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CONSTRUCTION AND USE OF IMITATING MODEL IN THE PROCESS OF MULTI-CRITERIA OPTIMIZATION OF THE SEPARATE-FLOW NOZZLE

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Abstract

There is considered a procedure of an optimum aerodynamic design of exhaust nozzles for high-bypass-ratio turbofans (BPR≥8) using numerical simulation of viscous flows on the base of RANS equations with q-ω turbulence model. Problem of multi-criterion optimization of nozzle configuration is considered in severe statement with taking into account real restrictions on the acceptable range of control parameters variation. In order to simplify solution of optimization problems and decrease volume of parametric calculations, functional correlation between nozzle characteristics and the control parameters is presented as a multi-factor simulation model.

Introduction

Designing both nozzle and other elements of high-bypass turbofan (HBT) has obvious compromise character since there is necessity to provide stable and effective work of cruising power plant for all flight regimes. In such situation, it is rather difficult to choose decisive criterion (objective function) because considered objective functions either have optimum in different zones of control parameter space or control parameters has opposite influence of these functions. For example, diminishing take-off noise gives diminishing the thrust at cruising flight regime and worsening mass-overall parameters.

One of the key items of proposed methodology is representation of parametrical calculation results as a simulation model that defines a functional connection between control and geometrical parameters and integral characteristics of nozzle. Using the simulation model essentially simplifies solution of multi-criterion optimization problem and, simultaneously, diminishes the volume of necessary parametrical calculations. Efficiency of developed methodology is demonstrated for shape optimization of high-bypass turbofan nozzle with bypass ratio BPR~9.4.

1 Choice of control parameters and optimization criteria

Performed analysis of numerical-experimental investigation results of optimal geometry of bypass nozzle has shown that effective thrust losses are influenced by about 10 geometrical parameters and that expected positive effect of choosing these optimal parameter values is often comparable with test errors of thrust characteristics of models in wind tunnel (WT). It is obvious that most of geometrical parameters of bypass nozzle elements can't be changed independently. For example, in the case of approximately constant engine length, variation of outer contour nozzle cowl length inevitable results in change of gas generator length and, hence, in change of its angle contraction.

Large difference of pressure drops in different contours is characteristic for HBT with bypass ratio BPR ≥8. Preliminary estimations show that gas flow in outer contour nozzle takes place at supercritical pressure drops (NPR_2>2) and, in inner contour nozzle, at subcritical pressure drops (NPR_1<1.8). Outer contour supersonic jet results in a complicated flow structure with interchange of rarefaction zones and shock waves. Their position influences on subsonic jet of inner contour is realized.

To develop an adequate simulation model of bypass nozzle thrust characteristics with taking into account 10 geometrical parameters, it is necessary to create a data bank included not less than 3^{10} points. Because it is impossible to test necessary number of model variants, numerical simulation methods of flow on basis of Navier-Stokes equations can be used as a tool of aerodynamic designing of bypass output device. But, even in this case, calculation of ~60 000 nozzle variants is unreal task. For correct solution of optimization problem, it is necessary to choose a set of control parameters among whole diversity of control parameters. They must correspond to flow physics, permit to develop an adequate simulation mathematical model of nozzle thrust characteristics and influence both on aerodynamic and on mass-overall characteristics of bypass nozzle. In the current work, following geometrical parameters have been chosen as control these, in accordance with the scheme in Fig. 1:

– theoretical inclination angle of nozzle outer contour exit section relatively the nozzle symmetry axis, θ_t,
– rounding radius of gas generator cowl near nozzle outer contour exit section, R,
– contraction angle of gas generator cowl at the exit section of nozzle outer contour, θ_f,
– contraction angle of conical part of inner contour central body, θ_c

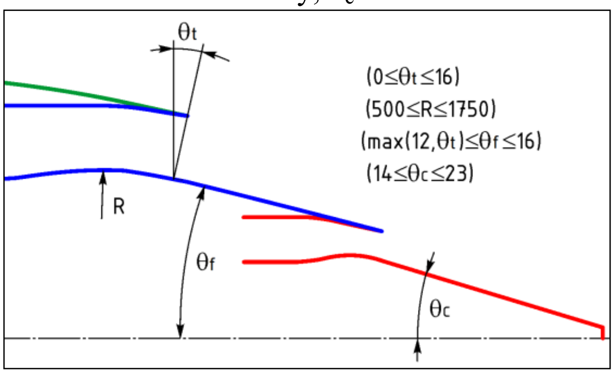

Fig. 1. Control geometrical parameters for bypass nozzle

Control parameters have been varied in following ranges:

$$\begin{aligned} &\theta_t^{\min} \le \theta_t \le \theta_t^{\max}, \\ &R^{\min} \le R \le R^{\max}, \\ &\max(\theta_t^{\min}, \theta_t) \le \theta_f \le \theta_f, \\ &\theta_c^{\min} \ge \theta_c \le \theta_c^{\max}. \end{aligned} \tag{1.1}$$

A classical problem of nozzle aerodynamic design is to search such combination of geometrical parameters that provides maximal coefficient of effective thrust $\overline{P}_{c\ eff}$ and, correspondingly, minimal effective thrust losses of nozzle $\Delta\overline{P}_{c\ eff} = 1 - \overline{P}_{c\ eff}$. It should be noticed that efficient thrust coefficient of bypass nozzles is defined as relation between sum of inner and outer contour thrusts (excluding external drag force of outer contour) and sum of inner and outer ideal thrusts.

As a rule, a designer, during HBT development, have to take into account both aerodynamic and mass characteristics of projected objects, i.e. a multi-criterion optimization problem of power plant elements is solved. Only an engine designer can perform a qualified calculation of nozzle mass characteristics. Therefore, it is useful to apply a simplified model of nozzle mass characteristics at the stage of testing the multi-criterion optimization methodology, when the engine designer doesn't take part into optimization process. In the scope of the current work, a "surface" mathematical model, where nozzle mass (G_c) is proportional to its surface area is proposed. Such model permits to compute change of structural mass in dependence of four control parameters. The mass of construction includes the mass of fixed part G_{fix} that is taken the same for all variants of nozzle, because it doesn't practically depend upon control parameters. The mass of construction also includes the mass of a variable part G_{var} that essentially depends upon control parameters:

$$G_c = G_{fix} + G_{var}(\theta_t, R, \theta_f, \theta_c) \tag{1.2}$$

Variation of construction mass relatively some basic variant of nozzle $G_{c\ base}$ that mass is calculated using similar methodology is defined as follows:

$$\Delta G = G_c / G_{c\ base} - 1 \quad (1.3)$$

For basic nozzle variant, values of control parameters at the center of accessible region of variation have been taken.

2 Development of mathematical model of bypass nozzle geometry, creation and automatic modification of calculation grid

All the calculations in the current work have been performed on basis of the code ZEUS EWT TsAGI [1] using $q-\omega$ turbulence model. Calculations have been performed only for cruising flight regime with Mach number M_∞=0.8. Using the numerical investigation results for basic configuration, a mathematical model of bypass nozzle geometry that is based on analytical dependences has been proposed. Generatrixes of axisymmetric nozzles of the inner and outer contours have been given by straight segments and circular arcs. First of all, the geometry of outer contour nozzle has been optimized. The geometry of subsonic part of outer contour nozzle hasn't been varied during the calculations. Preliminary estimations show that HBT with bypass ratio BPR~9 has inner contour nozzle thrust about 15% of outer contour nozzle thrust. Therefore, the main attention has been devoted to optimization of outer contour nozzle. Only contraction angle θ_c of center body has been varied among all geometrical parameters of inner contour nozzle. The diameter of bottom tip at central body was the same for all the variants.

3 Peculiarities of developing the multifactor simulation model of nozzle

To calculate the integral characteristics of nozzle, a simulation model is proposed. This model is one of possible variants for determination of the functional connection between control geometrical parameters (factors) and integral characteristics of nozzle (responses) – objective functions. The simulation model is analytical dependencies, which permit to estimate the response value Y with necessary accuracy at any point of accessible region of factor's variation Ω. The current work uses methods of linear regression analysis [2] in developing the simulation model of nozzle integral characteristics. In the framework of this analysis, model type is defined as follows:

$$Y(\vec{X}) = \vec{b} * \vec{\Phi}(\vec{X}) = \sum_{i=1}^{k} b_i \varphi_i(\vec{X}), \quad (3.1)$$

$$\vec{X} \in \Omega,$$

where $\vec{X}$ is a vector of factors, $\vec{\Phi}(\vec{X})$ is a functional vector; $\varphi_i(\vec{X})$ are basis functions – components of vector $\vec{\Phi}(\vec{X})$; k is dimension of coefficient vector $\vec{b}$.

The paper [3] describes an algorithm for developing the multifactor regression equation (3.1), when there is no a priori information about view of functional vector $\vec{\Phi}(\vec{X})$. In accordance with this algorithm, at first, "initial" calculations are performed to obtain 1D section for each factor x_j and to choose formulas that approximate these sections sufficiently

$$y(x_j) = \vec{b}_j \vec{f}_j(x_j) \quad (3.2)$$

After that, the view of functional vector $\vec{\Phi}(\vec{X})$ is determined on basis of formulas (3.2) using rather simple and well-defined procedures.

Because the simulation model of nozzle is directed, first of all, to solution of optimization problems, it seems useful to modify algorithms from [3] for choosing a series of 1D section that are necessary to define 1D approximation formulas (3.2). For that, at the first stage of data analysis, it is assumed to perform a preliminary one-criterion optimization using the coordinate descent method. It is known that this method determines the trajectory approaching to extremum using the analysis results of 1D sections $y(x_j)$ and that space position of each subsequent section factors is defined with taking into account optimal values of factors $x_j^{(opt)}$ that have been determined at the previous stages. During the preliminary optimization, several problems are solved at the same moment. Firstly, obtained 1D sections are used to define the form of equations (3.2). Secondly, a subregion with the best values of response is chosen in factor space and gravity center of initial data sample shifts to this subregion. It has a good influence on simulation model (3.1) adequacy. In addition, qualified information about zone of objective function extremum is useful both to correct the boundaries of accessible region of factor variation and in drawing the plan of parametrical calculations.

3.1 Preliminary optimization of nozzle shape, analysis of 1D sections

As an initial point for coordinate descent method, a center of accessible region of factor variation (1.1) has been chosen, where several values of effective thrust $\overline{P}_{c\ eff}$ have been calculated. These values have corresponded to different values of angle θ_t, i.e. points of 1D section $\overline{P}_{c\ эф}(\theta_t)$ have been determined. Using a dialogue system APPEX [4], an approximation formula has been chosen to provide adequate representation of 1D sections $\overline{P}_{c\ эф}(\theta_t)$. Obtained approximation formula has been used to determine the value $\theta_t^{(opt)}$ that corresponds to $\overline{P}_{c\ eff}$ maximum. Obtained $\theta_t^{(opt)}$ has determined a new point of factor space, where approximation formula for the section $\overline{P}_{c\ eff}(R)$ has been chosen and the value $R^{(opt)}$ has been determined. Further, the process has been repeated for remain control parameters θ_f and θ_c. As a result, a point in zone with high values of objective function $\overline{P}_{c\ eff}$ has been chosen in factor space and, at the same time, form of approximation equations (3.2) has been defined:

$$\overline{P}(\theta_c) \Rightarrow \vec{f}_1(x_1) = \{1, \mathrm{Sin}(x_1), \cos(x_1), \mathrm{Sin}(2x_1), \cos(2x_1), \sin(3x_1)\},$$
$$\overline{P}(R) \Rightarrow \vec{f}_2(x_2) = \{1, \mathrm{Ln}(x_2), \mathrm{Ln}^2(x_2), \mathrm{Ln}^3(x_2)\},$$
$$\overline{P}(\theta_c) \Rightarrow \vec{f}_3(x_3) = \{1, \mathrm{Sin}(x_3), \cos(x_3), \mathrm{Sin}(2x_3)\},$$
$$\overline{P}(\theta_\kappa) \Rightarrow \vec{f}_4(x_4) = \{1, x_4, x_4^{\ 2}\}. \tag{3.3}$$

Quality of 1D section approximation can be estimated in Fig. 2, where the solid line corresponds to approximation values and the symbol “star” shows position of effective thrust coefficient maximum $\overline{P}_{c\ eff}$.

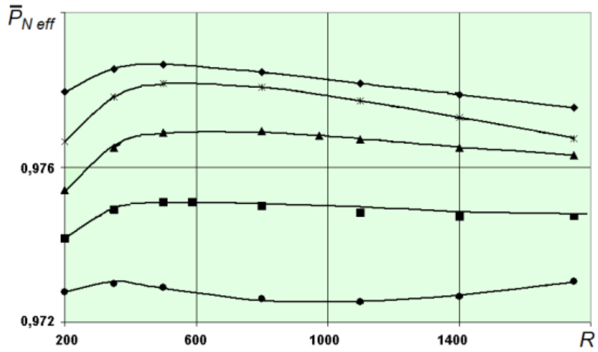

Fig. 2. Approximation of 1D sections

After triangulation of obtained points corresponded to 1D sections $\overline{P}_{c\ eff}(\theta_t)$ and $\overline{P}_{c\ eff}(R)$, levels of function $\overline{P}_{c\ eff}(\theta_t, R, \theta_f = \theta_f^{(opt)}, \theta_c = \theta_c^{(opt)})$ can be develop. These levels are shown in Fig. 3. Figure 3 shows that levels obtained using initial data (solid lines) and using 1D approximation (dashed line) are very close. It confirms correspondence of chosen approximation formulas.

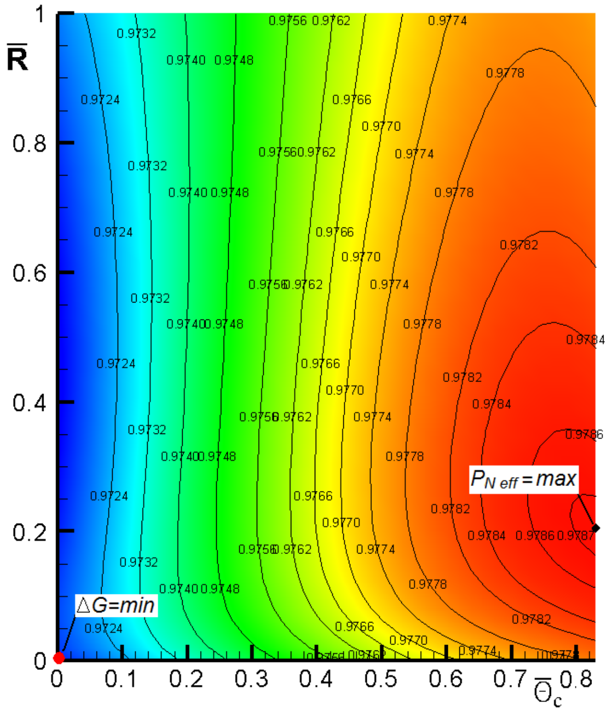

Fig. 3. Levels of function $\overline{P}_{c\ eff}(\overline{\theta}_t, \overline{R})$

3.2 Development of multi-factor simulation model type

According to [3], approximation formulas (3.3) are the basis for development of multi-factor regression equation. Let's define the symbol $\otimes$ as an operation that is used to two functional vectors and permits to obtain a vector, which components are various pairwise products of initial vector components. Then, following functional vector $\vec{\Phi}(\vec{X})$ is defined for the simulation model (3.1)

$$\vec{\Phi}(\vec{X}) = \vec{f}_1(\theta_c) \otimes \vec{f}_2(R) \otimes \vec{f}_3(\theta_z) \otimes \vec{f}_3(\theta_\kappa) \quad (3.4)$$

It should remember that such approach to development of simulation model type gives rather complicated resulting regression equation (3.1) (k=288 for formulas like (3.3)). It can include terms that inessentially influence on response behavior. But, if only significant terms remain in the equation at the stage of regression coefficient calculation, then the length of resulting regression equation can be essentially diminished and, appropriately, the volume of initial data that are necessary to develop the simulation model can be decreased.

One of the responsible moments of developing the simulation model is to choose a plan of parametrical calculations, i.e. determination of the number and position of calculation points in factor space. Methods of experiment mathematical planning [5] permit to choose an experiment plan for regression model of given type, so as necessary statistical properties of this model are provided and, at the same time, the volume of initial data sample is minimal. When the final type of simulation model can't be defined, methods of experiment mathematical planning [5] can't be used. To develop the plan of parametrical calculations, the current work proposes to use so-called LP_τ sequences [6], which permit the most uniform distribution (in terms of statistics) of calculation points in factor space. Parametrical calculation plans developed using LP_τ sequences has following advantages:

- values of each factors are varied at non-recurrent levels. It permits to obtain more full representation of factor influence on the response and to calculate acceptable statistical properties of resulting regression equation;
- the algorithm of developing LP_τ sequences provides expansion of initial data sample without necessity to recalculated the points included into plan at the previous stage;
- during the multi-criterion optimization, it is possible to choose approximately efficient points that are estimation Paretto's points.

The plan of parametrical calculations has been chosen with taking into account the limitations (1.1) that define the accessible region of factor variation. The distribution of points inside the accessible region of factor space has been defined using LP_τ sequences and has been added by "angular" points that define limiting values of factors. General number of plan points is equal to 78.

To determine the final form of simulation model (3.1), the current work uses a step-by-step regression procedure that permits essential diminishing the number of terms in the final regression equation. In the framework of this procedure, the most considerable terms of basic equation (3.4) are included into developed regression equation sequentially until growth of multiple correlation coefficient **r** and simultaneous decrease of mean-square deviation **ss** become negligible.

The next stage of step regression procedure completion is to estimate the adequacy of obtained variant of simulation model. Because the dispersion can't be defined for results of numerical calculations, it isn't possible to perform strict verification of regression relation, as it is accepted in framework of regression analysis [3]. In the current work, when a solution about adequacy of obtained simulation model is made, both approximation accuracy of plan points and predicting properties of model for control data sample are taken into account. The results of comparing the levels drawn using initial data and regression relations are also taken into account. As a control sample, data that have been obtained at the stage of 1D section analysis $\bar{P}_{c\ eff}(x_j)$ are considered. In Fig. 4, dashed lines show the approximation results of control data sample using multi-factor simulation model. It is visible that approximation curves are in good correspondence with calculation results and mean-square deviation is about ~0.005%. It is rather acceptable for practical applications.

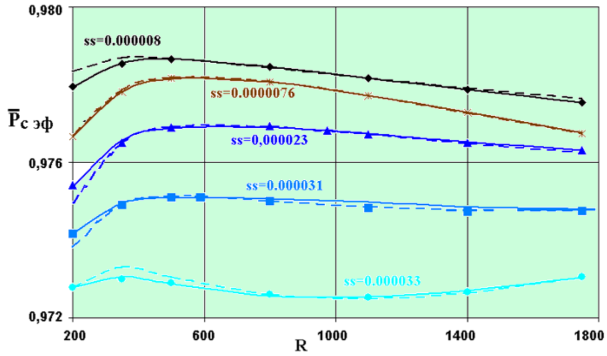

Fig. 4. Multi-factor approximation of 1D sections **ss**

For the purpose of further increase of simulation model quality, the plan of parametrical calculations has been added by points from control data sample and the step regression procedure has been repeated with taking into account the combined sample of initial data. Such procedure both enlarged the number of freedom degrees at the stage of estimation of regression coefficients and improves predictive properties of simulation model at the most interesting subregion of factor space – near maximum of effective thrust coefficient. The solid line in Fig. 4 corresponds to effective thrust coefficient values calculated using final variant of nozzle simulation model.

4 Algorithms of bypass nozzle geometry using simulation model

As it has been noticed above, replacement of discrete series of numerical calculation results by rather simple analytical expression both simplifies solution of different problems, including optimization problems, and excludes necessity of additional numerical calculations even in the cases, when problem formulation is essentially changed. Examples of combined using obtained simulation model and formulas (1.2)-(1.3) for solution of optimization problems in different formulation are presented below.

4.1 Conventional one-criterion optimization

The problem of conventional extremum search is formulated as follows. In the admissible region of varying the geometrical parameters (factors) (1.1), it is necessary to determine the points, where effective thrust coefficient and relative change of nozzle weight achieve their maximal and minimal values. For solution of this problem, direct method, so-called Box's complex-method, is used. This method permits to solve the problem of extremum search with taking into account limitations of both the first (1.1) and the second type.

Figure 3 demonstrates levels of the function $\overline{P}_{c\,eff}(\overline{\theta}_t,\overline{R})_{\theta_f\theta_c=opt}$ in relative coordinates $(\overline{\theta}_t,\overline{R})$. The symbol "rhomb" shows the position of effective thrust maximum and the symbol "circle" shows the point corresponded to minimum of nozzle mass increment.

4.2 Multi-criterion optimization of bypass nozzle

Usually, in development of real constructions, it is necessity to take into simultaneously account several contradictory requirements. Figure 5 shows that minimal nozzle weight and effective thrust maximum are realized in different points of control geometrical parameter space. It is obvious that, in such situation, it is impossible to design a nozzle with minimum possible mass and with maximal thrust efficiency. Therefore, it is necessary to search compromises, i.e. to solve a multi-criterion optimization problem. Quantity of works and its quality, when such problems are solved, are in direct relation with method of obtaining the characteristics of designed article and with the number of considered variants. Analytical representation of nozzle characteristics (in the form of simulation model) permits to solve in strict mathematical formulation almost all optimization problems. Examples of multi-criterion optimization problem solutions using bypass nozzle simulation model are presented below.

4.2.1 Determination of a united objective function

One of possible variants to solve a multi-criterion problem is to choose a united decisive criterion that is a combination of initial objective functions. As a result, the problem is slightly simplified and reduced to one-criterion optimization. As such criterion, a weighted sum of $\Delta\overline{P}_{c\ eff} = 1 - \overline{P}_{c\ eff}(\theta_t, R, \theta_f, \theta_c)$ – effective thrust losses and $\Delta G = G(\theta_t, R, \theta_f, \theta_c)/G_{base} - 1$ – nozzle relative mass change is proposed:

$$F(\theta_t, R, \theta_f, \theta_c) = \Delta\overline{P}_{c\ eff} + w\Delta G, \qquad (4.1)$$

where w is a weight coefficient.

Limiting situation for the function (4.1) is: *w=0* – one-criterion optimization of nozzle aerodynamics, *w>1* – one-criterion optimization of construction mass. Generally, the value of coefficient w depends upon HBT bypass ratio, upon fuel rate, required engine thrust and flight duration. It means that the coefficient w for each concrete aircraft modification must be defined separately, during the process of object investigations.

For each given value of *w*, the optimal form of the nozzle is defined by geometrical parameter values $\theta_t, R, \theta_f, \theta_c$, so as minimum of function (4.1) is achieved. Figure 5 shows values of $\Delta\overline{P}_{c\ eff}$ in optimal point in dependence of weight coefficient *w*.

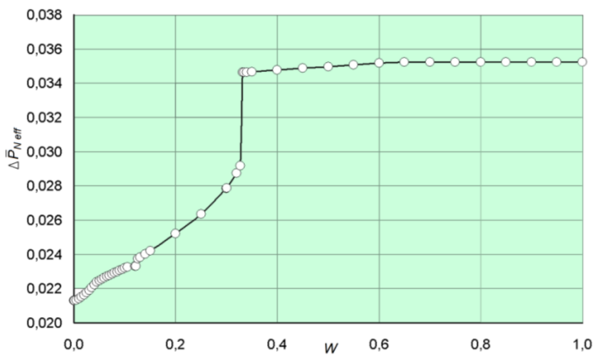

Fig. 5. Dependence of $\Delta\overline{P}_{c\ eff}$ upon weight coefficient *w* in the optimal point

Chart presented in Figure 5 permits both to choose optimal nozzle geometry and to define its characteristics for any w without using the simulation model and without any additional calculations. But, when form of equations (4.1) is changed, the procedure of conventional one-criterion optimization must be repeated.

4.2.2 Compromise curve calculation

One of commonly used variants of solving the multi-criterion optimization problem is to plot a compromise surface, which is defined as a set of effective or Paretto's points, in the space of decisive criteria (objective functions). In choosing the optimal geometry of bypass nozzle, a set of Paretto's points is monotone decreasing curve that is an envelope for the region of admissible points in the plane of decisive criteria на плоскости decisive criteria $(\Delta\overline{P}_{c\ eff}, \Delta G)$.

There are different methods to plot the compromise curve or Paretto's front. The current work proposes following algorithm to determine the points that belong to the compromise curve. As an objective function, effective thrust losses $\Delta\overline{P}_{c\ eff}(\theta_t, R, \theta_f, \theta_c)$ are taken. Limitations (1.1) that determine the admissible region of geometrical parameters are added by a limitation of the second type:

$$\Delta G(\theta_t, R, \theta_f, \theta_c) \le \Delta G_{\lim}. \qquad (4.2)$$

After that, a conventional extremum (minimum) of the function $\Delta\overline{P}_{c\ eff}(\theta_t, R, \theta_f, \theta_c)$ is searched for each given $\Delta G_{\lim}$ using Box complex-method. It is assumed that, during the search of extremum, intermediate values of $\Delta\overline{P}_{c\ eff}(\theta_t, R, \theta_f, \theta_c)$ and $\Delta G(\theta_t, R, \theta_f, \theta_c)$ are calculated using simulation model developed in advance and formulas (1.2)-(1.3) for estimation of nozzle mass. Admissible values of limitation of the second type $\Delta G_{\lim}$ (4.2) belong to the segment $(\Delta G_{\min}, \Delta G_{\max})$. Its boundaries have been defined in advance, at the stage of one-criterion optimization.

The results of compromise curve plotting using the algorithm above are presented in Fig. 6, where the symbol «*» designates Paretto's points obtained for different values of $\Delta G_{\lim}$ and the symbol «rhomb» corresponds to calculation points used in designing the simulation model.

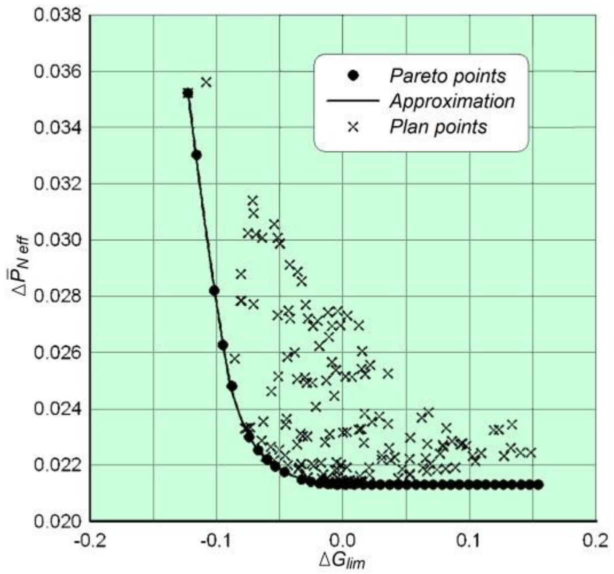

Fig. 6. Compromise curve

Figure 6 shows that obtained Paretto's points belong to monotone decreasing curve and all plan points are above them. Hence, this curve can be defined as an envelope for admissible region of objective functions $\Delta\overline{P}_{c\ eff}(\theta_t, R, \theta_f, \theta_c)$ and $\Delta G(\theta_t, R, \theta_f, \theta_c)$. As it has been mentioned above, one of the advantages of the parametrical calculation plan, which has been formulated using LP_τ sequences, is uniform distribution of points in the region of admissible values. It permits to choose approximately effective points among a set of initial data. Such points, which are marked by the black color in Fig. 6 and connected by the dashed line, can be treated as a zero approximation for the compromise curve. It should be noticed that such preliminary estimation of Paretto's point position can be performed at early stages of data analysis, before development of the simulation model and the multi-criterion optimization procedure.

Sample of Paretto's points can be approximated by a piecewise smooth curve using the dialogue system APPEX [4]. The solid line in Fig. 6 corresponds to the results of Paretto's point approximation using this formula. It should remember that each point of compromise curve corresponds to concrete values of both geometrical parameters and objective functions.

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