29th Congress of the International Council of the Aeronautical Sciences
St. Petersburg, Russia; September 7-12, 2014

SUPERSONIC NOZZLE THRUST AUGMENTATION

S.A. Takovitskii*
***Central Aerohydrodynamic Institute (TsAGI)**

Keywords: *supersonic, nozzle, thrust, insert*

Abstract

The problem of designing supersonic nozzle that provides maximum jet thrust is considered. To increase the thrust under dimensional constraints it is proposed to use the internal nozzle insert. The insert is located near the outlet cross section outside the region of aerodynamic interference with the nozzle edge. The insert shape is profiled by a direct CFD-based optimization method. Flow modeling and aerodynamic forces calculation are conducted within the frameworks of Euler and Navier-Stokes equations.

1 Introduction

Development of high-speed flying vehicles requires designing optimal nozzles of power plants that ensure maximum thrust. The effectiveness of the nozzle, as a gas-dynamic device, is restricted to a narrow range of changes in key parameters. An engine with a Laval nozzle provides the maximal possible thrust on the design conditions with equality of the pressures at the nozzle exit and in the environment. In the outlet section the jet is parallel to the axis of symmetry with the same value of the velocity at any point. On off-design conditions the flow in the nozzle is characterized by the formation of intensive wave structure which leads to loss of thrust.

One of the classical problems of supersonic aerodynamics is to determine axisymmetric nozzle which ensure the maximum thrust for specified constraints on the overall size [1, 2]. When there are not other constraints the method of control contour provides extremum conditions of exact solution within the framework of Euler equations by the method of characteristics.

The main disadvantages of the Laval nozzle are concerned with its large dimensions. If the requirement for uniformity of flow is not determinative, the length and weight of the nozzle can be reduced without a significant worsening of thrust characteristics. Proposals to increase thrust at a given constraints on dimensions are particularly relevant to vehicles with high altitude of flight. At low values of the atmospheric pressure the underexpansion mode is realized. The length of the nozzle is insufficient to ensure the design conditions of the jet efflux. The flow in the outlet section is characterized by irregularity in distribution of gas-dynamic parameters on the radial coordinate. This indicates a non-optimality of the nozzle due to dimensional constraints. On the other hand, it enables to increase the thrust by means of additional jet deflection. For this purpose it is proposed to use the nozzle insert which is located near the nozzle exit and does not influence on the flow near the nozzle wall. The key parameters of the insert geometry, including length and divergence ratio, are determined by the direct optimization method. The question concerning nozzle thrust augmentation through a placement of additional bodies into flow was analyzed in [2].

2 Direct optimization method

Aerodynamic design applications are reduced to constrained minimization of a function of many variables. The variation problem is simplified on the base of local linearization. In case of small perturbations of supersonic flow the linearized theory allows to connect change of pressure at given surface point with form deflection in its vicinity. This connection can be established both theoretically and through numerical calculation. A summation of aerodynamic loading over all elements of the aircraft surface gives a quadratic approximation of the objective function. On the base of the information on derivatives of first and second order the shape variations ensuring a quadratic rate of convergence to the optimum are determined.

The nozzle generatrix (fig. 1) is represented by a set of segments joining the points (x_i, r_i), i=1…n, where x_i are the distances from the nozzle throat and r_i are the distances from the axis of symmetry [3]. Only the radial coordinates r_i are varied in the optimization process. The radius of the nozzle entry section is taken as the scale length: $r_1 = 1$, $x_1 = 0$. The nozzle length is defined as $L=x_n - x_1$. If the exit section diameter D is fixed, then the parameter $r_n = D/2$ is excluded from the set of independent variables. In general, the nozzle diameter is expected to be large enough, so the condition $r_n \le D/2$ is ensured. It is assumed that on the after end ($r_n \le r \le D/2$, $x = x_n$) the pressure p_b is constant and independent of the nozzle shape.

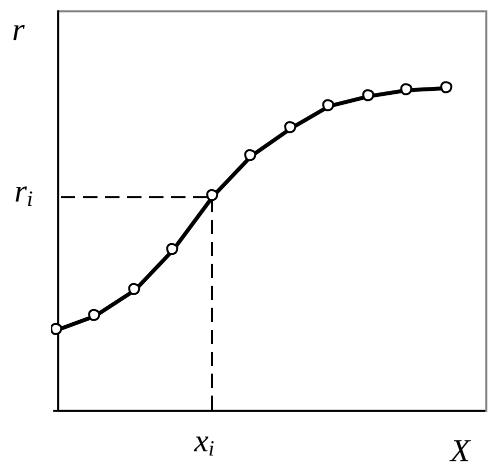


Fig. 1 Nozzle generatrix

The objective function F is the aerodynamic force acting on the nozzle in the longitudinal direction

$$F = \pi \sum_{i=1}^{n-1} p_i \left(r_{i+1}^2 - r_i^2 \right) + \pi p_b \left(0.25 D^2 - r_n^2 \right)$$

Here, p_i is the pressure on the generatrix segment $x_i \le x \le x_i$+1. It is more convenient to use the representation in which the variable and constant components are separated out

$$F = \pi \sum_{i=1}^{n-1} (p_i - p_b)\left(r_{i+1}^2 - r_i^2 \right) + \\ + \pi p_b \left(0.25 D^2 - r_1^2 \right)$$

It is required to determine the maximum value of the objective function under the geometric constraints. Additional restriction of the inequality type is the condition imposed on the surface curvature near the throat.

The optimization process consists of a number of iteration procedures. For the current nozzle the characteristics and the surface distributions of the gas-dynamic parameters are determined using the methods of computational aerodynamics. This information makes it possible to represent the objective function as a quadratic form. It is assumed that the local relations between the geometric and gas-dynamic parameters are valid. The pressure variation is related with the increment of the angle of inclination of the corresponding segment in the plane wave approximation

$$p_i = p_{i0} + k_{i0} \left(\Delta r_i - \Delta r_{i+1} \right)$$

$$k_{i0} = \frac{\gamma M_{i0}^2 p_{i0}}{\sqrt{M_{i0}^2 - 1}\left(x_{i+1} - x_i \right)}$$

Here, the pressure p_{i0} and the Mach number M_{i0} on the segment between the x_i and x_{i+1} sections correspond to the original nozzle geometry specified by the parameters r_{i0}, $\Delta r_i = r_i - r_{i0}$ is the radius increment in section i, and γ is the specific heat ratio.

The total aerodynamic force increment due to the nozzle shape variation contains components which depend linearly and quadratically on the increments of the radii in the corresponding cross-sections

$$\Delta F = \pi \sum_{i=1}^{n-1} \{[-2(p_{i0} - p_b) r_{i0} + k_{i0}(r_{i+1,0}^2 - r_{i0}^2)]\Delta r_i + \\ + [2(p_{i0} - p_b) r_{i+1,0} - k_{i0}(r_{i+1,0}^2 - r_{i0}^2)]\Delta r_{i+1} + \\ + [-(p_{i0} - p_b) - 2k_{i0} r_{i0}]\Delta r_i^2 + \\ + [(p_{i0} - p_b) - 2k_{i0} r_{i+1,0}]\Delta r_{i+1}^2 + \\ + 2k_{i0}(r_{i0} + r_{i+1,0})\Delta r_i \Delta r_{i+1}\}$$

For the quadratic form ΔF the gradient and the Hessian matrix are determined and the extremum is found by the Newton method. In the final stage of the iteration procedure the solution obtained is checked using the methods of computational aerodynamics. A one-dimensional search of the objective function maximum is carried out in the direction given by the extremum values of the increments Δri in the space of the geometrical parameters.

It should be noted that for the optimal nozzle the Busemann condition [2] is fulfilled. The condition relates the geometric and gas-dynamic parameters in the exit section $x=L$

$$1 - \frac{p_b}{p} = \frac{\gamma M^2}{\sqrt{M^2 - 1}} r_x'$$

This equation follows from the extremum condition $\Delta F'_{\Delta r_n} = 0$.

The method of local linearization provides a fast convergence to the optimum due to simplification of the variation problem statement. On the other hand the method gives opportunity to establish analytical solution of the problem and study characteristic features of optimal configurations. It is determined that near to optimal nozzle generatrix represented by a power-law dependence of the radius on the longitudinal coordinate. The solution is a power-law function of the form $\frac{r}{r_1} = \left(A + B\frac{x}{L}\right)^{2/3}$.

The nondimensional coefficients A and B are determined from the boundary conditions. In case of absence of a constraint on the nozzle contour curvature A=1. Coefficient B defines the radii ratio $B = \left(\frac{r_n}{r_1}\right)^{3/2} - 1$.

3 Optimal nozzles

The optimization study was performed for nozzles differing in length. The following cases were considered: L = 10, 14, 20 and 30. It was assumed that the base pressure p_b is zero, and the roundness radius is 0.5. For the long nozzles the roundness radius near the throat has a slight effect on the thrust characteristics [3].

The nozzle flow was studied within the framework of the Euler model for an inviscid gas. The equations of motion written in a cylindrical coordinate system were integrated using the marching difference scheme [4]. In the initial section the parameters of a uniform supersonic flow with a near-unity Mach number were preassigned. The specific heat ratio γ= 1.4. The computation grid had 201 points in the direction from the axis of symmetry to the nozzle surface.

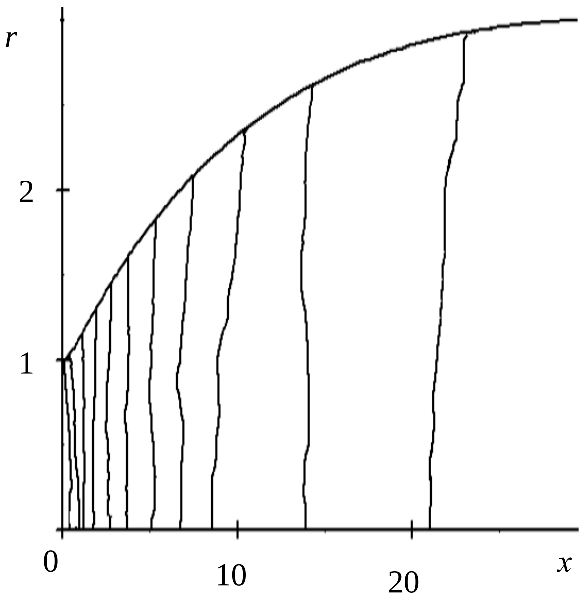
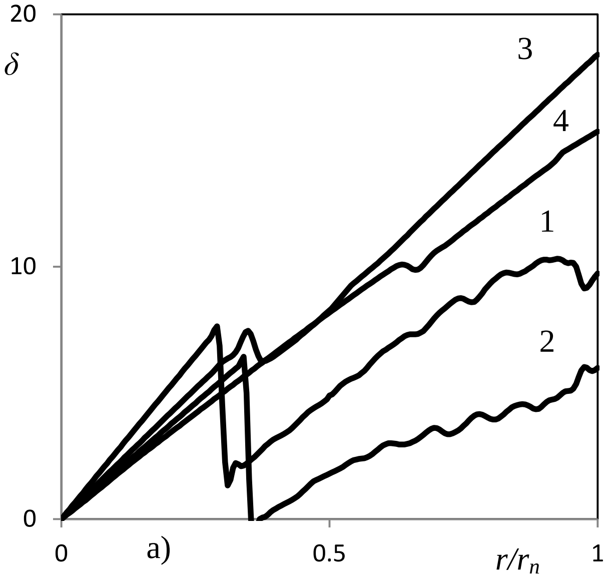


Fig. 2 Mach number contours (ΔM=0.25)

The internal flow field in the Laval nozzle is presented in fig. 2 as Mach number contours. The contours are plotted with the step ΔM=0.25, the contour nearest to the initial section corresponding to the Mach number 1.25.

The nozzle has a large length with a small expansion ratio, thus ensuring substantially isentropic flow acceleration. The angle between the velocity vector and the symmetry axis does not exceed tenths of a degree.



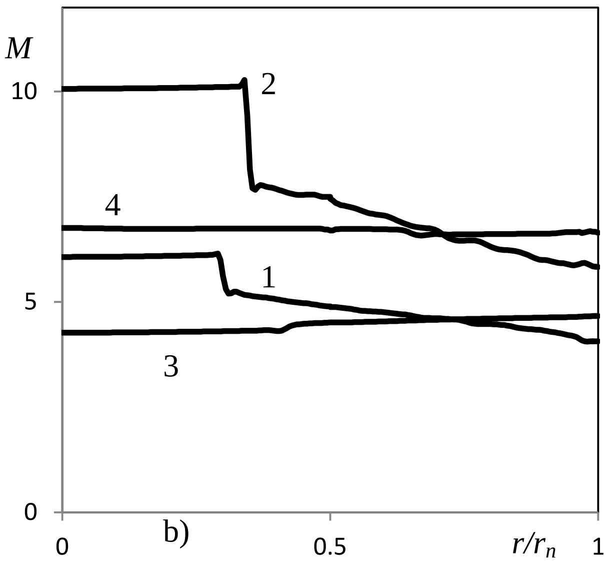


Fig. 3 Flow parameters distribution
a) Jet angle; b) Mach number
1 – L=10, optimal nozzle
2 – L=30, optimal nozzle
3 – L=10, conical nozzle
4 – L=30, conical nozzle

Along with curved generatrixes, straight generatrixes were considered. The expansion ratio of conical nozzles was not varied. The nozzles have equivalent radii in exit section.

Increasing length of the nozzle increases the optimum expansion ratio, defined as the ratio of the radii of the output r_n and critical r_1 sections. The optimal nozzles with length L = 10 and 30 have the radius of the outlet section 4.3 and 9.2, respectively. The curvature and slope angle of generatrix decrease with increase of distance from critical section. In the considered range of lengths L = 10 ÷ 30 the generatrix slope at exit section varies as 0.17 ÷ 0.1.

The constraint putted on the length of the nozzle results in uneven distribution of flow parameters in the radial direction. In the central part of the flow an extended low-pressure region is located. In the central part of the flow an extended low-pressure region is located. Jet angle δ distribution in the exit section is shown in fig. 3a. The angle δ increases with increasing distance from the axis of symmetry. The maximum value of the angle δ is defined by the nozzle wall slope and decreases with increasing length of the nozzle. In the conical nozzles Mach number depends insignificantly on the radial coordinate (fig. 3b).

This characteristic feature of the flow with underexpanded jet suggests the possibility of the thrust augmentation through additional flow deflection in the vicinity of the nozzle exit. For this purpose the internal insert is used, that located outside the region of aerodynamic interference with the nozzle wall.

3 Nozzle thrust augmentation

The effect of the thrust increasing on the internal insert was confirmed for both axisymmetric and two dimensional nozzles. The nozzle jet thrust is created by the wall and the insert. Geometry of zero thickness insert is defined by a set of nodal points $\left(x_{B_j}, r_{B_j}\right)$, $j = 1...m$. Position of the exit section of the insert is fixed $x_{Bm} = x_n$. The insert generatrix has the set of m = 101 nodal points.

The objective function – the total aerodynamic force due to the pressure forces acting on the wall and the insert of the nozzle in the longitudinal direction:

$$F = \pi \sum_{i=1}^{n-1} p_i \left(r_{i+1}^2 - r_i^2 \right) +$$
$$+ \pi \sum_{j=1}^{m-1} \left(p_{B.B_j} - p_{B.H_j} \right) \left(r_{B_{j+1}}^2 - r_{B_j}^2 \right)$$

Here $p_{\text{B.B}j}$ and $p_{\text{B.H}j}$ – the pressure and the inner and outer sides of the insert in the interval $x_{\text{B}j} \le x \le x_{\text{B}j+1}$, respectively.

The insert is located near the nozzle exit. The angle of inclination of the generatrix is about two times less than the corresponding value at the nozzle wall. Optimum length and diameter of the insert is determined by compromise solution. Increasing the length results in a decrease of the lateral dimensions of the insert due to requirement on lack of aerodynamic interference with the nozzle wall. It should be noted that the area of the insert surface is largely dependent on the lateral dimensions. Also, when approaching the axis of symmetry of the nozzle the radial component of velocity and pressure decrease (Fig. 3).

The longitudinal sections of the flow field in the nozzle are shown in Fig. 4 as lines of equal values of Mach number. The contours are plotted with the step ΔM = 0.5, the contour nearest to the initial section corresponding to the Mach number 1.5. Optimized nozzle (Fig. 4a) and conical nozzle (Fig. 4b) are compared. Bottom parts of the figures correspond to the nozzles without the insert, and the upper parts - nozzles with internal insert. Flow in the optimal nozzle has a complicated structure. In the central part of the stream there is an extended low pressure region, which is bounded by shock wave front. The region of the insert influence is bounded by the shocks (the inner side of the insert) and rarefaction waves (the outer side of the insert). Rarefaction waves do not get on the nozzle wall, indicating the absence of aerodynamic interference. The increasing pressure on the inner surface and the reducing pressure on the outer surface of the insert create the additional thrust.

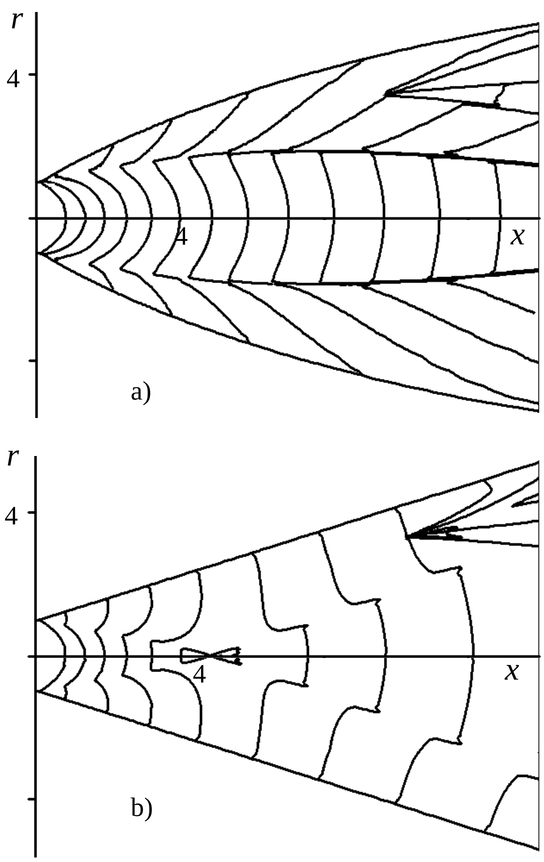


Fig. 4 Mach number contours (M=0.5)
a) optimized nozzle
b) conical nozzle

The Insert deflects the jet to the axis of symmetry. The jet angle δ in the exit section is reduced in the vicinity of the insert (Fig. 5).

The nozzle thrust includes two components. The constant component I related with flow momentum in the critical section was excluded from consideration in the formulation of the optimization problem. Only the second component of the thrust was considered, the force F due to pressure acting on the wall of the expanding part of the nozzle and on the insert. The effectiveness of the nozzle thrust characteristics is evaluated by the dimensionless parameter $F_I = F/I$ defined as the ratio of the two components of the thrust.

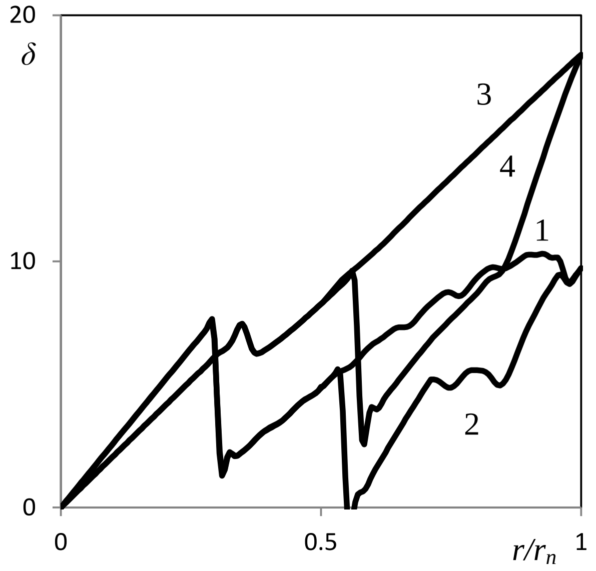


Fig. 5 Jet angle distribution
1 – L=10, optimal nozzle
2 – L=10, optimal nozzle with insert
3 – L=10, conical nozzle
4 – L=10, conical nozzle with insert

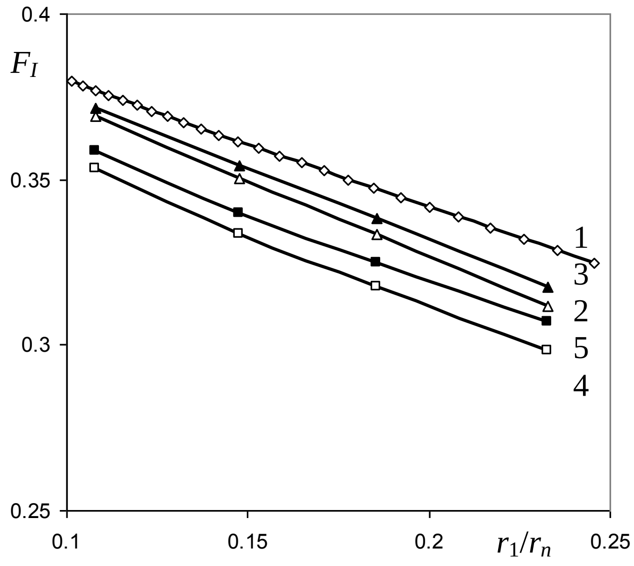


Fig. 6 The nozzle thrust parameter
1 – ideal nozzles, 2,3 – optimized nozzles; 4,5 – conical nozzles; 2,4 – without insert; 3,5 – with insert

Comparison of the thrust characteristics for the ideal nozzles, the nozzles without inserts and the nozzles with internal inserts is shown in Fig. 6. The axis of abscissas is defined by the ratio r_1/r_n inversed to the expansion ratio of the nozzle. The conical nozzles have slightly worse thrust characteristics in comparison with the optimized nozzles. In their turn, the optimized nozzles do not provide the maximal thrust because of the constraint on the external dimensions. To achieve the ideal thrust at fixed expansion ratio it should be used the nozzle of high lengthening. In this case the relative increase of the thrust parameter F_I is 2 ÷ 5 %.

Application of the nozzle inserts results in the thrust augmentation. The relative thrust created by the insert increases with decreasing length of the nozzle. For the nozzle with L=10 the insert thrust is approximately 2% of the thrust received on the wall of the nozzle. The greatest relative increase of the parameter F_I is achieved for nozzles having conical wall. It is ranging from 1.5 to 3% depending on the expansion ratio of the nozzle. It should be noted that the nozzle insert profiling does not provide a significant increase of the jet thrust.

Additionally, the nozzle with the insert is studied within the framework of the Reynolds averaged Navier-Stokes equations. The insert has symmetric parabolic profile with relative thickness of 3%. Reynolds number calculated on the radius of the inlet throat is $Re = 2.33\ 10^6$. The Mach number contours in the longitudinal section of the nozzle with L = 10 are shown in Fig. 7. The contours step is ΔM = 0.5, and the critical section closest level line corresponds to the Mach number M = 1.5. The level lines are condensed to the surfaces of the nozzle wall and the insert within the boundary layers. The surface friction drag conduces to slight reduction of the thrust augmentation created by the nozzle insert.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research, project No. 13-01-00854a.

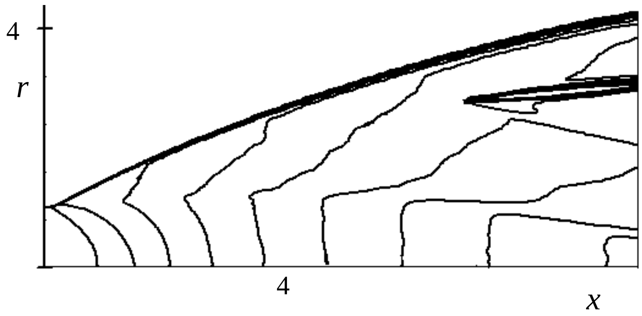


Fig. 7 Mach number contours (ΔM=0.5)

References

[1] Guderley K.G., Armitage I.V. A general method for the determination of best supersonic rocket nozzles. *Symposium on external problems in aerodynamics.* Boeing Sci. Res. Laboratories. Flight Sci. Laboratory. Seatle. Washington. 1962.
[2] Kraiko A.N. *Variational Problems of Gasdynamics* [in Russian] - Nauka, Moscow, 1979.
[3] Takovitskii S.A. Optimal supersonic nozzles with power-law generators. Fluid Dynamics, Vol. 44, No. 1, 2009.
[4] Takovitskii S.A. Method for calculating supersonic flow past flight vehicles using multi-zone computation grids. Tr. TsAGI, No. 2590, 1997.

Contact Author Email Address

c.a.t@tsagi.ru

Copyright Statement