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COMPUTATIONAL INVESTIGATION OF SIMULATION ON THE DYNAMIC DERIVATIVES OF FLIGHT VEHICLE

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Abstract

Dynamic derivatives are key parameters to flight vehicle design, which directly affect aircraft flying qualities and control rate, and it has become a hot topic to obtain dynamic derivatives through various methods. Here systematic research is developed with Computational Fluid Dynamic (CFD) method to calculate each dynamic derivative, and longitudinal dynamic derivative is chose to represent this method. Firstly, the combined dynamic derivatives can be calculated by using two unsteady methods, which are small amplitude pitching oscillation and differential methods. Then the lag of wash derivatives can also be obtained by using the unsteady method of small amplitude plunging oscillation, and the damping derivatives are simply the difference between the combined and lag of wash derivatives. Finally, the Finner missile is taken as an example to testify the methods. The results obtained from the methods demonstrate to be consistent with the experimental data and those from references. Further verification of this systematic method is applied in the Canard Rotor/Wing Aircraft designed by our team; each longitudinal and lateral directional dynamic derivative is simulated, and the results fit the flight tests data well. The systematic simulating methods are testified to be reliable and useful in engineering.

1 Introduction

With the increasing demand for aircraft design, the research of the dynamic stability characteristics has attracted much more attention. Dynamic stability derivatives [1, 2], as key parameters indicating flight dynamic stability, are generally a measurement of how many changes will occur to the forces or moments acting on the vehicle with a fluctuation of parameters in the flight condition, such as angle of attack, airspeed, altitude, etc. These derivatives are the indispensable original aerodynamic parameters for the aircraft navigation system, control system and dynamic quality analysis as well.

Currently, there are several ways to acquire dynamic derivatives, including theoretical estimations, wind tunnel tests, flight tests or Computational Fluid Dynamics (CFD) methods [3]. Combined with the basic parameters of aircraft, the theoretical estimation method can efficiently calculate the dynamic derivatives based on empirical or semi-empirical formulas, charts, etc. However, the application of this method is limited for lack of accuracy and invalid in transonic flow. Wind tunnel tests and flight tests can measure all the dynamic derivatives accurately, and can also capture the details of flow, but it is difficult to perform systematic research with such huge cost and long design period. With the development of CFD, especially in unsteady aerodynamics theory and computation, it is possible to acquire dynamic derivatives with CFD methods, which have become a promising way for aircraft design [4, 5, 6]. The methods to obtain dynamic derivatives mainly rely on identifying the unsteady aerodynamic forces and moments calculated by CFD code. Researchers have done much work and developed many methods in terms of CFD in recent years, of which the unsteady Euler method [7], the dual-time

DADAI method [8] and the nonlinear reduced frequency method [9] are the most three famous ones.

The modern aircraft design needs detailed dynamic derivatives so that the aerodynamic configuration and control rate can be designed more effectively, which has not been satisfied as most research can only achieve the combined derivatives. This paper focuses on systematically simulating the dynamic derivatives using CFD. Firstly, methods to calculate combined and single dynamic derivatives are built and testified by using Finner missile, and further verification of the calculating methods of dynamic derivatives are done in the Canard Rotor/Wing Scale Aircraft model. Compared with results from the system parameter identification using the flight tests data, we can complete more analysis of the methods

2 Methods to calculate dynamic derivatives

Based on resonant perturbation theory, we propose a systematic method to calculate dynamic derivatives through CFD code. Now take longitudinal as an example to introduce this method in detail.

2.1 Combined dynamic derivatives

Two unsteady methods can be applied to calculate the combined derivatives, namely the small amplitude oscillation method and differential method.

2.1.1 Small amplitude oscillation method

This method forces the model to oscillate around the centre of gravity. With Taylor expansion, the unsteady moment can be expressed as

$$\begin{aligned} M_y &= M_{y0} + M_y^{\alpha}\Delta\alpha + M_y^{\dot\alpha}\Delta\dot\alpha + M_y^{\omega_y}\omega_y \\ &+ M_y^{\dot\omega_y}\dot\omega_y + \hat\Delta(\Delta\alpha, \omega_y) \end{aligned} \tag{1}$$

Where M_y^{α}, $M_y^{\dot\alpha}$ are the zero and first order pitching moment dynamic derivatives to the angle of attack(AOA), $M_y^{\omega_y}$, $M_y^{\dot\omega_y}$ are the zero and first order pitching moment dynamic derivatives to pitching angular velocity ω_y, $\hat\Delta$ denotes the higher order derivatives.

When rigid flight vehicles oscillate with circular frequency ω in sinusoidal form, the motion equations can be described as

$$\begin{cases} \theta = \theta_0 \sin(\omega t) \\ \dot\theta = \omega\theta_0\cos(\omega t) = \omega_y \\ \ddot\theta = -\omega^2\theta_0\sin(\omega t) = \dot\omega_y \\ \Delta\alpha = \theta = \theta_0\sin(\omega t) \\ \Delta\dot\alpha = \dot\theta = \omega\theta_0\cos(\omega t) \end{cases} \tag{2}$$

Combined with formula (1) and omit the higher order values, the unsteady pitching moment is simplified as

$$\begin{aligned} M_y &= M_{y0} + (M_y^{\alpha} - \omega^2 M_y^{\dot\omega_y})\theta_0\sin\omega t \\ &+ (M_y^{\dot\alpha} + M_y^{\omega_y})\omega\theta_0\cos\omega t \end{aligned} \tag{3}$$

When $\omega t = 2n\pi$, the initial effect can be ignored, and the unsteady moment will be periodical changing, formula (1) can be further interpreted as

$$M_y^{\dot\alpha} + M_y^{\omega_y} = \frac{\bar M_y \sin\lambda}{\omega\theta_0} = \frac{M_{y\omega t=2n\pi} - M_{y0}}{\omega\theta_0} \tag{4}$$

Using reduced frequency $k = \omega l / 2V_*$ to nondimensionalize this method, we finally get the formula to calculate the combined derivatives of pitching moment with small amplitude oscillation method as

$$C_{m\dot\alpha} + C_{mq} = \frac{M_{y\omega t=2n\pi} - M_{y0}}{kqsl\theta_0} = \frac{C_{M_{y\omega t=2n\pi}} - C_{M_{y0}}}{k\theta_0} \tag{5}$$

2.1.2 Differential method

The differential method solves the combined derivatives by enforcing the aircraft moving upwards at same velocity to the same AOA at different angular velocities ω_{y1}, ω_{y2}. Expand the moment equation and omit high-order terms, then the pitching moments are

$$M_{y1} = M_{y0} + M_y^{\alpha}\Delta\alpha + M_y^{\dot\alpha}\Delta\dot\alpha_1 + M_y^{\omega_y}\omega_{y1} \tag{6}$$

$$M_{y2} = M_{y0} + M_y^{\alpha}\Delta\alpha + M_y^{\dot\alpha}\Delta\dot\alpha_2 + M_y^{\omega_y}\omega_{y2} \tag{7}$$

According to the theory of small disturbance in flight mechanics and subtract the two equations, we get

$$M_y^{\dot\alpha} + M_y^{\omega} = (M_{y1} - M_{y2}) / (\omega_{y1} - \omega_{y2}) \tag{8}$$

The non-dimensional pitching angular velocity is

$$\bar\omega_y = \omega_y l / 2V_* \tag{9}$$

Then the differential method to simulate pitching dynamic derivatives is simplified as

$$C_{m\dot{\alpha}} + C_{mq} = \frac{C_{my1} - C_{my2}}{\bar{\omega}_{y1} - \bar{\omega}_{y2}} \tag{10}$$

2.2 Single dynamic derivatives

The first order dynamic pitching moment derivative of AOA $C_{m\dot{\alpha}}$ is also named as the lag of wash derivative. If this derivative can be acquired by CFD, then the combined derivatives minuses $C_{m\dot{\alpha}}$ will get the other dynamic derivative C_{mq} , which is the aim of this systematic CFD method.

We use small plunging oscillation method to identify $C_{m\dot{\alpha}}$, the unsteady pitching moment is

$$M_y = M_{y0} + M_y^{\alpha}\Delta\alpha + M_y^{\dot{\alpha}}\Delta\dot{\alpha} \tag{11}$$

While the motion equation of flight vehicle is

$$z(t) = z_m \sin(\omega t) \tag{12}$$

$$v_z = \dot{z}(t) = \omega z_m \cos(\omega t) \tag{13}$$

$$\dot{v}_z = \ddot{z}(t) = -\omega^2 z_m \sin(\omega t) \tag{14}$$

When the AOA is α , the additional AOA $\Delta\alpha$ due to the motion of flight vehicle is

$$\Delta\alpha = \frac{\omega z_m \cos(\omega t)\cos\alpha}{V} \tag{15}$$

$$\Delta\dot{\alpha} = \frac{-\omega^2 z_m \sin(\omega t)\cos\alpha}{V} \tag{16}$$

Now the pitching moment changes to

$$\begin{aligned} M_y = M_{y0} + M_y^{\alpha}\frac{\omega z_m \cos(\omega t)\cos\alpha}{V} \\ + M_y^{\dot{\alpha}}\frac{-\omega^2 z_m \sin(\omega t)\cos\alpha}{V} \end{aligned} \tag{17}$$

When $\omega t = 2n\pi + \pi/2$, we get

$$M_y^{\dot{\alpha}} = \frac{M_y - M_{y0}}{-\frac{\omega^2 z_m \cos\alpha}{V}} \tag{18}$$

Similar to the small amplitude oscillation method, we can finally nondimensionalize this formula to

$$C_{m\dot{\alpha}} = \frac{C_{my} - C_{my0}}{-k\frac{\omega z_m \cos\alpha}{V}} \tag{19}$$

3 Dynamic derivatives calculation of Finner missile

The Finner missile is one of the most famous dynamic derivative simulation models. The geometry is showed in Fig.1. We will first use this model to calculate the longitudinal dynamic derivatives to verify the CFD method. The Mach number is 1.58 and the initial AOA is 0, Fig.2 is the surface grid generated by ANSYS ICEM CFD.

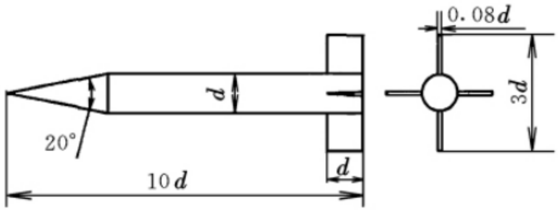


Fig.1. Geometry of Finner missile

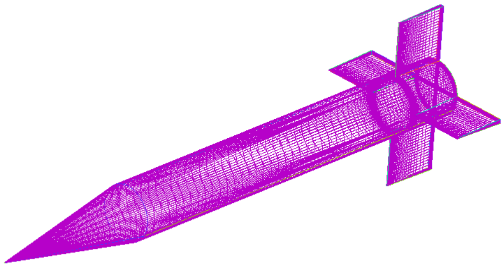


Fig.2. Surface grid of Finner missile

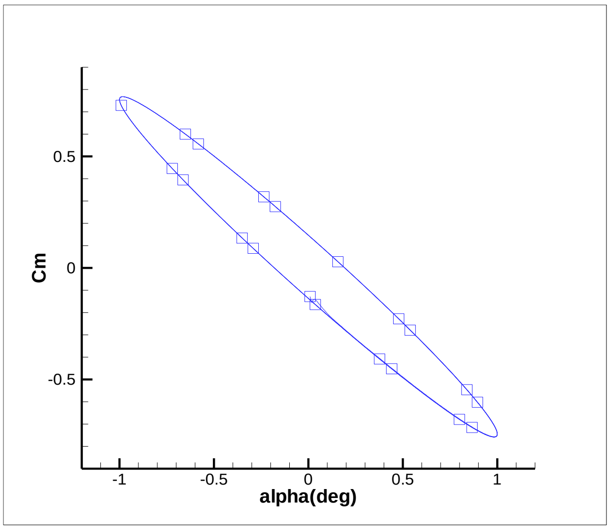
3.1 Combined dynamic derivatives

3.1.1 Small amplitude oscillation method

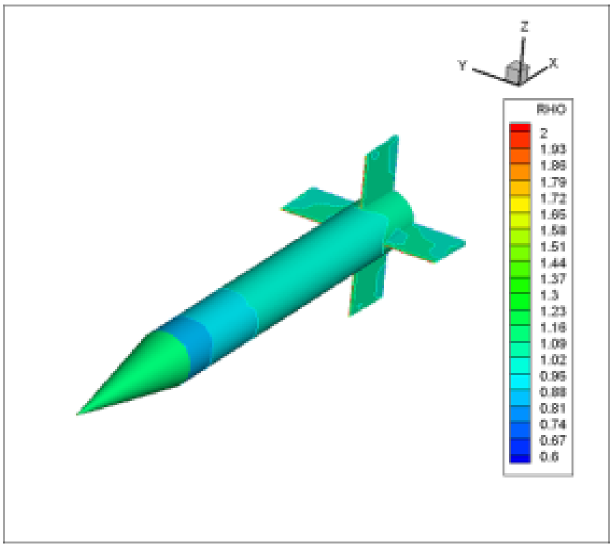
Enforce the model to oscillate around the centre of gravity with motion law

$$\alpha = \alpha_0 + \alpha_m \sin(\omega t) = 1^{\circ} \sin(17t)$$

The reduced frequency k is 0.0158226, the hysteresis loops of moment coefficient and the pressure contour of wall when transient angle of attack is 0.986 degree are showed in Fig. 3.



a. Hysteresis loops of pitching moment coefficient



b. Pressure coefficient contour of wall

Fig.3. Results of small amplitude oscillation method

Based on this method we can finally obtain the combined dynamic derivative in Table 1, the result indicates good consistency with experiment data.

Table 1. Longitudinal combined dynamic derivative

$C_{m\omega t=2n\pi}$	C_{mz0}	Calculation	Experiment	Error
-0.136386	0.004852	-511.48	-526	2.76%

3.1.2 Differential method

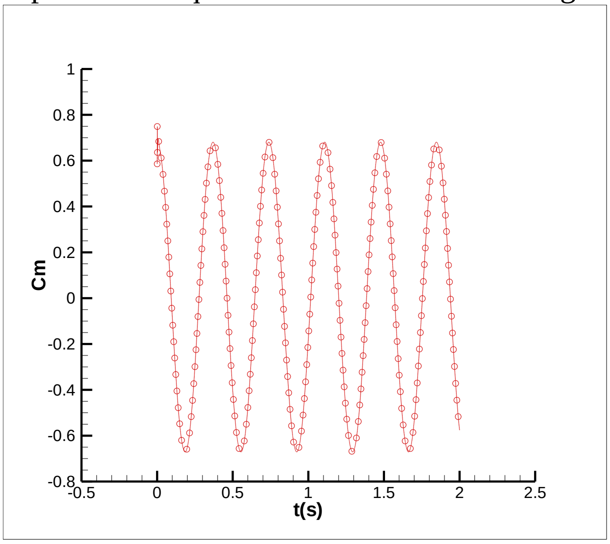
The Finner missile model is forced to move to 5° in two different constant angular velocities $5^\circ/s, 10^\circ/s$. Then the combined dynamic derivative can be obtained by the transient pitch moment at 5° AOA, Table 2 shows the details of the process, and the differential method is effective to simulate combined derivatives.

Table 2. Longitudinal combined dynamic derivative

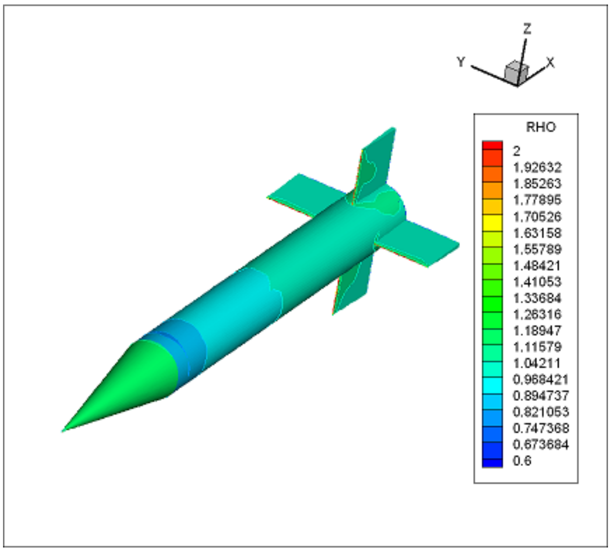
Angular velocity	Moment coefficient	Calculation	Experiment	Error
$5^\circ/s$	-3.7501	-493.7	-526	6.13%
$10^\circ/s$	-3.7902			

3.2 Single dynamic derivatives

The combined derivative is the summation of two single derivatives marked as $C_{m\dot{\alpha}}$ and C_{mq}, and the single dynamic derivative $C_{m\dot{\alpha}}$ can be identified by small plunging oscillation method. The motion equation is $z(t) = 0.2\sin(17t)$ with reduced frequency 0.0158226. The unsteady pitching moment coefficient and the pressure contour of wall when the transient vertical displacement equals 0.13m are showed in Fig. 4.



a. Unsteady pitching moment coefficient



b. Pressure coefficient contour of wall

Fig.4. Results of small plunging oscillation method

The lag of wash derivative $C_{m\dot{\alpha}}$ computed is -53.31, and the pitching damping derivative C_{mq} equals -458.17 with the difference between the combined and the lag of wash derivative, which matches ref. [10] well.

4 Dynamic derivatives calculation of Canard Rotor/Wing Aircraft Scaled model

Further verification of the calculating method of dynamic derivatives is performed on the Canard Rotor/Wing Aircraft Scaled model designed by our team, which is shown in Fig. 5. The flight vehicle consists of prepositive canard, postpositive horizontal tail and central rotor/wing. Canard Rotor/Wing Aircraft shares the flying character of both rotorcraft and fixed wing aircraft. During take-off and landing, it flies like a helicopter, and when flying in high-speed cruising, it works like a fixed-wing aircraft which has three lifting-surfaces. During the transformation from helicopter mode to fixed-wing mode, rotor/wing slows down gradually and then it will be locked. Compared to general flight vehicle, the CRW aircraft is a STOL (short take-off and landing) and economical flight vehicle.

Fig.5. Canard Rotor/Wing Aircraft Scaled model

As a vital approach in aerodynamic research and new-tech validation, flight tests with scaled model will play a more significant role in the future, not only for the reason it can get a lot of data with less costs, but also that the combination of flight tests with this reusable scaled model and CFD calculation is also beneficial for minimizing risks and reducing design period.

Based on the CFD methods developed before, the dynamic derivatives of scaled model in fixed-wing mode are simulated both in longitudinal and lateral directional. The surface gird of simplified scaled model is shown in Fig. 6. Parameters are determined according to the flight tests at an ensemble of isolated points given in Table 3. The centre of gravity is chosen as the moment center and rotation points when simulating dynamic derivatives.

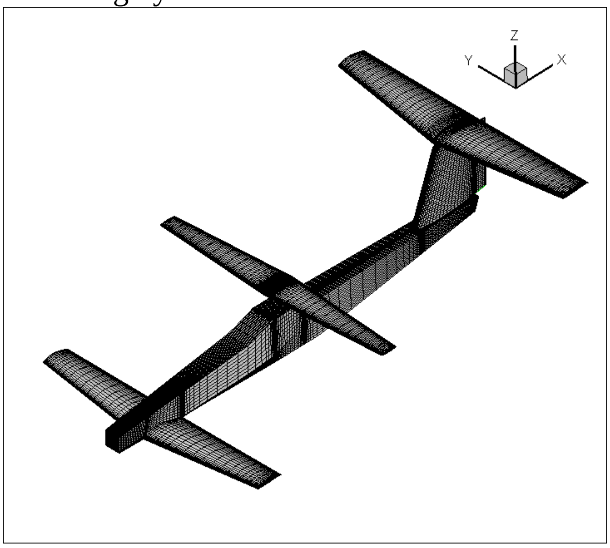


Fig.6. Surface grid of simplified scaled model

Table 3. Parameters for CFD calculation

Parameter	Value
h(altitude)	500m
V_∞ (velocity of far field)	32m/s
c (mean aerodynamic chord)	0.13m
b (span of rotor/wing)	1.3m
S (reference wing area)	0.17m^2

4.1 Dynamic derivatives of longitudinal

4.1.1 Combined dynamic derivatives

The small amplitude oscillation method is used with enforcing the model to oscillate around the centre of gravity as

$$\alpha = \alpha_0 + \alpha_m \sin(\omega t) = 1^\circ \sin(24.615385t)$$

The reduced frequency is $k = \omega c / 2V_\infty = 0.05$, while the differential method requires model ascend to 1° of AOA at two different constant angular velocities $1^\circ/s, 2^\circ/s$. Table 4 shows the CFD solutions of the combined derivatives by these methods.

Table 4. Combined dynamic derivatives

Method	Calculating value	Flight test value
oscillation	-10.8	-10.0
differential	-10.73	

4.1.2 Single dynamic derivatives

With the reduced frequency 0.05, the motion equation when using small plunging oscillation method to obtain $C_{m\dot{\alpha}}$ is $z(t) = 0.1\sin(24.615385t)$, then the longitudinal single dynamic derivative can be calculated. The results are given in Table 5.

Table 5. Longitudinal dynamic derivatives

Dynamic derivative	Calculating value	Flight test value
$C_{m\dot{\alpha}}$	-2.6	-3.1
C_{mq}	-8.2	-6.9

4.2 Dynamic derivatives of lateral directional

4.2.1 Combined dynamic derivatives

Similar to longitudinal, the combined dynamic derivatives of lateral directional can be obtained by small amplitude oscillation method and differential method, the motion law is $\psi = 1^\circ \sin(2.46153846t)$ with reduced frequency $k = \omega b / 2V_\infty = 0.05$ for oscillation method. When using differential method, the model is forced to roll or yaw to 1° at constant angular velocities $1^\circ/s, 2^\circ/s$, and the combined dynamic derivatives from these two methods are showed in Table 6.

Table 6. Lateral directional combined dynamic derivatives

Dynamic derivative	Oscillation method	Differential method	Flight test value
$C_{l\dot{\beta}}\sin\alpha + C_{lp}$	-4.1	-4.4	-5
$C_{n\dot{\beta}}\sin\alpha + C_{np}$	0.5	0.6	0.4
$C_{lr} - C_{l\dot{\beta}}\cos\alpha$	1.8	1.78	1.5
$C_{nr} - C_{n\dot{\beta}}\cos\alpha$	-8	-7	-10.5

4.2.2 Single dynamic derivatives

In order to calculate the single dynamic derivatives of lateral directional, we need to excite the model to move with sinusoidal oscillation to get the single dynamic derivatives $C_{l\dot{\beta}}$ and $C_{n\dot{\beta}}$, then the other dynamic derivatives will be separated. The model moves as $y(t) = 0.1\sin(2.46153846t)$ with reduced frequency $k = \omega b / 2V_\infty = 0.05$. The result can be seen in Table 7

Table 7. Lateral directional single dynamic derivatives

Dynamic derivative	Calculating value	Flight test value
C_{lp}	-4.1	-5
C_{np}	0.5	0.4
C_{lr}	2	1.8
C_{nr}	-10	-13

The longitudinal and lateral directional dynamic derivatives calculation values of Canard Rotor/Wing Aircraft Scaled model match well with the flight test results, which shows that the method is effective.

5 Conclusions

Systematic simulation method with CFD of dynamic derivatives is one key aspect of unsteady aerodynamics, and fundamental methods are developed in this paper for calculating the combined and single dynamic derivatives. The Finner missile and Canard Rotor/Wing Aircraft scale model have been applied to verify the method.

In conclusion, the small amplitude oscillation and differential methods to obtain combined dynamic derivatives as well as the small plunging oscillation method to identify single dynamic derivatives have been validated to be effective, which can also be used for dynamic derivative identification of longitudinal and lateral directional. The combination of CFD methods and flight tests with scale model may provide technical support for flight vehicle design with less costs and time. However, more research is still needed due to the large complicated work for calculations and tests, while it is also in pressing demand to improve the accuracy of obtaining dynamic derivatives.

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