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APPLICATION OF SUBOPTIMAL 4-D NAVIGATION ALGORITHMS FOR FLIGHT PLANNING AND CONTROL CONSIDERING WEATHER CONDITIONS

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Abstract

Algorithms for airliner trajectory control in accordance with 4-D navigation principles are considered. Vertical component of control is designed with the suboptimal algorithms. These algorithms are based on the results of the solution of problem about minimum fuel consumption in a flight of given range within specified time. Energy approximation and singular perturbation method are used in solving. The algorithms make available to take into account probable wind on a route.

Particular attention is given to arrival of an aircraft in the required time. In connection with this, special control trajectory algorithm of the continuous descent is examined. This algorithm relies on the approximation of optimal continuous descent by the profile "CAS – constant number M – constant CAS" (here CAS is calibrated air speed). The algorithm provides some time window of arrival at the cost of the profile parameter variation. It may be used also for compensation of probable wind impact.

Horizontal component of control is constructed according to a preplanned route. A route can be replanned at flight in connection with the necessity of fly-around of weather dangerous or prohibited air areas that had not been taken into account at flight planning. The method of trajectory generation is proposed to provide safe fly-around of such areas. This method relies on using so called "safety map".

1 Introduction

Automation of flight management presumes the availability of corresponding algorithms in FMSC (Flight Management System Computer) of aircraft. Algorithms of trajectory control are primary. They need to provide safety, efficiency. At the same time they must be sufficiently fast-acting. The algorithms of formation of management in vertical control play the dominant role in providing efficiency. In the present paper the possible technique of airplane control synthesis based on the solution of the problem about minimum fuel consumption at flight from one point in another in energy approximation is considered.

Apparently, for the first time the energy approximation for optimization of trajectories of flying vehicles has been offered in [1]. After that it was used widely for solving several problem of airplane trajectory optimization. In particular, this approach has been applied with success to the solution of the problem about optimum flight on the given range within required time (for example, in [2]–[9]). Let's notice that this problem is called as the problem of 4-D navigation problem. In the solving process the whole trajectory is divided into three segments: climb, cruise, descent. These segments are connected among themselves through trajectory parameters and adjoint variables.

By analogy, in proposed algorithm the decomposition of trajectory onto the same three segments is presumed. Trajectory control variables on each segment are determined on the basis of minimization of its particular so called cost function. These functions are including mass and polar of airplane, engine performances, atmosphere parameters. For transition from one segment to another the special transition algorithms are provided. Output parameters of algorithm are lift and tangential load factors. They are sufficient for airplane control in vertical plane. Proposed algorithm may be used for trajectory management both without any restriction on flight altitude and with the restriction caused given flight level.

In horizontal plane usually airplane is flying according statutory route. But at appearance of unforeseen dangerous/prohibited area on a route the problem of avoiding this area arises. In this paper the feasibility approach to trajectory generation providing safe and efficient fly-around of no-go areas. Corresponding algorithms for constructing fly-around routes have been elaborated. Background of routes is so called "digital safety map". Fly-around routes are defined by pseudo-WPTs on this map. According to the approach all possible fly-around routes are constructed. Active route is chosen either automatically in accordance with specified criterion or by a crew.

Considered approach and corresponding algorithms may be used not only for fly-around of stationary dangerous areas but also for conflict resolution with moving no-go areas including area of dangerous proximity with another aircraft.

2 Solution of 4-D Navigation Problem

The construction of suboptimal algorithm of aircraft control providing arrival to specified point with minimum fuel consumption within given time is presented in this paper. The algorithm is based on the solution of 4-D navigation problem considered in [2]–[9]. For solving this problem the energy approximation was used in these references. This approximation is used in present paper too.

Longitudinal motion of aircraft mass center is described by equations

$$\begin{aligned} \dot{E} &= n_{xa} V, \\ \dot{m} &= -f, \\ \dot{x} &= V_g. \end{aligned} \qquad (1)$$

Here E is specific mechanical energy, n_{xa} is tangential load factor, V is true airspeed, f is fuel rate, x is current range, V_g is ground speed.

Specific mechanical energy is regarded in moving medium (in air) and equals

$$E = h + \frac{V^2}{2g} \qquad (2)$$

where h is flight altitude.

Tangential load factor is expressed by equation

$$n_{xa} = \frac{P}{mg} - \frac{qSC_D}{mg} \qquad (3)$$

where P is thrust of engines, q is dynamic pressure, S is reference area, C_D is drag coefficient, g is gravitational acceleration.

In accordance with energy approximation the lift load factor n_{za} is assumed to be equal to 1:

$$n_{za} = \frac{qSC_L}{mg} = 1$$

where C_L is lift coefficient.

The value of drag coefficient C_D in equation (3) is defined according to polar of aircraft $C_D(C_L, M)$. Here M is Mach number.

Ground speed V_g is defined by the equation

$$V_g = \sqrt{V^2 - W^2 \sin^2 \Psi_W} + W \cos \Psi_W$$

where W is wind speed, Ψ_W is wind angle relatively to the ground speed direction.

At time $t = 0$ the initial conditions are known:

$$E(0) = E_0,\ m(0) = m_0,\ x(0) = x_0.$$

At specified final time T the conditions are

$$E(T) = E_f,\ x(T) = L. \qquad (4)$$

Here the values $E(0)$ and $E(T)$ may be calculated in according to specified altitude and airspeed in corresponding times, using the equation (2).

The objective of trajectory optimization is fuel consumption. Criterion of the problem may be specified in the form

$$J = -m(T) \to \min.$$

Control variables are flight altitude and the thrust as it is appropriate in energy approximation. Control variables are subject to constraints

$$P_{\min}(h,M) \leq P \leq P_{\max}(h,M), \quad (5)$$
$$h_{\min}(E) \leq h \leq h_{\max}(E). \quad (6)$$

Boundary conditions (5) are engine characteristics. Boundary conditions (6) is defined by minimum $h_{\min 1}$ and maximum $h_{\max 1}$ allowed values of altitude as well as lower and upper restrictions on calibrated air speed (*CAS*). The last restrictions may be transformed in the restrictions on the dynamic pressure determined with taking into account air compressibility q_{com}. Appropriate boundary values of altitude h_{min2}, h_{max2} may be determined by solving the equations

$$q_{com\max} = p\left(h_{\min 2}\right) \times \times \left[\left(1 + \frac{0.4g\left(E - h_{\min 2}\right)}{a_s^2\left(h_{\min 2}\right)}\right)^{3.5} - 1\right],$$

$$q_{com\min} = p\left(h_{\max 2}\right) \times \times \left[\left(1 + \frac{0.4g\left(E - h_{\max 2}\right)}{a_s^2\left(h_{\max 2}\right)}\right)^{3.5} - 1\right]$$

where p is atmospheric pressure, a_s is sound speed.

Hereafter the values $h_{\min} = \max(h_{\min 1}, h_{\min 2})$ and $h_{\max} = \min(h_{\max 1}, h_{\max 2})$ are defined.

So as to avoid the dependence of boundary values on the state variable in (6) the conversion

$$\overline{h} = \frac{h - h_{\min}(E)}{h_{\max}(E) - h_{\min}(E)}$$

is performed.

The boundary values of control variable $\overline{h}$ don't depend on the state variable E:

$$0 \leq \overline{h} \leq 1.$$

Pontryagin's maximum principle is used for the solution of the optimization problem. Subject to the condition (5) the Hamiltonian has the view

$$\mathrm{H} = \lambda_E n_{xa} V - \lambda_m f + \lambda_x V_g + \lambda_t + + \mu_1 \left(P_{\min}(h,M) - P\right) + \mu_2 \left(P - P_{\max}(h,M)\right)$$

where λ_E, λ_m, λ_x are adjoint variables, λ_t is constant.

The adjoint variables satisfy the differential equations

$$\dot{\lambda}_E = -\frac{\partial \mathrm{H}}{\partial E}, \; \dot{\lambda}_m = -\frac{\partial \mathrm{H}}{\partial m}, \; \dot{\lambda}_x = -\frac{\partial \mathrm{H}}{\partial x} = 0. \quad (7)$$

The transversality conditions have the view

$$\lambda_E(T) = \nu_E, \; \lambda_m(T) = \nu_m < 0, \; \lambda_x = \text{const}.$$

The values of λ_x and constant λ_t determine range and duration of flight. They should be chosen so that the conditions (4) must be satisfied. Parameter ν_m must be any negative value, parameter ν_E is found from the condition that Hamiltonian H equals 0 on an optimum trajectory.

Lagrange's multipliers μ_1, μ_2 are defined as follows:

$$\mu_1 \begin{cases} = 0 \text{ if } P_{\min} - P < 0, \\ > 0 \text{ if } P_{\min} - P = 0, \end{cases}$$
$$\mu_2 \begin{cases} = 0 \text{ if } P - P_{\max} < 0, \\ > 0 \text{ if } P - P_{\max} = 0. \end{cases}$$

Optimum values of control variables P, $\overline{h}$ are found from the condition of Hamiltonian minimum that is

$$\min_{P,\overline{h}} \mathrm{H} \to P, \overline{h}.$$

Original optimization problem is reduced to two points boundary value problem (TPBVP). At solving the latter the direct (1) and adjoint (7) equations may be integrated both in direct and in inverse time. Let it emphasize that the value of $\lambda_m(T)$ may be any negative. The calculation practice is showing that the value of λ_m don't change a sign on whole trajectory. Therefore at integrating in direct time the value of λ_m may be accepted any negative, for example $\lambda_m(0) = -1$.

So for solving of this TPBVP the values of two constants λ_x and λ_t should be found for satisfying the conditions (4).

Since at initial time the tangential load factor is positive the value of λ_E in this time may be determined by the expression

$$\lambda_E(0) = -\min_{\bar{h},P} \left. \frac{-\lambda_m(0)f + \lambda_x V_g + \lambda_t}{n_{xa}V} \right|_{E=\text{const}}.$$

Computational solution of considered TPBVP may be performed only for not long flight trajectories. For example the computational program on language C with variable of type double enables to find an optimum trajectories of duration no more than 1.2 hour.

For determination of optimum trajectories with greater duration of flight their decomposition onto 3 segments is needed: climb, cruise and descent. At optimization of climb the control variables may be obtained from the expression

$$\left(\bar{h},P\right) \sim \min_{\bar{h},P} \left. \frac{-\lambda_m f + \lambda_x V_g + \lambda_t}{n_{xa}V} \right|_{E=\text{const}} = -\lambda_E \quad (8)$$

since the condition $n_{xa} > 0$ is fulfilled on this segment. The value of λ_E is needed for integrating adjoint variable λ_m. Optimization of control variables for the descent segment may be performed similarly taking into account the condition $n_{xa} < 0$:

$$\left(\bar{h},P\right) \sim \max_{\bar{h},P} \left. \frac{-\lambda_m f + \lambda_x V_g + \lambda_t}{n_{xa}V} \right|_{E=\text{const}} = -\lambda_E. \quad (9)$$

Optimum parameters of flight on the cruise may be found from the expression

$$(h,V) \sim \min_{h,V} \left. \frac{-\lambda_m f + \lambda_t}{V} \right|_{\substack{n_{za}=1 \\ n_{xa}=0}} = -\lambda_x. \quad (10)$$

As it was said above the value of λ_x is constant on continuous trajectory (actually it is changing slightly from step to step because of calculating discrecity). Moreover according to singular perturbation method this value should be equal in all three expressions (8), (9) and (10). In some above-mentioned papers the value λ_m is assumed constant on a whole trajectory. The calculation results have shown that this assumption influences on optimum trajectory insignificantly. At using decomposition for the problem solving the time of descent beginning and the value of λ_t must be found to satisfy the conditions (4).

3 Construction of suboptimal algorithm

The expressions (8) – (10) are applied for determination of trajectory control providing a fuel consumption close to minimum value. Control variables are computed at the supposition that $\lambda_m = 1$ and the value of λ_x is found from (10) provided the values of f and V_g are calculated for aircraft mass m_{cr0} in the beginning of cruise segment. The value of m_{cr0} is estimated according to the formula:

$$m_{cr0} = m_0 - \Delta m^{pr}_{fcl}$$

where Δm^{pr}_{fcl} is predictable fuel consumption on the climb segment. It depends on initial aircraft mass, initial and final conditions of the climb segment. Calculation results have shown that the value of m^{pr}_{fcl} may be approximated by the function of initial aircraft mass and initial altitude.

Considered algorithm is intended for determination of desirable values of trajectory parameters: lift and tangential load factors. These parameters are determined with taking into account restrictions on flight envelope and capabilities of aircraft control.

At first for determining lift load factor the needed flight-path angle γ_n is calculated. This applies equally to all segments of trajectory. For climb segment it is defined by the equation

$$\sin\gamma_n = n_{xa}\frac{dh_{opt}}{dE} + \frac{k_h\left(h_{opt} - h\right)}{V} \quad (11)$$

where h_{opt} is the optimum value of altitude obtained from (8) at current values of specific energy E and aircraft mass m, $\frac{dh_{opt}}{dE}$ is derivative of optimum climb program, n_{xa} is current value of tangential load factor, k_h is algorithm parameter. Here and further the subscript 'n' indicates 'needed value'.

The determination of derivative is carried out numerically that is the increment ΔE is added to current value of specific energy and optimum value altitude h_{opt1} is determined for augmented value of specific energy, after that the value of derivative is defined:

$$\frac{dh_{opt}}{dE} = \frac{h_{opt1} - h_{opt}}{\Delta E} \tag{12}$$

where ΔE is the specific energy increment. At calculation the value of ΔE was equal to 200 m.

Predictable value of $\sin\gamma_n^{pr}$ is determined in much the same way for specific energy $E + \Delta E$:

$$\sin\gamma_n^{pr} = n_{xa}\left(\frac{dh_{opt}}{dE}\right)^{pr} + \frac{k_h\left(h_{opt}^{pr} - h\right)}{V} \tag{13}$$

where pr is the superscript for 'predictable value'.

If signs of the differences

$$\left(\frac{dh_{opt}}{dE}\right)^{pr} - \frac{dh_{opt}}{dE} \text{ and } \sin\gamma_n^{pr} - \sin\gamma_n \tag{14}$$

do coincide then

$$\sin\gamma_n = \sin\gamma_n^{pr}. \tag{15}$$

Obtained value γ_n is limited below to avoid the violation of upper boundary of calibrated air speed (CAS). Boundary value $\gamma_{n\min}$ is defined with using equivalent linear method. In this case the method consists in the following. Desirable process of compensation of the difference $\Delta V = V_{max} - V$ between maximum value of airspeed

$$V_{\max} = a_s(h)\times \times\sqrt{5\left[\left(\frac{q_{com\max}}{p(h)} + 1\right)^{2/7} - 1\right]} \tag{16}$$

and its current value is assigned in the form

$$\Delta\dot{V} + k_V\Delta V = \dot{V}_{\max} - \dot{V} + k_V\Delta V = 0 \tag{17}$$

where k_V is algorithm parameter.

The value of $\dot{V}_{\max}$ is defined by differentiating the expression (16):

$$\dot{V}_{\max} = \frac{dV_{\max}}{dh}\dot{h} = \frac{dV_{\max}}{dh}V\sin\gamma_{n\min}. \tag{18}$$

The value of $\dot{V}$ is given by the equation of the speed with $\gamma = \gamma_{n\min}$:

$$\dot{V} = g\left(n_x - \sin\gamma_{n\min}\right). \tag{19}$$

After substituting (18), (19) into (17) and transforming obtained expression the following equation for $\gamma_{n\min}$ may be derived:

$$\sin\gamma_{n\min} = \frac{n_{xa} - k_V\Delta V/g}{1 + \frac{dV_{\max}}{dh}V/g}. \tag{20}$$

If the value of $\sin\gamma_n$ obtained in (11), (12), (13), (14) and (15) is less the value of $\sin\gamma_{n\min}$, then it is accepted

$$\sin\gamma_n = \sin\gamma_{n\min}.$$

In the process of aircraft flying in accordance with described algorithm the current value of the airspeed is sufficiently far from its bottom limit therefore this limitation isn't provided specially. This limitation is held at satisfying inequality $h \le h_{\max}(E)$. At approaching to cruise altitude h_{cr} (it may be altitude of given flight level or optimum altitude obtained according to (10)) the flight-path angle needed for capturing this altitude is defined so:

$$\sin\gamma_{crn} = k_{h1}\frac{(h_{cr} - h)}{V} \tag{21}$$

where k_{h1} is algorithm parameter.

While the condition

$$\sin\gamma_n > \sin\gamma_{crn}$$

is fulfilled the needed flight-path angle is found in accordance with (21) till descent.

At climb the needed value of tangential load factor n_{xan} is determined by the equation

$$n_{xan} = \frac{P_{opt}}{mg} - \frac{qSC_D}{mg} \tag{22}$$

where

$$P_{opt} = \arg\min_P \frac{f + \lambda_x V_g + \lambda_t}{n_{xa}V}$$

is computed at current values of h, V, n_{xa}. In equation (22) the value of drag coefficient C_D is found from aircraft drag polar of airplane at $C_L = mg/qS$

When an airplane approaches to cruise altitude and in cruise the needed value of tangential load factor is defined by the equation

$$n_{xacrn} = k_V\left(V_{cropt} - V\right) + \sin\gamma$$

where V_{cropt} is found from (10). When value n_{xacrn} is less than value of n_{xan} determined by (22) it is accepted

$$n_{xan} = n_{xacrn}.$$

At absence of wind the descent distance R_{des} may be approximated by polynomial function of aircraft mass, the values of specific energy in descent top and descent terminal points. If wind is present the term proportional to wind speed and descent time is added to this approximation.

The value of needed flight-path angle for descent is determined with using equations (12) - (15). The distinction with a case of climb consists in the following: formula (15) is valid in case the signs of differences in (14) are differ.

Needed flight-path angle is constrained from the bottom by the value calculated according to equation (20). This constraint is introduced for avoidance of violation of *CAS* upper bound.

In addition, the bottom constraint on altitude must be taken into account. This constraint is fulfilled by limitation on flight-path angle

$$\sin\gamma_n \geq \frac{k_{h1}\left(h_{\min 1} - h\right)}{V}.$$

The thrust in descent segment is accepted idle regime.

Needed value of the lift load factor n_{za} is determined by the equation

$$n_{za} = \frac{k_\gamma V\left(\sin\gamma_n - \sin\gamma\right)}{g} + \cos\gamma \qquad (23)$$

for all three flight segment. Here k_γ is algorithm parameter.

For validation of proposed technique the comparison of the results of trajectory calculation with using described algorithm and the solution of optimum problem has done. The comparison was performed for whole trajectory in the vertical plane starting from climb and finishing at capturing glide path. The range of the trajectories in all presented below cases was equal to 1000 km.

At calculating trajectory with using described suboptimal algorithm the aircraft motion is simulated by airspeed, flight-path angle, altitude, mass and range. Lift coefficient C_L and thrust P were assumed as control variables. The value of C_L was determined by the formula

$$C_L = \frac{mgn_{za}}{P/C_L^\alpha + qS}.$$

Here needed value of n_{zan} was calculated by the equation (23), C_L^α is derivative of lift coefficient with respect to angle of attack.

Thrust control was executed in accordance with integral law that at digital realization has the form

$$P^{i+1} = P^i + k_P m^i\left(n_{xa}^i - n_x^i\right)\Delta t$$

where Δt is the discrecity of computing, k_P is algorithm parameter, the superscript $'i'$ indicates the number of integration step. The restrictions (5) was taken into account of course.

Trajectory optimization was carried out with using energy approximation. Flight level wasn't specified that is cruise altitude was optimum. Aerodynamic characteristics of aircraft and characteristics of its power plant corresponded to typical medium range airliner. Atmosphere condition was accepted standard.

At first the trajectories were compared without specified time. The value λt was equal to 0 for both compared trajectories. The comparison of flight profiles in the coordinates "Mach number – altitude" is presented in Fig. 1. Optimum profile obtained with energy approximation is shown by blue color, profile obtained with using suboptimum algorithm is done by red color. In this case the profiles coincide practically.

The flight times and fuel consumptions coincide also. The flight time is equal to 78.5 min. The second comparison was performed with specified flight time. It was equal to 76 min. For

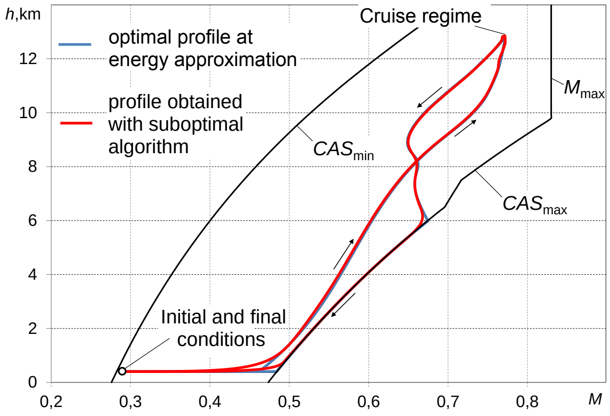

Fig. 1 Comparison of flight profiles in the coordinates "Mach number vs altitude"

providing specified time the value of parameter λ_t is needed to choose for both compared trajectories.

The results of comparison are given in the table 1. In the second row the results of optimization at energy approximation (EA) are given, in the third row the results obtained with using suboptimal algorithm (SO) are shown. On the

Table 1

	λ_t	$m_{f\,opt}$,kg	Δm_f,kg	Δm_f, %
EA	0.0629	3746.5	–	–
SO	0.07263	3571.1	4.6	0.12

trajectory computed with using suboptimal algorithm the fuel consumption exceeds on 0.12% the fuel consumption on optimum trajectory at energy approximation. It is worth to note that the values of λ_t for compared trajectories differ.

4 Descent within specified time

An arrival to terminal condition at specified time is provided by appropriate value of λ_t as it was said above. Using the considered algorithm the distance R_{des} of descent top is estimated. If the values of λ_t and/or R_{des} are incorrect slightly or an unforeseen wind happens at descent then arrival to terminal condition may be provided by descent profile described below.

Optimal descent profile may be approximated by profile of the type "constant *CAS* – constant *M* – constant *CAS*" as it is presented in Fig. 2. In this case Mach number in the medium part of profile equals $M^* = 0.67$. At that the arrival time increases by 4 sec, fuel consumption is up 1.2 kg. Both profiles is computed at thrust corresponding idle regime of engines. Descent time T_{des} may be varied at the cost of the value of M^* as it is shown in Fig. 3. At varying the value of M^* from 0.44 to 0.77 the descent time changes from 42.4 min till 22.2 min respectively. The range of time change is 20.7 min. Actually the time window of arrival with taking into account flight time in cruise is less and equals 9 min.

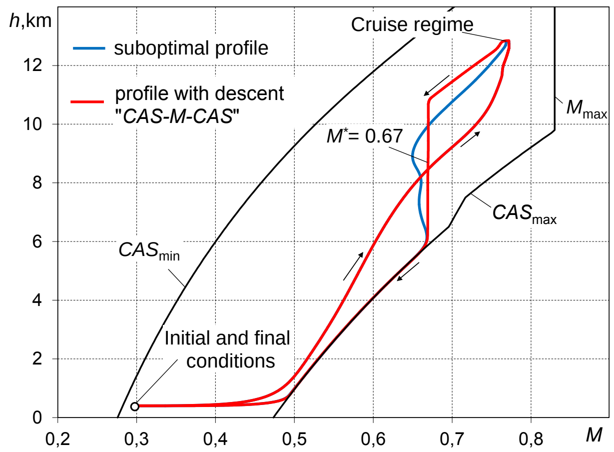

Fig. 2 Approximation optimum descent profile

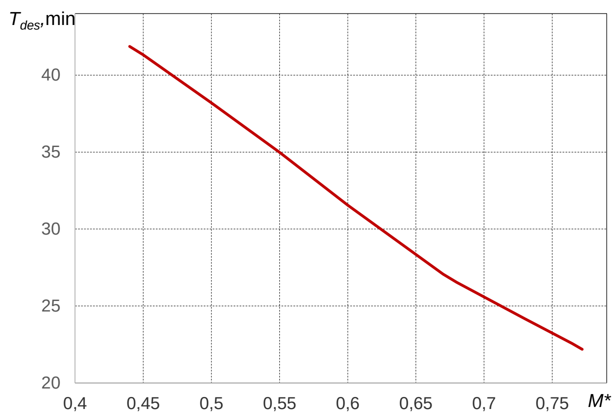

Fig. 3 Dependence of descent time upon parameter M^*

Key problem of concerned method of descent control consists in the assessment of the descent top distance R_{des} and the value of M^* while aircraft is in a cruise segment. Having dependencies $T_{des}(M^*)$ and $R_{des}(M^*)$ these values can be found

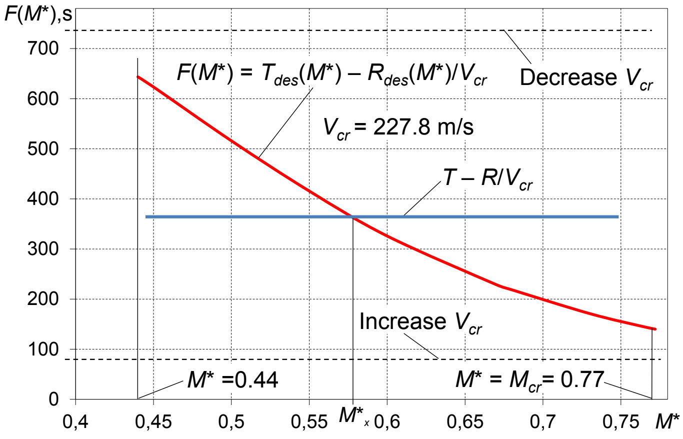

Fig. 4 Graphic illustration of finding the value of M^*

from the equations

$$T = T_{des}(M^*) + T_{cr},$$
$$R = R_{des}(M^*) + T_{cr}V_{cr}$$

where T is specified time of arrival in given point, R is current distance to given point.

Expression for determination of the value M^* has the view

$$T_{des}(M^*) - R_{des}(M^*)/V_{cr} = T - R/V_{cr}. \quad (24)$$

The descent top distance may be obtained after determination of M^*.

Graphic illustration of finding the value of M^* is presented in Fig. 4 where M^*_x is required value of M^*. In addition, equation (24) may be used for correction of cruise velocity. If the value of right side in (24) exceeds possible value of the left side the cruise velocity must be decreased for compensation of a discrepancy between specified and predicted time of arrival. And vice versa, if the right side in (24) is less than the possible value of the left side then cruise velocity must be increased before descent.

It should be noted that descent profile of considered type may be used for compensation of impact of a wind, which has been ignored at flight planning.

5 Safe fly-around of no-go areas

Usually pre-flight planning is using statutory airways for generation of a trajectory. At that known weather conditions, air traffic, NOTAM directions are taken into account. But the problem of operative re-planning may be appeared in a flight. That may be connected with the necessity of conflict resolution for weather/traffic/terrain dangerous that hasn't been foreseen at planning. Considered method is intended for trajectory generation of safe and efficient fly-around of dangerous areas. Fly-around trajectories are assumed to be in horizontal plane. The background of the method is "digital safety map".

5.1 "Digital safety map"

"Digital safety map" is located in tangent plane to Earth in the point of aircraft position. The rectangular coordinate system is placed on the map. In principle map dimensions should correspond to measures of an area that may be observed from an aircraft. Fly-around of dangerous areas is carried out in map boundaries. If fly-around trajectory is come out these boundaries the dimensions of the map should be extended.

Proposed method is effective for any shape of dangerous areas. They may be approximated by figures with smooth boundaries or by polygons. Approximating figures may be both convex and concave.

The grid with an identical discrecity value Δl along both axes is put in the coordinate system. Grid discrecity corresponds to necessary precision of dangerous area description. Each grid node is assigned minimum distance to the nearest dangerous area. Minimum distance is shown in relative measurement unit

$$\bar{r}_{\min} = [r_{\min}/\Delta l]\,.$$

Here brackets indicate integer of a value.

Grid nodes within dangerous areas are assigned 0 (null). The set of assignments for all grid nodes may be considered as the "digital safety map". The value $\bar{r}_{\min}$ characterizes safety level. The line connecting the nodes with identical value represents the line of the equal safety level. Fly-around trajectories are generated so as to avoid coming into an area constrained by a line with specified safety level.

5.2 Generation of fly-around trajectories

The manner of generation of fly-around trajectories is explained on the example of fly-around of dangerous areas in Fig. 5. Let an aircraft fly from the point S to the point F. Rectilinear trajectory is blocked by the dangerous area 1.

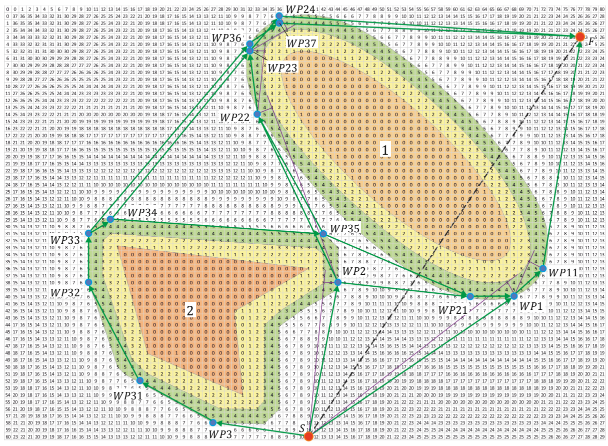

Fig. 5 All feasible fly-around routes

Let the distance of safety fly-around be equal to 15 km. That is fly-around trajectories should not come into an area constrained by the line passing nodes with the value 3 (15 km/5 km = 3). Fly-around trajectories are defined by conventional manner for a route planning that is with using WPT (waypoints). Besides the safety line the width of fly-around corridor is given. Let the value of corridor width be equal to 10 km. In this case WPT location doesn't go out of the area constrained by the line passing the nodes with the value 5.

Tangents from point S to the line passing the nodes with the value 3 around dangerous area 1 are drawn. These tangents are found by means of direct search of nodes with the value 3. The normals to these tangents in the contact points are drawn. The intersections of normals and external boundary of corridor give the first potential WPT for forming fly-around trajectories. If no-go areas are absent on the segment from point S to obtained WPT then this segment may be considered as a leg of fly-around trajectory. In Fig. 5 segment S – WP1 satisfies this condition. But the segment for fly-around of dangerous area 1 on the left intersects no-go area 2. This area should be also avoided.

The trajectories for fly-around of dangerous area 2 should be found. The tangents from point S to the line passing nodes with the value 3 around area 2 are drawn and contact points are defined. Normals are drawn to the tangents in these points. Intersections of these normals and outer boundary of fly-around corridor are the new potential WPT. In Fig. 5 they are designated WP2 and WP3. The check is fulfilled to detect occurrence of dangerous areas on the segments S – WP2 and S – WP3. In this case dangerous areas are absent on both segments. Thin lines demonstrate auxiliary plotting: tangents and normals.

Construction of the following segments of fly-around trajectories is made from points WP11, WP21 and WP31. That is done by analogy as from point S with one difference: if the direction of following segment differs from the direction of previous segment more than 120° then construction of this fly-around trajectory is accounted unpromising and is ceased. Thus the full fly-around trajectories are generated up to point F.

Several WPT may be needed to circumnavigate one dangerous area as in the presented example for fly-around of areas 1 and 2 on the left. It is obvious while a fly-around corridor is wider then necessary number of WPT is less.

5.3 Rectification of trajectories

Since a segment from each WPT is designed as segment from initial point (with taking into account above difference) then trajectories with unreasonable waypoints and turnings are probable. The trajectory portions with these peculiarities are S–WP1–WP11, S–WP2–WP21–WP1, WP33–WP36–WP37, WP34–WP23–WP24. Procedure of trajectory rectification is implemented in these cases. It consists in removing intermediate dispensable WPTs. In the result above cited trajectory portions become the legs S–WP11, S–WP1, WP33–WP37, WP34–WP24. After that the check of absence of dangerous areas on the legs obtained in the result of removing some WPTs. The procedure of merger of waypoints is provided if they are at a very short distance.

Final set of fly-around trajectories obtained after implementing procedures of trajectory rectification is shown in Fig. 6. Active fly-around trajectory may be chosen automatically in accordance with specified criterion or manually by a crew.

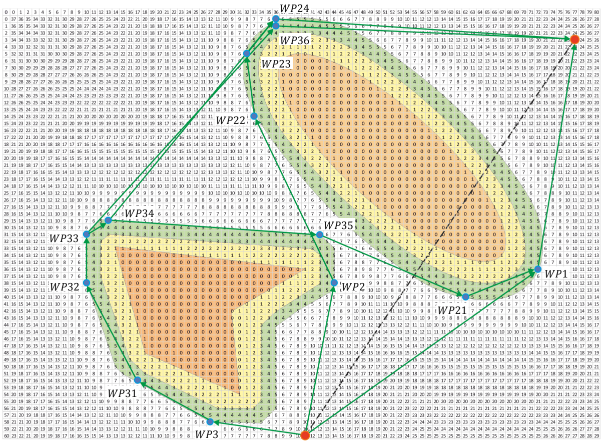

Fig. 6 Set of fly-around routes after rectification

5.4 Fly-around of a moving dangerous area

Proposed method can be used for generation of trajectories for fly-around of dangerous area moving with constant velocity.

Let an aircraft fly with the constant velocity V_g. Velocity vector of dangerous area is equal to $\vec{V}_0$. Time of an aircraft flight up to any node of the map is equal to

$$T = D/V_g$$

where D is the path length up to this node.

Here the time of an aircraft turn in node direction isn't taken into account. The calculation experience has shown it is not necessary to take into account this time.

The dangerous area will shift for the time T in the position

$$\vec{S}(T) = \vec{S}(0) + \vec{V}_0 T$$

where $\vec{S}(0)$ is the position of dangerous area in the current moment. That is each point of the area moves according this equation.

"Digital safety map" is designed by the following way: each node is assigned minimum distance up to dangerous areas in the position at time T. Generation of fly-around trajectories are performed as in the case of stationary no-go areas. But at maneuvering of an aircraft the "digital safety map" is needed to recalculate.

Proposed approach and algorithms may be implemented for traffic conflict resolution. In this case air space around another aircraft is considered as moving dangerous area. The example of fly-around trajectories to prevent dangerous proximity with another aircraft and moving bad weather area is presented in Fig. 7. Here " digital safety map" has dimensions 200 km × 200 km with the grid discrecity 2 km. The range of dangerous proximity is assumed to be 10 km. Highlighted areas in the "digital safety map" are the dangerous proximity areas. Their boundaries pass nodes with the value 5 (10 km/2 km = 5).

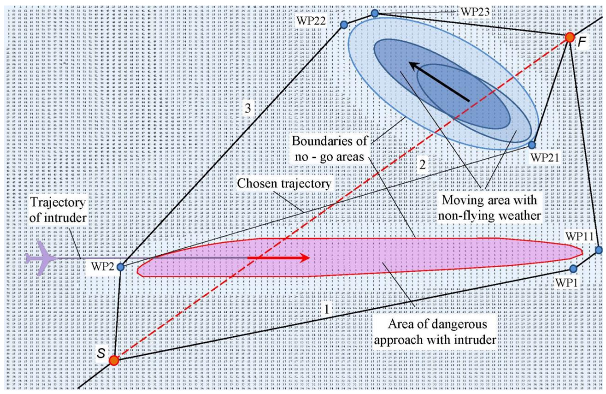

Fig. 7 Routes of fly-around of moving objects

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