the truth of normality hypotheses for fatigue logarithms of specimens with different stress concentrators. In accordance with this, let us consider the fatigue logarithms before the cracks arise in UE in aircraft RZ segments. These logs are to be treated as the samplings of *independent random normally distributed values* with $a^i_{lg\,N}$ averaged values and $\sigma^i_{lg\,N}$ RMSDs the spaces which of (the number of UEs in *i*-th segment)) are known and are equal to n_i.. Principally for UE the Weibull distribution may be taken as for the model under consideration the selection is not a closing argument.

The *independence* is the second requirement that consists in estimation of the crack length, up to which the damage effect in adjacent UEs remains unchangeable. To define this length the criterion of change of stress state in adjacent UEs may be used [4]. Let $2l = l_1 + d + l_2$ be a crack length, where l_1 and l_2 are crack paths towards the both sides of UE, d –is a hole diameter; $2b$ is a distance between adjacent UE centers. From [4] it follows that the stress in the neighboring hole zone increases not more than by 5%, if $l/d \le 0.3$,and not more than 13%, if $l/d \le 0.5$. In dependence of the geometry and the segment stress levels the first requirement or the second one is possible to be adopted to define the l_0 required crack length, up to which the fatigues, before cracks arise in adjacent elements, may be treated as independent random values. Moreover, it is to be noted that the condition for the fatigue independence before the first crack appears is nor very tough. Eg, the paper [5] demonstrates that the limiting distributions for the dependent values sampling extrema under specified conditions are the same as for the independent ones. Under these assumptions let the probabilistic model of RZ fatigue before the first crack with the length not more than l_0 in any RZ segment arises be formulated.

2.1 Probabilistic model of RZ fatigue

Under the calculated substantiation of RZ designed life it required to show that the operating time before the l_0 crack arises in each RZ segment is to be not less than the N designed life under the probability, which is not less than the p normative specified value.

Based on the UE fatigue lognormal distribution assumption it follows that the Φ_e UE fatigue logarithm distribution integral function *is a function that for any lgN value defines that the probability of that the fatigue logarithm before the crack arises in each UE will be* $\le lgN$. It is formulated as follows:

$$\Phi_{ei}\left(lg\,N, a_{lg\,N}, \sigma_{lg\,N}\right) = \frac{1}{\sqrt{2\pi}\sigma_{lg\,N}N}\int_{-\infty}^{\frac{\lg N - a_{lg\,N}}{\sigma_{lg\,N}}} e^{-\frac{t^2}{2}}dt. \quad (1)$$

This function depends on two parameters: the averaged value and the UE fatigue logarithm RMSD. If the both parameters are known then for any N operating time value it is possible to calculate the p probability of crack appearance in UE under $\le N$ operating time and the 1-p probability of crack appearance under $> N$ operating time.

The integral FDF of the i-th F_{yi} *segment is defined as the function that specifies the probability that the crack will arise at* $\le \lg N$ *fatigue logarithm in the i-th segment at least from one of the UEs.* This definition is conformed by the followed function:

$$F_{yi}\left(lg\,N, a^{i}_{lg\,N}, \sigma^{i}_{lg\,N}, n_i\right) = 1-\left[1-\Phi_{ei}\left(lg\,N, a^{i}_{lg\,N}, \sigma^{i}_{lg\,N}\right)\right]^{n_i}. \quad (2)$$

It depends upon the fatigue logarithm parameters in the *i*-th segment and the n_i quantity of UE in it. If the both parameters are known it is possible for any N to calculate the p-probability of crack appearance at least in one UE before N or the 1-p probability that the no crack will arise in any UE of the i-th segment before the N operating time.

Let the F_{rz}, integral FDF be defined as the function that specifies the probability that the fatigue logarithm will be ≤ lgN before the crack arises from UE at least in one of the k segments that have n_i quantity of UE. The function is formulated as:

$$F_{P3}\left(lg\,N, \Theta_a, \Theta_s, n_i, k\right) = 1-\prod_{i=1}^{k}\left(1-\mathit{\Phi}_{ei}\left(lg\,N, a^{i}_{lg\,N}, \sigma^{i}_{lg\,N}\right)\right)^{n_i}. \quad (3)$$

The number of parameters in this function is equal to 2*k*, where $a^{i}_{lg\,N} \in \Theta_a \; (i = 1, 2, \ldots$ is a set of averages fatigue logarithms values for all RZ segments and $\sigma^{i}_{lg\,N} \in \Theta_\sigma \; (i = 1, 2, \ldots$ is a set of fatigue logarithms RMSD for all RZ segments. The k-constant is a number of RZ segments and $n_i \in \Theta_n \; (i = 1, 2, \ldots$ i is a set of UE numbers in all segments.

The values of the both parameters depend on the operational factors, the material and semi-products strength characteristics, skins, geometrical correlation of joints, the surface treatment, the fastenings properties, the riveting regimes and riveting equipment. The *k* and Θ_n constants are specified at the design stage when selecting the geometric characteristics and the allowable mode of deformation. The part of factors that are in effect not only when producing the materials but also when assembling the product and mounting the fastening elements is impossible to reproduce when manufacturing the specimens in laboratory. Therefore the estimation of UE FDF are to be obtained based on results of the tests of full-scaled panels which are routinely produced at the plant in total conformity with the serial technology of aircraft panels manufacturing. The RZ model proposed allows, based on the samplings that are assembled of the fatigue values that are obtained by the obligatory tests for the life substantiation and by use of statistical methods to obtain the $\mathit{\Phi}_e$ parameters estimations that enclosed the implications of all the production and technology factors.

2.2 Methods to obtain the original data to estimate the UE FDF parameters

The large-size full-scale panels that are tested at the design stage, as a rule, have joints segments. Moreover, the full-scale joints elements are specially tested to select material, to make a decision on geometrical and technological parameters, to study the cracks growth rate and to obtain the crack growth resistance characteristics. The results of these tests contain the information on the *fatigue* of each UE that was achieved by this UE under the termination of test without damage if the crack did not arise in UE (*the fatigue before the censoration*) or the information on *the fatigue before the crack of a certain length arises*. The full-scale tests make it possible to form the samplings that have both the first UE fatigues and the second ones in order to obtain the $\mathit{\Phi}_e$ estimations.

Each sampling may be formed based on results of tests of identical full-scale elements with RZ segments (it concerns the material and semi-product of the parts being connected, the type and technology of mounting the fastening elements, presence or absence of gaskets/spacers and all joints characteristics that influence the fatigue). To make it the following procedures are used:

– Recalculation of all tests results for the equivalent pulsing cycle, for which the UE parameters will be obtained. When considering the possibility of different tests combination it is to be taken into account which inaccuracies may be introduced into recalculation [6].

– The unriveting and the instrument post-test inspection of the joint area allow estimating the crack length (if it presents) for each UE of the panel specimen at this fatigue or getting the information that no crack is in UE. The instrument inspection results allow generating the samplings which are to obtain by statistical methods the Φ_e parameters for cracks of different length and to recalculate for one crack length at this that length is chosen which is observed the most frequently under the instrument inspection. The Φ_e parameters calculation method for such a censoring is given in [7].

– The fractographic analysis of fracture that allow obtaining the small cracks growth rate.

– The cutting of joint area after the panel failure into plates, which of the samples are made and tested.

The cutting of joint area after tests allows to increase significantly the poor operational time before the cracks arise from UE due to full-scale panels tests. The paper [8] proposes the methodology to production, tests and processing the results of these tests. One of the samples variants is given in Fig. 2. The practice of testing such samples demonstrated that the failure occurs not only in the center but in all cuts, more often in three central UE and less often in peripheral ones. By use of these samples, the fatigue curve of tested joint UE may be obtained. (see Example 3.1). In this case, the attention must be paid to the selection of panel stress levels in order to minimize the recalculation inaccuracies. It is better when the stresses level are higher in all sample sections than those when testing this UE in panel.

The joint plates with equal longitudinal width present another variant of sample. In this case, the operating time is increased up to failure; the sample's fragments are tested after the failure by one of the holes. All the fatigues are recorded before the failure and the censoring with taking into account all previous tests. The issue, which of variants is optimal in terms of time, costs and minimal calculation inaccuracy is to be investigated further.

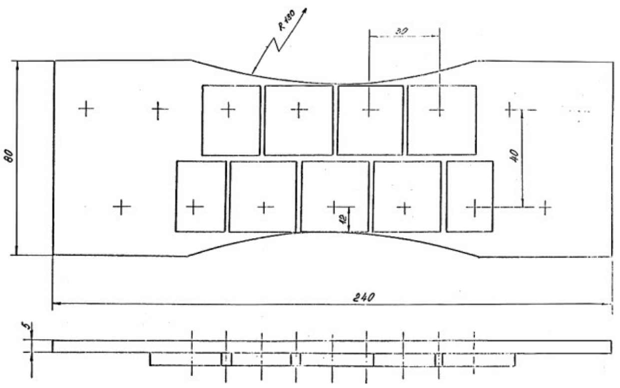

Fig. 2. Test sample sketch.

The worthiness of such tests is not only the possibility to take into account the industrial and technological factors but also to obtain rather good (near to the true values) the estimations of RZ segments parameters by regulation of censoring by means of selecting the stress levels and UE number when manufacturing the specimens. The parameters estimation quality depends not only upon the total sampling amount (number of tested UEs) and the number of fatigues before the first crack arises but in significant degree upon the uniformity of operating time alteration before the cracks arise and before the censoring within the entire fatigues range obtained.

The UE fatigues that are obtained when testing the n full-scale elements (or the specimens) with identical joints segments and are recalculated for the equivalent pulsing cycle stress are shown as follows:

- N_1, N_2, …, N_m – are recalculated values of UE fatigues;
- l_1, l_2, …, l_m – number of UEs where under N_i $(1 \le i \le m)$ fatigue a crack (failure) is present;
- n_1, n_2, …, n_m – number of UEs where under N_i $(1 \le i \le m)$ fatigue a crack (failure) is absent.

Then $\sum_{i=1}^{m} l_i + \sum_{i=1}^{m} n_i = \sum_{i=1}^{n} k_i$, где $\sum_{1}^{n} k_i$ is a total number of tested UEs and m is a number of fatigues before the crack (failure) arises that were recorded during tests. M-number of fatigues obtained during the testing may be significantly higher that n-number of tested full-scale elements and/or specimens if there is a unriveting and NDC before the detecting the cracks in the course or after the tests or the cutting and additional testing the specimens.

2.3 Statistical methods to estimate the Φ_e parameters bases on censored samplings

Two methods are presented to calculate the Φ_e parameters estimations: the least-squares method (LSM) by Johnson and the maximum likelihood method (MLM) [9]. The calculations are carries out of identical joints tests results that are recalculated for one equivalent stresses level and for one crack length.

Calculation of $\tilde{\iota}$ $\tilde{}$ *estimations of* Φ_e *parameters (LSM by Johnson)*

To plot the empirical distribution function (by Johnson method) [10] the logarithms of all N_i UEs recalculated fatigues are ranged in general variational series, numbered throughout. Then the N_i before failure (or with crack) is marked (e.g. by figure 1 in the next column). If the N_i before failure coincides by value with N_i before censoring, then the fatigue before failure is to take the first place in variational series. For the N_i before failure the r_i frequency is estimated with taking into account the UEs without failure by the recurrent formula:

$$r_i = r_{i-1} + \frac{\sum_{i=1}^{n} k_i + 1 - r_{i-1}}{\sum_{i=1}^{n} k_i + 2 - i},$$

where $\sum_{1}^{n} k_i$ – is a total number of UEs in tests; i –is an ordinal number in variational series of UE with crack, $r_0 = 0$.

For any value of $lg\, N_i$ $(1 \le i \le m)$ for UE with crack the values of p_i which are empirical distribution function ordinates with taking into account the UEs without crack) are calculated by the formula:

$$p_i = \frac{r_i}{\sum_{i=1}^{n} k_i + 1}.$$

By these ordinates the u_i values are calculated which are quantiles of $\Phi(u_i, 0, 1) = p_i$. normalized normal distribution. The u_i quantile value for the p_i specified value may be found in any of statistical software packages. The u_i – value obtained is the upper limit of integral in equation (1).

Conditioned by the minimal deviation square sum method: $\frac{lg\, N_i - \tilde{\iota}}{\tilde{}}$ from u_i the $\tilde{\iota}$ $\tilde{}$ parameters estimations are calculated by the formulae:

$$\tilde{\iota} \quad \frac{\sum^{m} lg\, N_i \sum_{i=1}^{m} u_i\, lg\, N_i - \sum_{i=1}^{m} (lg\, N_i)^2 \sum_{i=1}^{m} u_i}{m \sum_{i=1}^{m} u_i\, lg\, N_i - \sum_{i=1}^{m} u_i \sum_{i=1}^{m} lg\, N_i}$$

$$\tilde{} \quad \frac{m \sum (lg\, N_i)^2 - \left(\sum_{i=1}^{m} lg\, N_i \right)^2}{m \sum_{i=1}^{m} u_i\, lg\, N_i - \sum_{i=1}^{m} u_i \sum_{i=1}^{m} lg\, N_i}.$$

Calculation of $\tilde{\iota}$ $\tilde{}$ *estimations of* Φ_e *parameters (by MLM method)*

The likelihood function is a probability to obtain in tests that very $N_1, N_2, \ldots, N_m$ sampling that is obtained. Under the assumptions adopted for the N_i logarithms normality and independence this probability is written by the formula (the indices 1 и lg$N$ for parameters estimations are omitted for the sake of formula writing simplification):

$$L(N_i, \tilde{\iota}\ \tilde{} \quad \prod_{i=1} \left(\frac{0.4343}{\sqrt{2\pi}\,\tilde{}} \exp\left(\frac{(lg\, N_i - \tilde{\iota}}{2\tilde{}} \right.\right. .$$

$$\times \left[1 - \frac{1}{\sqrt{2\pi}\,\tilde{}} \int^{lg\, N_i} \exp\left(-\frac{(t - \tilde{\iota}}{2\tilde{}} \right.\right. .$$

The values under which the logarithm of this function (the logarithm may be taken as the function is monotonous) achieves the maximum may be regarded as the parameters estimations. This condition leads to the following set of equations to define the parameters:

$$\left. \begin{array}{l} \sum_{1}^{m} l_i\, lg^2 N_i - \tilde{} \\ \sum_{1}^{m} l_i\, lg\, N_i - \tilde{\iota} \quad n_i R_i = 0 \end{array} \right\}$$

here $R_i = \frac{(1/\sqrt{2\pi})\exp\left[-(lg\, N_i - \tilde{\iota}\right.}{1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(lg\, N_i - \tilde{\iota}} e^{-t^2/2} dt}$

The solution of this set of equations may be found out by any numerical method. The estimations that are obtained by Johnson method may be taken as the initial approximation. If the unriveting and the instrumentation inspection were made then the

Φ_e parameters are estimated for cracks of different lengths that do not contradict the appearance independence condition. In this case in the likelihood function and in the equations the factor is introduces that takes into account those UEs (by use of probability to get into the space/interval) where the length of crack is longer than that one of the crack for which the parameters are estimated (details are in [7]).

The parameters estimations that are calculated by MLM by *complete samplings* are asymptotically normal. This means that the estimations scattering as compared with other estimations of the same parameter and obtained by other methods is minimal. The estimation of the averaged value is not shifted and it is possible by a minor correction to make non-shifted the estimation of the dispersion for normal distribution.

But the application of this *method for censored samplings* demonstrated that these estimations properties may be kept only under additional conditions. The statistical simulation was carried out to investigate the estimations features of parameters that are obtained by censored samplings by different methods [11]. The samplings were simulated from normal distribution per 100, 200, 400 and 600 space. The percentage of fatigues with crack is 5-30; the $a^1_{lg N}$ are general distribution parameters in range from 5.3 to 6.1, and $\sigma^1_{lg N}$ is from 0.15 to 0.35.

The relative inaccuracy value was calculated by the formulae:

$$\Delta(\tilde{\ } \quad \left| \frac{\tilde{\ }}{a^1_{lg N}} \right|, \quad \Delta(\tilde{\ } \quad \left| \frac{\tilde{\ }}{\sigma^1_{lg N}} \right|$$

The simulation demonstrated that the estimations shift value depends upon RMSD general value and does not depend upon general averaged value. *Under the growth of total number of UEs and the percentage of fatigues with crack the shift is quickly decreased.* When the space is 100 UEs for estimations which were obtained by LSM starting from 10% the truncations $\Delta(\tilde{\ }$ and $\Delta(\tilde{\ }$ % take place; the $\Delta(\tilde{\ }$ and $\Delta(\tilde{\ }$ % are for MLM but at this the LSM overestimates averagely the RMSD and the MLM underestimates it. When simulating the censoring depth at all truncation percentages and sampling spaces was not specially addressed and was random. Further it is to carry out the simulation with controllable censoring depth i.e. it is required to define what kind of alteration of fatigues before the crack and censoring allows obtaining the estimations of the same quality like the complete sampling provides due to greater total amount of observations in the course of tests.

3 Consideration of the number of one-type concentrators

If the $\tilde{\ }$ $\tilde{\ }$ estimations of Φ_e parameters are obtained, the relation between them and the averaged value and the RMSD of the segment with k-identical to them UEs is established by the $\tilde{\ }$ $\tilde{\ }$ and the $\tilde{\ }$ $\tilde{\ }$ $\tilde{\ }$, correlations where: k_σ for $k < 10$ and k_a for $k < 50$ is defined by tables of ordinal statistics for normal distribution . [11], and for $k \geq 10$ and $k \geq 50$ correspondently they are possible to be calculated by formulae [12]:

$$k_\sigma = \frac{\pi}{\sqrt{12\,ln\,k}} \text{ и } k_a = \sqrt{2\,ln\,k} - \frac{ln\,ln\,k + 1{,}3766}{2\sqrt{2\,ln\,k}}$$

The relative inaccuracy is not more than 2% for k_σ and not more than 2.5% for k_a. These formulae are possible to be used when there is still no tests results but it is necessary to estimate the fatigue value of the structure segment or of the zone with several segments. To estimate in the first approximation the UE parameters it is possible to use the $\tilde{\ }$ от $\tilde{\ }$ dependencies (See Fig. 3). The red dots represent the UEs that are calculated by MLM based on high number of results of testing the full-scale panels with joints segments and specimens that were cut from them. The elements under joining are made basically of D16 and V95 type alloys; the fastening items (rivets and rivet-bolts) are made of V65 alloy In 60s-70s of the last century the standard dependence was obtained by results of testing mainly the plate with hole and the few full-scale elements. The red and blue dashes represent the

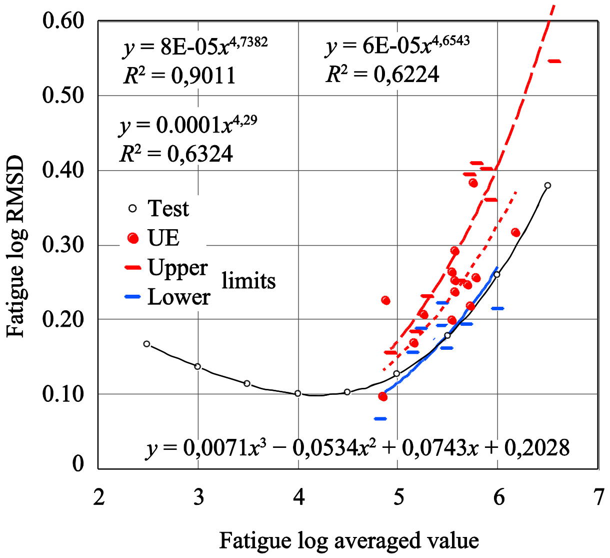

Fig. 3. RMSD dependence on fatigue log averaged value.

confidence interval limits for parameters estimations that are calculated by likelihood ratio [14]. The fatigue of the RZ segment or the zone with a number of segments may be calculated, under the specified probability and at the Φ_e known parameters estimations, by formulae (2) and (3).

The example as follows:

– calculation of Φ_e parameters MLM based on testing results of two panels with identical joints segments and of specimens that were cut out of these panels (Fig. 2);

– plotting the S-N diagram based on the parameters obtained;

– plotting the F_{yi} and F_{P3} construction on the lower surface of the wing that is composed by 13 RZ segments (A/c type #3 in Table 1).

3.1 Example

The specimens (Fig. 2) are cut from two panels. In the regular part the panels consisted of skin and stringers that were attached to skin by two-row of rivets. The skin is made of D16AT alloy. The skin thickness (5 mm) is constant by cross-section. The joints are pressurized by sealant before riveting the stringers. The rivets made of V-65 alloy with diameter of 5 мм of ZUK-2 type are riveted by press by the batch method. When testing the panel the rivets in the regular part practically are free of shear load.

One panel is tested under pulsing tension with σ_{max}^{gross}=128MPa maximal stress in failed cross-section (frequency1–2 Hz). The panel failure took place after 71 144 cycles on the regular zone. The second panel is tested under pulsing tension with σ_{max}^{gross}=116MPa stress (frequency 0.8 Hz). The panel failure took place after 192 346 cycles. The crack arose in skin near the rivet hole for fastening the stringer. Totally 17 specimens made of the first panel were tested under pulsing tension with σ_{max}^{gross}=128MPa in the middle part. Twenty specimens were made of the second panel, 15 of them were tested under pulsing tension with σ_{max}^{gross}=128MPa stress in the middle cross-section and 5 specimens were tested under pulsing tension with σ_{max}^{gross}=186MPa stress. The overall statistical data and the Φ_e FDF parameters calculation results are given in Table 2 and Fig. 3. The columns ##1…5 (Table 2) contain the maximal stress values in middle cross-section of specimen under test; the stresses gross in each UE of specimen and number of UEs by which the specimen failed (or where the crack took place) and which were censored. UEs fatigues are recalculated for the stress in cross-section by which the specimen failed. The columns ## 6...11 contain the averaged value and the UE fatigue log RMSD that are calculated by MLM with considering the fatigues before censoring all UEs of the specimen (1). The fatigues before the failure of specimens by UE in one of the cross-sections are marked by red (Fig. 3). The black color is used to denote the fatigues before the censoring the other specimens UEs which are recalculated for the stress in cross-section by which this specimen failed. The red line is a S-N diagram that is superposed on the fatigues before the specimens failure without consideration of censoring (this exponential quantity is adopted for fatigues recalculation for censoring). The blue line is a S-N diagram (Fig. 3a) that is plotted based on all averaged UE fatigue logs values that are calculated by MLM at each level with consideration of censoring. This S-N diagram at Fig. 3b is presented as two paths: the first one is plotted based on stress levels from 185MPa up to 126MPa incl. and the second one is plotted based on stress levels from 126MPa up to 93MPa. The calculated cross-point at 126MPa stress level is given in Table 2. One can see that

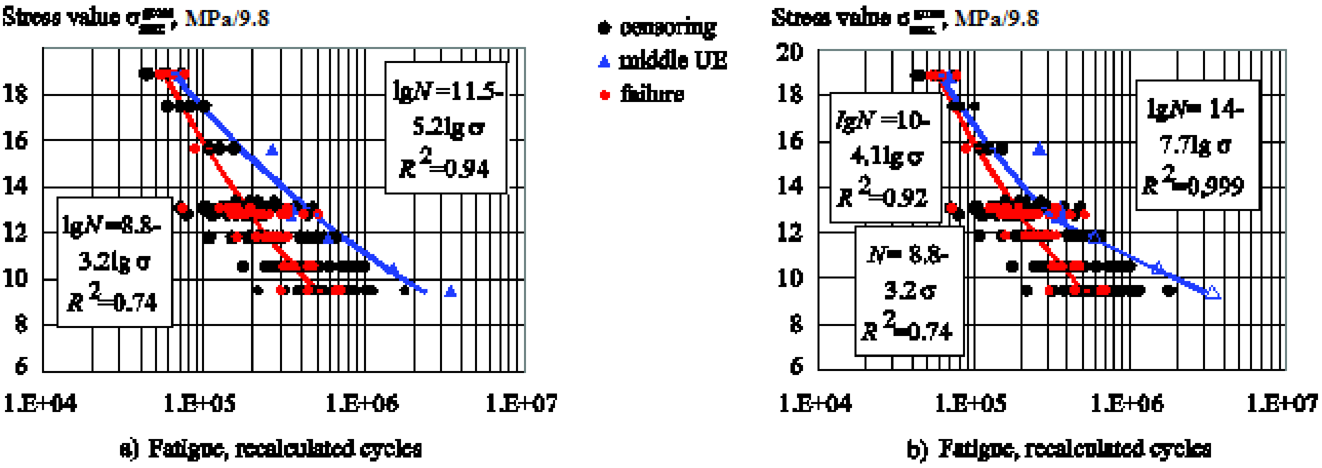

Fig. 4. S-N diagrams of 9UEs specimen failure when taking into consideration the censoring a) as one path б) as two paths.

Table. 2.

σ_{max}^{gross}, MPa	UEs totally	UEs number		σ^{gross}, MPa	UE fatigue log DF parameters					
		By failure	By censoring		Left limit 80% confiden. interval	Estimation $a^1_{lg\,N}$	Right limit 80% confiden. interval	Left limit 80% confiden. interval	Estimation $\sigma^1_{lg\,N}$	Right limit 80% confiden. interval
1	2	3	4	5	6	7	8	9	10	11
186	45	5	10	185	4,81	4,84	4,90	0,061	0,087	0,138
		0	10	172						
		1	9	154		5,42				
		0	10	131						
127	290	8	24	128	5,44	5,55	5,73	0,232	0,317	0,466
				126						
		15	49	125	5,49	5,54	5,61	0,161	0,198	0,251
		9	55	116	5,69	5,78	5,92	0,194	0,256	0,256
		4	60	103	6,00	6,18	6,56	0,213	0,317	0,545
		4	62	93,1	6,27	6,54	7,14	0,325	0,445	0,794

two paths describe more accurately the fatigue dependence on stress.

The corresponding S-N curves are required to plot the RZ FDF for the wing lower surface that has segments with different materials joints, for the semi-products with different geometric characteristics and fastening elements of different type. The UE joints S-N curve correspond to the concrete zone (Fig. 4). The calculation is shown here of RZs composed of 13 identical equally loaded segments that correspond to one S-N curve (Fig.4b).

The averaged fatigue values are to be estimated for required stress levels based on the equations of the both paths (Fig. 4b) based on N results obtained we calculate $lg\,\bar{N}$ and $RMSD_1$ $RMSD_1$ by use of the dependence equation (Fig.3). The required $\overline{lg\,N} = lg\,\bar{N} - \dfrac{RMSD_1^2}{0.8686}$ and the scattering ($RMSD_2$) based on the same dependence we define by the $\overline{lg\,N}$. The values obtained are given in columns 4…8 (Table 3). The stress levels are given in columns 2 and 3. Fig. 5 shows in normal probabilistic scale the Φ_e, F_{yi} ($i = 1,\ldots, 13$) and F_{P3}, by which for any fatigue at X-axis the probability may be found at Y-axis, before which the first crack may appear in any of segments or in the RZ as a whole.

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Table 3

№ *i* segments	*n* number of UEs	$\sigma_{прив}$, MPa	$\bar{N}$	$lg\,\bar{N}$	$RMSD_1$	$\overline{lg\,N}$	$СКО_2$
1	2	3	4	5	6	7	8
1	110	87,2	5 116 940	6,71	0,35	6,57	0,32
2	484	95,1	2 525 521	6,40	0,29	6,31	0,27
3	316	101	1 590 962	6,20	0,25	6,13	0,24
4	1196	106	1 104 470	6,04	0,22	5,99	0,22
5	1506	111	779 511	5,89	0,20	5,85	0,19
6	3639	116	544 162	5,74	0,18	5,70	0,17
7	7813	121	405 766	5,61	0,16	5,58	0,16
8	8014	125	298 564	5,48	0,15	5,45	0,14
9	3921	130	253 256	5,40	0,14	5,38	0,14
10	2012	135	217 775	5,34	0,13	5,32	0,13
11	1049	141	182 984	5,26	0,12	5,24	0,12
12	740	146	159 142	5,20	0,12	5,19	0,12
13	656	150	142 801	5,15	0,11	5,14	0,11

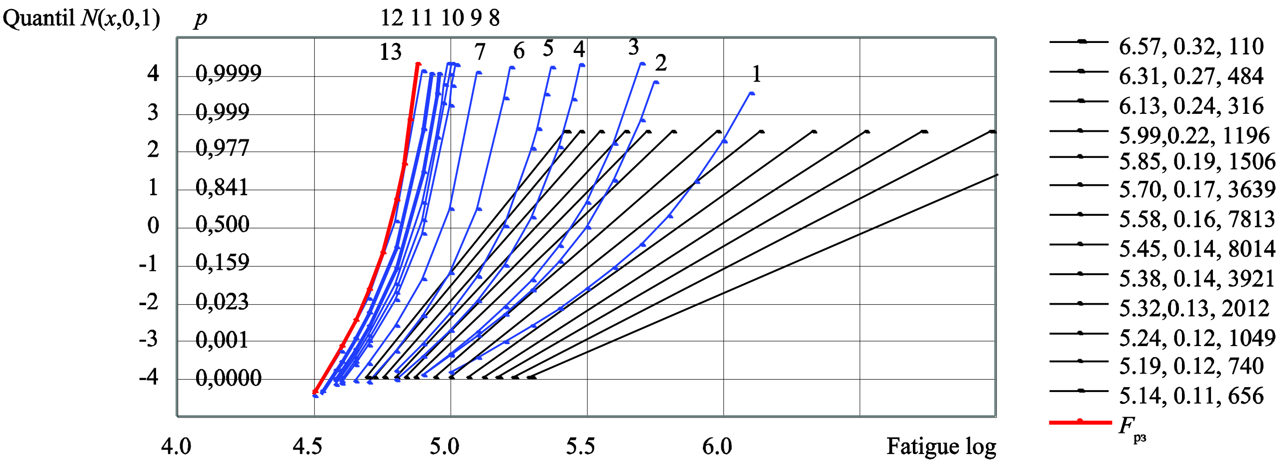

Fig. 5. Fatigue distribution function of UE (direct), segments 1,…, 13 F_{yi} (blue), F_{P3} (red).

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