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TURBULENCE MEASUREMENTS IN A FLOW PART OF TURBOMACHINES USING A HIGH-FREQUENCY PRESSURE PROBE

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Abstract

In the present paper the method for determining turbulent flow characteristics in a flow part of turbomachines using a high-frequency pressure probe is presented.

In the paper is shown how to use a high-frequency pressure probe to determine the Reynolds stress tensor and the turbulence intensity of a gas flow, to find an one-dimensional power spectrum, to find turbulent length scales and the dissipation rate of a turbulent kinetic energy in locally homogeneous and locally isotropic flows.

Some drawbacks of the method for determining turbulent flow characteristics related to the probe head's geometry are pointed.

1 Introduction

One of the main problems arising in the designing of turbomachines is a correct assessment of a gas flow turbulent characteristics. The proper assessment of turbulent characteristics of a gas flow allows more reliably predicts an efficiency of turbomachines. Having known the real turbulent characteristics of a gas flow in turbomachines it could permit to verify CFD codes that will enable to predict characteristics of a turbomachines more correctly.

A certain number of papers concerned measurements of turbulent flow characteristics using high-frequency pressure probes were published in literatures. For example, in [1] the method of measurement the Reynolds stress tensor in the flow part of the axial turbine using the two-sensors high-frequency pressure probe and some its results are presented. The main drawback of the method [1] is that it based on using the Bernoulli's equation, and hence is suitable only for research incompressible flows. Also, the method [1] does not allow to determine the turbulent length scales, a turbulent power spectrum and the dissipation of turbulent kinetic energy. In [2] and [3] are shown results of measurements of turbulent flow characteristics in a wind tunnel and at the outlet of a rotating nozzle by means of the four-sensor high-frequency pressure probe. The design of the four-sensor high-frequency pressure probe [2] is made so that sensors are placed outside a measurement area and are connected with the probe head by means of tubes. The probe made under this scheme [2] does not provide an opportunity to measure high frequency pulsations of three components of a velocity vector.

This paper presents a method for determining turbulent characteristics of a gas flow in a flow part of turbomachines using the five-sensor high-frequency pressure probe. The presented method of measuring turbulent flow characteristics is based on using a calibration characteristic of the five-sensor high-frequency pressure probe for determining discrete oscillograms of three components of a velocity vector followed by extraction of their turbulent constituents.

2 The method's description

2.1 Defining the probe's calibration characteristic

Fig. 1 shows the scheme of the five-sensors high-frequency pressure probe Kulite FAP-HT-250 [4] which can be used for measurement of turbulent characteristics of a gas flow.

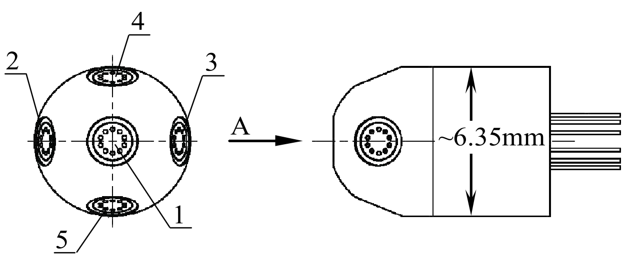

Fig. 1. The scheme of the five-sensors high-frequency pressure probe FAP-HT-250.

At the probe head of ~ 6.35 mm diameter are five piezoresistive pressure sensors (Fig. 1). Each sensor is designed as the four arm Wheatstone bridge with transducers based on silicon [4] and has two contact inputs and two outputs. The sensors have a high sensitivity and enable to make a registration of pressure fluctuations in the flow with a frequency not less than 200 kHz.

The pressure probe's calibration characteristics are essential to make measurements of flow's parameters. The calibration characteristic allows to establish a connection between signals from the probe's five pressure sensors and the flow parameters such as the total pressure p^* and the static pressure p, flow velocity c, the incidences at the probe head in the plane of the sensors 1, 2, 3 - α_0 (in the circumferential direction), and the sensors 1, 4, 5 - β_0 (in the radial direction). Calibration characteristic is obtained experimentally by testing the pressure probe in the wind tunnel at different regimes of the reduced flow velocity λ_{is} and at the different incidences at the probe head α_0, β_0.

The incidence at the probe head in the circumferential direction α_0 is the function of the regime λ_{is} and the dimensionless parameter h_1.

$$\alpha_0 = f(h_1, \lambda_{is}) \qquad (1)$$

$$h_1 = \frac{p_2 - p_3}{p_1 - \frac{p_2 + p_3}{2}} \qquad (2)$$

The incidence at the probe head in the radial direction β_0 is the function of the regime λ_{is} and the dimensionless parameter h_2.

$$\beta_0 = f(h_2, \lambda_{is}) \qquad (3)$$

$$h_2 = \frac{p_4 - p_5}{p_1 - \frac{p_4 + p_5}{2}} \qquad (4)$$

The total pressure p^* and the static pressure p of a flow at the probe head are the function of the incidences α_0, β_0, and a signal from pressure sensor.

$$p^* = f(\alpha_0, \beta_0, p_1) \qquad (5)$$

$$p = f\left(\alpha_0, \beta_0, \frac{p_i}{p_1}\right) \qquad (6)$$

In (6) p_i is a signal from one of the peripheral sensor (i = 2, 3, 4, 5) which is selected depending upon an incidence at the probe head.

The reduced velocity λ_{is} of the flow is the function of the total pressure p^*, static pressure p, and the isentropic exponent k of a gas.

$$\lambda_{is} = f(p^*, p, k) \qquad (7)$$

The flow velocity c is a function of the reduced velocity λ_{is}, isentropic exponent k of a gas, the gas constant R, and the average total flow temperature T^*.

$$c = \sqrt{\frac{2 \cdot k}{k+1} \cdot R \cdot T^*} \cdot \lambda_{is} \qquad (8)$$

The equations (1) - (8) are determined by the time-averaged signals form the sensors 1, 2, 3, 4, 5 of the pressure probe at the different stationary regimes λ_{is} and at the different stationary incidences α_0, β_0 at the probe head.

It can be assumed that the calibration characteristic (1) - (8) obtained by the time-averaged signals form the probe's sensors will be valid for determining the dynamically changing parameters of a gas flow.

2.2 Definition of turbulent oscillograms of three components of a velocity vector

To solve the problem of measuring dynamically changing parameters of a turbulent flow the information-measuring system of registration of pressure fluctuations must have a sufficiently large sampling frequency per one channel (sensor). To measure the characteristics of turbulent gas flow in the flow part of turbomachines the minimum sampling frequency should be several times higher than the frequency of passing rotor's blades by the high-frequency pressure probe.

Having applied a calibration characteristic of the pressure probe using the equations (1) – (8) to discrete recorded signals from probe's sensors we obtain discrete oscillograms of the flow parameters (changing parameters on time).

$$p^* = p^*(t);\ p = p\,(t),\ c = c\,(t);\quad \alpha_0 = \alpha_0\,(t);\ \beta_0 = \beta_0\,(t) \qquad (9)$$

According to the known velocity c and the incidences to the probe head at the circumferential α_0 and radial β_0 directions we can define the velocity components in the Cartesian coordinate system (in the axial, circumferential and radial directions).

$$c_1 = c\cdot\cos(\alpha_0);\ \ c_2 = c\cdot\sin(\alpha_0);\quad c_3 = c\cdot\sin(\beta_0); \qquad (10)$$

The next step is to apply the discrete Fourier transform for all three components of a velocity vector (velocity components) c_i ($i = 1 \ldots 3$) and construct their amplitude-frequency characteristics. As an example, Fig. 2 shows the amplitude-frequency characteristics of one component of a velocity vector.

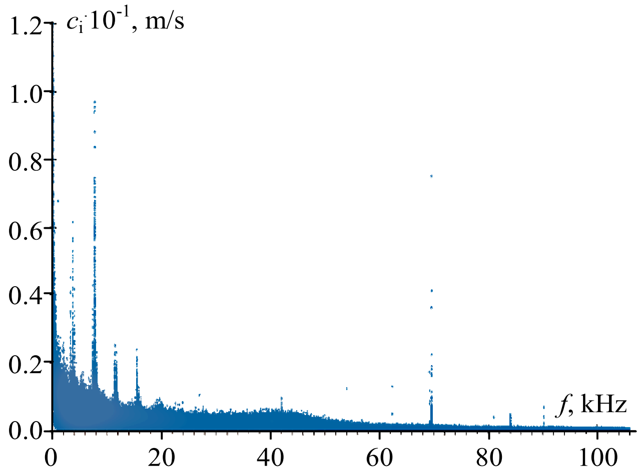

Fig. 2. The amplitude-frequency characteristic of a velocity component.

Fig. 2 shows that the oscillogram of a velocity component c_i contains as a turbulent constituent and an unsteady constituent associated with the presence of wakes in a gas flow from the blades' trailing edges of a turbomachine rotor. Unsteady components is visible on the amplitude-frequency characteristic as the sharp peaks at frequencies proportional to the speed of a turbomachine rotor and the number of blades at different stages of a turbomachine, as well as their superposition pointing to the impact of individual stages to the entire flow pattern of a turbomachine's flow part.

Sharp peaks may be present on the amplitude-frequency characteristic as a result of electrical interference on the measuring channels. These interferences can be easily removed by filtration of the amplitude-frequency characteristic.

The Fourier transform has the property of linearity so it's possible to extract the turbulent constituent of a velocity component from the initial oscillogram. Fig. 3 shows the amplitude-frequency characteristic obtained after applying the filtration of the initial amplitude-frequency characteristic of a velocity component.

The upper area of the amplitude-frequency characteristic (highlighted in blue) is the area of unsteady effects, the lower area (highlighted in red) is the area of the turbulent constituent of a velocity component.

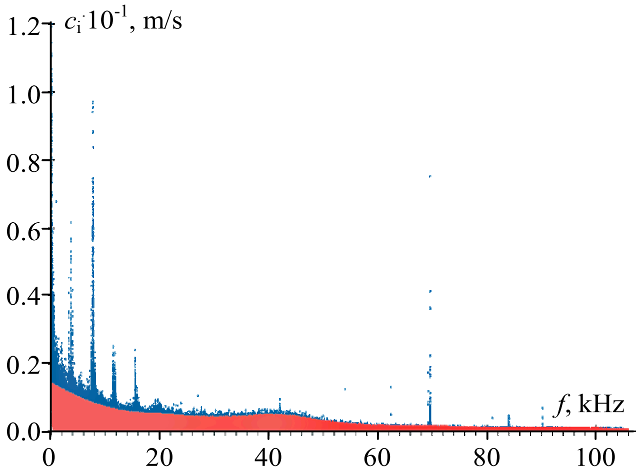

Fig. 2. The filtered amplitude-frequency characteristic of a velocity component.

The Inverse Fourier transform from the upper area of the amplitude-frequency characteristic will give the unsteady oscillogram of a velocity component. The Inverse Fourier transform from the lower area of the amplitude-frequency characteristic will give the turbulent oscillogram of a velocity component.

Thus, it is possible to determine the turbulent oscillograms of all three velocity components c'_i ($i = 1\ldots3$) of a flow.

2.3 Definition of the turbulent flow characteristics

Using the defined turbulent oscillograms of three velocity components c'_i ($i = 1\ldots3$) we can find their double-point correlation C_{ij} (Reynolds stress tensor) on a certain period T.

$$C_{ij} = \overline{c'_i c'_j} = \cdot\frac{1}{T}\int_0^T c'_i c'_j \mathrm{d}t \qquad (11)$$
$$i, j = 1\ldots3$$

Turbulent kinetic energy of a flow we can be defined as the first invariant of the Reynolds stress tensor C_{ij} (half the sum of the tensor's diagonal elements).

$$k = \frac{1}{2}\cdot\overline{c'_i c'_i} \qquad (12)$$

The turbulence intensity of a flow is defined by using turbulent kinetic energy k and modulo of an averaged velocity vector U of a flow.

$$Tu = \frac{\sqrt{2\cdot k}}{U} \qquad (13)$$

Reynolds stress tensor C_{ij} written in the principal axes has the following form:

$$C_{ij} = \begin{bmatrix} \overline{c'_1 c'_1} & 0 & 0 \\ 0 & \overline{c'_2 c'_2} & 0 \\ 0 & 0 & \overline{c'_3 c'_3} \end{bmatrix} \qquad (14)$$

If the equality of the tensor components (14) lying on the main diagonal is performed then the turbulence of a flow is isotropic.

Hereinafter the method of definition of the turbulent flow characteristics is assumed that turbulence of a flow is isotropic.

The oscillogram of the velocity component $c_u(t)$ along the direction of an averaged velocity vector U on a certain period T can be found using the angles α_{iu} between the averaged velocity vector U with the axes of the Cartesian coordinate system and three components of a velocity c_i(t) defined by (10).

$$c_u = c_i \cdot \cos\alpha_{iu} \qquad (15)$$

To find the turbulent flow characteristics the turbulent constituent c'_u of the velocity component c_u must be extracted. It could be done by using a direct Fourier transform, filtration and inverse Fourier transform (see above).

The Euler temporal correlation R_E (normalized autocovariance of a time parameter $\tau > 0$) [5] of the turbulent constituent c'_u can be written as

$$R_E = \frac{\overline{c'_u(t)c'_u(t+\tau)}}{\overline{c'_u(t)}\cdot\overline{c'_u(t+\tau)}} \qquad (16)$$

If the oscillogram of the turbulent constituent c'_u is an ergodic stationary random function then the Euler temporal correlation R_E can be defined as following integral:

$$R_E = \frac{1}{\overline{c'_u(t)}\cdot\overline{c'_u(t+\tau)}}\frac{1}{T}\int_0^T c'_u(t)c'_u(t+\tau)\mathrm{d}t \qquad (17)$$

In a locally homogeneous turbulence of a flow the equation (17) has the following form:

$$R_E = \frac{1}{\overline{c_u^{'}(t)}^2} \frac{1}{T} \int_0^T c_u^{'}(t) c_u^{'}(t+\tau) dt \qquad (18)$$

As an example, Fig. 3 shows the Euler temporal correlation R_E of the turbulent constituent c'_u of the velocity component c_u.

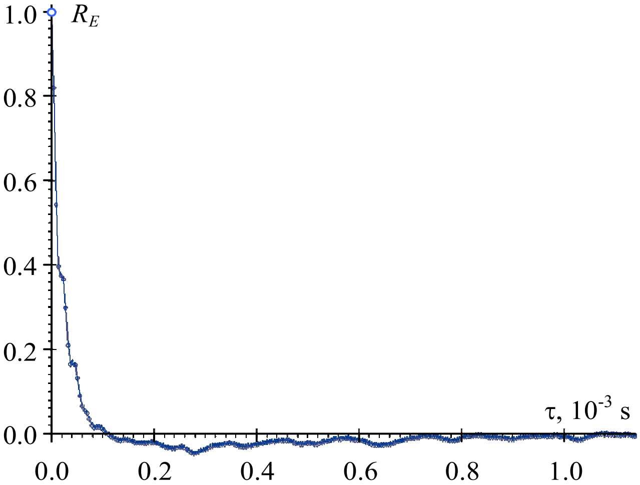

Fig. 3. The Euler temporal correlation R_E of the turbulent constituent c'_u of the velocity component c_u.

The Euler turbulent integral scale L_E along the direction of an averaged velocity vector U [5] is defined by integrating the Euler temporal correlation L_E.

$$L_E = \int_0^\infty R_E d\tau \qquad (19)$$

The integration of the Euler temporal correlation R_E (19) could be carried out either numerically or analytically. In the second approach is necessary to approximate Euler temporal correlation R_E using an analytic function.

Let's further assume that the turbulence field is locally homogeneous and locally has a constant averaged velocity vector U. Then using Taylor's hypothesis we can determine the longitudinal turbulent integral scale Λ_f [5].

$$\Lambda_f = U \cdot L_E \qquad (20)$$

Longitudinal turbulent integral scale Λ_f can be interpreted as the largest size of turbulent eddies in the flow [5].

The one-dimensional power spectrum $E(f)$ of the turbulent constituent c'_u of the velocity component c_u along the direction of an averaged velocity vector U can be written as [5]:

$$E(f) = 4\overline{c_u^{'}}^2 \cdot \int_0^{+\infty} R_E(\tau) \cdot \cos(2\pi f \tau) d\tau \qquad (21)$$

The one-dimensional power spectrum $E(f)$ shows the change of the energy of the turbulent constituent c'_u with a frequency f.

The Euler temporal correlation R_E is the inverse Fourier transform of the power spectrum $E(f)$ [5].

$$R_E(\tau) = \frac{1}{\overline{c_u^{'}}^2} \cdot \int_0^{+\infty} E(f) \cdot \cos(2\pi f \tau) df \qquad (22)$$

If we take into account the identity of Euler temporal correlation R_E and longitudinal normalized autocovariance R_f we obtain the same equations (21) and (22) for the longitudinal normalized autocovariance R_f along the direction of an averaged velocity vector U [5]:

$$E(f) = \frac{4\overline{c_u^{'}}^2}{U} \cdot \int_0^{+\infty} R_f(x) \cdot \cos\left(2\pi f \frac{x}{U}\right) dx \qquad (23)$$

$$R_f(x) = \frac{1}{\overline{c_u^{'}}^2} \cdot \int_0^{+\infty} E(f) \cdot \cos\left(2\pi f \frac{x}{U}\right) df \qquad (24)$$

As an example, Fig. 4 shows the one-dimensional theoretical power spectrum $E(f)$ obtained using (23) wherein the longitudinal normalized autocovariance R_f is taken as the exponential function (25).

$$R_f(x) = \exp\left(-\frac{x}{\Lambda_f}\right) \qquad (25)$$

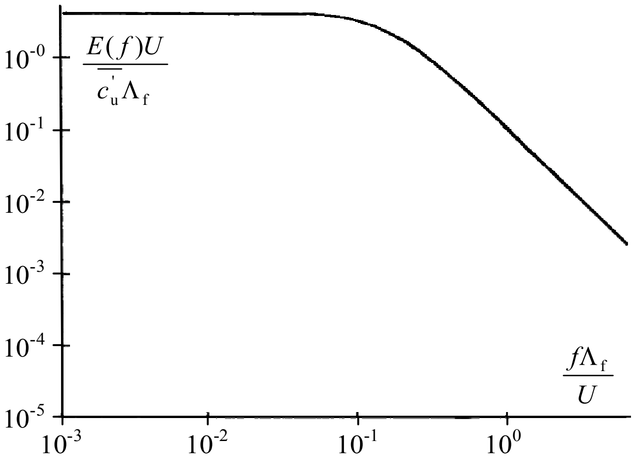

Fig. 4. The one-dimensional theoretical power spectrum $E(f)$ of the turbulent constituent c'_u of a velocity component c_u.

The longitudinal turbulent microscale λ_f is defined as the value of the second partial derivative of the normalized longitudinal autocovariance R_f at $x = 0$ [5].

$$\frac{1}{\lambda_f} = -\frac{1}{2}\cdot\left[\frac{\partial^2 R_f}{\partial x^2}\right]_{x=0} \qquad (26)$$

The longitudinal turbulent microscale λ_f can be interpreted as a size of turbulent eddies through which the dissipation of a turbulent kinetic energy is proceeded [5].

Taking into account the equation (24) the longitudinal turbulent microscale λ_f can be rewritten as follows:

$$\frac{1}{\lambda_f} == \frac{2\pi^2}{\overline{c_u'^2}\,U^2}\cdot\int_0^{+\infty} f^2 E(f)\,df \qquad (27)$$

The equation (27) shows that the longitudinal turbulent microscale λ_f is determined by the one-dimensional power spectrum $E(f)$ at high frequencies f.

2.4 Definition of the dissipation rate of a turbulent kinetic energy

The main method of a mathematical modeling of dynamic processes in turbomachines is based on a system of the RANS equations. This system of the RANS equations requires an additional equation or a system of additional equations (turbulence models) to set connection between turbulent parameters (Reynolds stress tensor) and averaged parameters of a flow. Most popular turbulence models such as k-e, k-ω, q-ω etc. are based on the isotropic nature of a turbulence. Therefore as parameters of such models are usually taken the turbulent kinetic energy (for one-parameter models) or a turbulent scale and the turbulent kinetic energy (for two-parameter models). As a turbulent scale is usually used the dissipation rate ε of a turbulent kinetic energy or the frequency turbulent scale ω.

The dissipation rate ε for compressible viscous gases is proportional to the kinematic viscosity of a gas ν and a correlation of partial derivatives of turbulent constituents of three velocity components [5].

$$\varepsilon = \nu\cdot\overline{\left(\frac{\partial c_i'}{\partial x_j}+\frac{\partial c_j'}{\partial x_i}\right)\cdot\frac{\partial c_j'}{\partial x_i}} \qquad (28)$$

In an incompressible gas with an isotropic turbulence the equation (28) can be rewritten as follows [5]:

$$\varepsilon = \nu\cdot 30\cdot\frac{\overline{c_u'^2}}{\lambda_f^2} \qquad (29)$$

The equations (27) and (29) show that the magnitude of the dissipation rate ε is determined by small-scale high-frequency velocity fluctuations.

The frequency turbulent scale ω is directly proportional to the dissipation rate ε and inversely proportional to the turbulent kinetic energy k [6]:

$$\omega = \frac{\varepsilon}{k} \qquad (30)$$

The coefficient of the kinetic turbulent viscosity is written as follows (up to a constant) [6]:

$$\nu_t = \frac{k}{\omega} \qquad (31)$$

3 Conclusion

In the present paper the method for determining turbulent flow characteristics in a flow part of turbomachines using the high-frequency pressure probe is presented. The method is based on the calibration characteristics of a high-frequency pressure probe to determine the discrete oscillograms of three components of a velocity vector with the next extraction their turbulent constituents.

The method allows:

- to determine the turbulent oscillograms of three velocity components c'_i ($i = 1\ldots3$) of a flow;
- to determine the Reynolds stress tensor C_{ij} and the turbulence intensity Tu of a flow;
- to determine the longitudinal turbulent integral scale Λ_f and the longitudinal turbulent microscale λ_f in a locally homogeneous and locally isotropic turbulence as well as the one-dimensional power spectrum $E(f)$ of the turbulent constituent c'_u of the velocity component along the direction of an averaged velocity vector U;
- to determine the dissipation rate ε of a turbulent kinetic energy and the frequency turbulent scale ω in a locally homogeneous and locally isotropic turbulence.

The use of the high-frequency pressure probe for measuring the turbulent characteristics of a flow has the following drawbacks:

- the probe head is a blunt body which would distort the turbulence intensity of an oncoming flow. Therefore the pressure probe have to be calibrated in flows with known turbulent characteristics before using it in turbomachines;
- the use of the pressure probe is limited in flows with a high turbulence intensity. In such flows on the probe head may occurs local separation which may lead to a distortion of measured parameters. A confidential turbulent intensity range of the pressure probe can be determined experimentally by testing the probe in flows with known turbulent characteristics.

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