PROFILING OF OPTIMAL ANNULAR NOZZLES WITH POLYPHASE WORKING MEDIUM

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Abstract

The problem of profiling optimal (i.e. securing maximum value of thrust) configurations of annular nozzles of external expansion with a polyphase (or multicomponent) working medium is considered. Direct methods of a calculus of variations are applied to the solution of a problem in view. The variational problem of search of an optimal configuration of annular nozzle is reduced to a problem of nonlinear programming. Optimal configurations of annular nozzles for different working mediums and operating conditions were built as a result of optimization.

1 Introduction

One of the major problems occurred at designing of the annular nozzles is problem of profiling optimal (i.e. securing maximum value of thrust) configurations of annular nozzle with a polyphase (or multicomponent) working medium [1]. The configuration of this nozzle presented in figure 1.

Search of an optimal configuration of an annular nozzle is carried out by the solution of a variational problem which can be realized by means of different methods, for example, by a method of a control contour or method of Lagrangian multipliers. However, as noted by A.N. Kraiko [2], in some cases application of such methods is very complex, laborious and does not guarantee success. In this connection to one of possible decisions can become application of direct methods of a calculus of variations [1, 3]. In variational problems of gas dynamics the essence of these methods consists in the following.

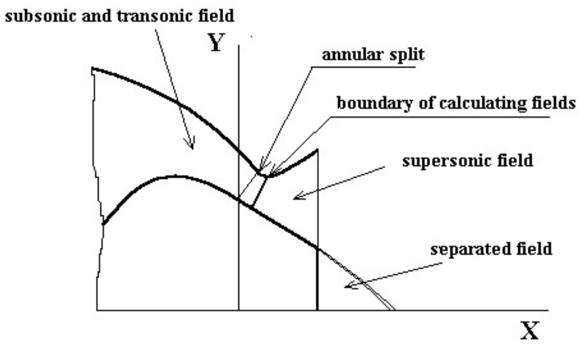

Fig. 1. The scheme of annular nozzle with indication of computational fields

Each smooth section required optimal generatrix nozzles is approximated by linear combinations of some known functions with beforehand unknowns coefficients. Existence of additional isoperimetrical conditions of geometrical character, defined the given restrictions on geometry of the optimized nozzle, imposes some connections on coefficients, reducing number of free parameters. All free parameters indicated as c_1, … c_n, generate n-dimensional space in which in direct methods the minimum or maximum of the optimized functional J being a function $c_1,\dots c_n$ is searched. Transition from a functional to a function represents effect of replacement of the smooth sections generatrix final combinations of approximating functions.

Considered variational problem at which statement the number of optimized parameters includes geometrical characteristics of a nozzle (thus parameters of a condensed phase and isentropic index *k*, and also the have available pressure differential in the nozzle p_o/p_H are considered as stationary values), is reduced to a problem of nonlinear programming [3]. Basic elements of the given approach are direct calculations of a field of flow and a method of search of an extremum of functions of many variables that makes it applicable to all gas dynamic problems for which methods of calculation of a field of flow including for annular nozzles with multicomponent flows are known.

2 Procedure of optimization and statement of variational problem

The procedure of optimization of a geometrical configuration of an annular nozzle represents a combination of analytical methods of construction of an optimized functional and the representation geometrical profile of an annular nozzle, with methods of search of the extremum of function being a function several variables, and direct calculations of a field of flow with the help of computational methods. Profiling optimal on trust annular nozzle on is carried out in conditions of the set restrictions on its geometrical characteristics.

The arbitrary stationary axisymmetric flow of gas in the nozzle is considered. Let it is required to construct a nozzle contour $y = \xi(x)$ supplying extremum to a functional:

$$J = \int_A^B \Phi(x, \xi, \xi', u_1, \ldots, u_n) dx, \qquad (1)$$

where Φ – a known function, $\{u_i\}$ ($i = 1, \ldots, n$) – a system of the functions, satisfying to flow equations; A and B - initial and final points of a nozzle contour. The stroke designated derivatives on x along a nozzle contour.

Following isoperimetric conditions are considered:

$$K_j = \int_A^B G_j(x, \xi, \xi') dx, \quad j = 1, \ldots, m, \qquad (2)$$

where $G_j(x, \xi, \xi')$ and K_j - known functions and constants.

The required optimal contour can be determined as follows:

$$y'(x) = \xi_o(x) + \Delta\xi(x), \quad y(x_o) = y_o, \qquad (3)$$

where $\xi_o(x)$ – known function, x_o – the initial point of a contour, value $\Delta\xi(x)$ is approximated by a segment of series:

$$\Delta\xi(x) = \sum_{k=0}^{l} c_k \varphi_k(x), \qquad (4)$$

where $\{\varphi_k\}$ - a system linearly independent basis functions, c_k – coefficients. Then for a contour, the given as (4), we shall have:

$$J = J(c_1, \ldots, c_r), \; r < l, \qquad (5)$$

and r it is selected so that $l - r$ coefficients in (3) it was possible to satisfy to isoperimetric conditions (2).

Thus, the variational problem of search of optimal geometry of a nozzle contour under the given conditions is reduced to a problem of search of a point $(c_1, \ldots, c_r)$ in which the value of a function J is extreme. Methods of nonlinear programming are applied to a searching of extremum of function. Components of a gradient of a function J are calculated under formulas:

$$\frac{\partial J}{\partial c_k} \approx \frac{J(c_1, \ldots, c_k + \Delta c_k, \ldots, c_r) - J(c_1, \ldots, c_k, \ldots, c_r)}{\Delta c_k}, \qquad (6)$$

$$\frac{\partial J}{\partial c_k} \approx \frac{J(c_1, \ldots, c_k + \Delta c_k, \ldots, c_r) - J(c_1, \ldots, c_k - \Delta c_k, \ldots, c_r)}{2\Delta c_k}, \qquad (7)$$

in which the function J is determined after calculation of a field of flow for a nozzle contour, the given as (3).

In this research, such approach is applied to search of optimal annular nozzles of different configurations with the multicomponent working medium, profiled in conditions of different restrictions on overall dimensions. The main attention is given to a problem of profiling of an optimal configuration of an annular nozzle of external expansion (without an external contour in supersonic field of nozzle) with a shortcut center body (fig. 2), implementing maximum thrust at the given restrictions on geometrical characteristics of the nozzle.

As of the main criterion of optimization, the thrust coefficient is used:

$$K_T = \frac{R}{F_* \cdot P_o}, \qquad (8)$$

where R – thrust of the nozzle, F_* – the square of throat nozzle, P_o – total pressure on an entrance in the nozzle.

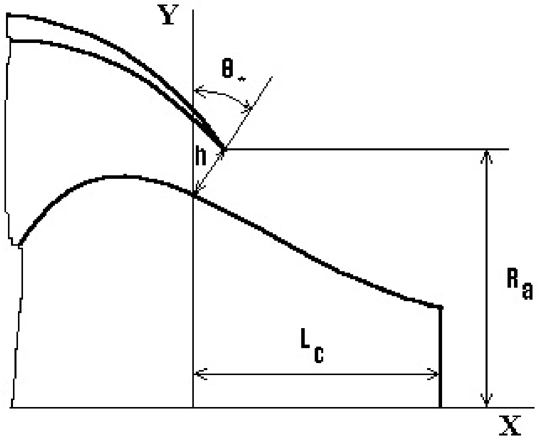

Fig. 2. A configuration of an annular nozzle for optimization

At statement or variational problem for the investigated nozzle, the thrust coefficient can be recorded as:

$$K_T = K_{T*} + \alpha \int_A^B P(x, y(x); \Theta_*) dF + K_{T_D}, \quad (9)$$

where K_{T*} – a thrust coefficient, created by a flow in throat nozzle, A, B – initial and final points of a profile of a center body, Θ_* - angle of inclination of a plane of annular throat to a axis of the nozzle, $P(x, y(x); \Theta_*)$ – pressure distribution along the center body, α – dimension a multiplier, K_{T_D} – a thrust coefficient, created of butt of a shortcut center body.

The integration is carried out on the area of a projection of a surface of a center body on a plane $X = const$.

The contour of a center body is divided into segments with some allocation of clusters and the smooth fulfilment between them. Therefore, the variational problem of a determination of a function $y(x)$ and the angle Θ_*, ensuring a maximum of a thrust coefficient in conditions of the given dimensional restrictions, is reduced to a problem of a searching of extremum of a function several variables:

$$K_T = f(\Theta_*; c_1, c_2, ..., c_n), \qquad (10)$$

where c_k – coefficients, defined a shape of a center body, which can be solved by methods of nonlinear programming.

Direct calculations of a field of flow are carried out for a multicomponent medium with monodisperse and polydisperse condensed phase.

3 Mathematical modeling of flows in annular nozzles

3.1 Mathematical modeling of two-phase monodisperse flow

Two-phase monodisperse flow in an annular nozzle is described by equations set of a axisymmetric flow of a two-phase mixture in the integral form [1, 4], permitting to realize the «transparent» calculation without preliminary selection of discontinuity in computational field.

The system is recorded for a plane of flow XY for the fixed square Ω, restricted a contour of G:

$$\begin{aligned}
&\frac{d}{dt}\iint_\sigma \rho u y dxdy + \iint_\sigma \rho_s f_x y dxdy + \\
&+ \oint_G y\left(\left(p + \rho u^2\right)dy - \rho u v dx\right) = 0; \\
&\frac{d}{dt}\iint_\sigma \rho y dxdy + \oint_G \rho y (u dy - v dx) = 0; \\
&\frac{d}{dt}\iint_\sigma \rho_s y dxdy + \oint_G \rho_s y (u_s dy - v_s dx) = 0; \qquad (11)\\
&\frac{d}{dt}\iint_\sigma \rho v y dxdy + \iint_\sigma y\left(\rho_s f_y - \frac{p}{\rho}\right)dxdy + \\
&+ \oint_G y\left(\rho u v dy - \left(p + \rho v^2\right)dx\right) = 0; \\
&\frac{d}{dt}\iint_\sigma \rho_s u_s y dxdy - \iint_\sigma \rho_s f_x y dxdy + \\
&+ \oint_G y \rho_s u_s (u_s dy - v_s dx) = 0;
\end{aligned}$$

$$\frac{d}{dt}\iint_{\sigma} \rho_S v_S ydxdy - \iint_{\sigma} \rho_S f_x ydxdy +$$
$$+ \oint_G y\rho_S v_S (u_S dy - v_S dx) = 0;$$
$$\frac{d}{dt}\iint_{\sigma} \rho y\left(\frac{V^2}{2} + e\right)dxdy + \iint_{\sigma} \rho_S y\left(\vec{V}_S \cdot \vec{f} + q\right)dxdy +$$
$$+ \oint_G \rho y\left(\frac{V^2}{2} + i\right)(udy - vdx) = 0;$$
$$\frac{d}{dt}\iint_{\sigma} \rho_S e_S ydxdy - \iint_{\sigma} \rho_S qydxdy +$$
$$+ \oint_G \rho_S e_S y(u_S dy - v_S dx) = 0,$$

where ρ and ρ_s − densities gas and particles, p − pressure of gas, u and v − axial and radial components of a vector of a velocity of gas $\vec{V}$; u_S and v_s − axial and radial components of a vector of a velocity of particles $\vec{V}_S$, x and y − axial and radial coordinates, $e = e(p,T)$ − specific internal energy of gas, $e_S = e_S(T_S) = c_в T_S$ − specific internal energy of substance of particles, $p = \rho RT$ − constitutive equation of perfect gas, t – time, $\vec{f} = \frac{d\vec{V}_S}{dt}$, $q = \frac{de_S}{dt}$.

Calculation of a field of flow is carried out by the relaxation method with use of the computational algorithm base on the scheme of Godunov–Kolgan [1, 5]. Boundary conditions of set of equations are received as follows:

- on rigid walls the impermeability condition v_n=0 is accepted;
- on input subsonic boundary: entropy S=const, total enthalpy H_o=const; allocation of angle of inclination of velocity vector $\Theta_* = \Theta_*(x, y)$.

The output boundary is selected so that normal to boundary the component of a velocity would be supersonic, in this case statements of boundary conditions it is not required.

The two-phase flow is characterized by essential interaction of gas and condensed phases among themselves. Calculation of interaction of particles of a condensed phase with gas is carried out with the help of the exchange terms of equations considering an exchange of an impulse and energy between considered phases. For the description of exchange terms in a two-phase monodisperse medium the ratios offered in research efforts [4] for flows in Laval nozzles were used. According to this approach, parameters of viscous interaction of gas and particles, and also heat exchange parameters between gas and particles are considered. For gas and particles on input boundary, the balance condition is realized.

Flow in an annular nozzle is «mixed», that is in various subareas of the solution of a problem of an equation of a gas phase can belong to the elliptical or hyperbolic type that results in necessity to apply different methods for the solution of a problem in each of them. Complexity of the solution of a problem is caused by that boundaries of the indicated subareas are beforehand unknown. Therefore, to calculate parameters of the «mixed» flow rationally also to apply a relaxation method, as well as at modeling of flows of perfect gas.

3.2 Mathematical modeling of flows of a multicomponent medium

More difficult case is application in annular nozzles as a working medium of a multicomponent medium.

Within an offered approach, for mathematical modeling of flows of a multicomponent medium in annular nozzles the equations of continuous model recorded for the discrete distribution function (original positions of the applied discrete approach for Laval nozzles are shown in [4, 6]) were used.

The set of equations of a non-stationary axisymmetric flow of a multicomponent medium in an annular nozzle looks like:

$$\frac{\partial \rho r}{\partial t} + \nabla \cdot r\rho\vec{u} = 0;$$
$$\frac{\partial r\rho u}{\partial t} + \nabla \cdot r\rho u\vec{u} + \frac{\partial rp}{\partial x} = r\sum_{i=1}^{l} N_i C_{R_i}(u_{si} - u);$$
$$\frac{\partial r\rho v}{\partial t} + \nabla \cdot r\rho v\vec{u} + \frac{\partial rp}{\partial r} - p = r\sum_{i=1}^{l} N_i C_{R_i}(v_{si} - v); \quad (12)$$

$$\frac{\partial u\rho\left(h-\frac{p}{\rho}\right)}{\partial t}+\nabla\cdot r\rho h\vec{u}=$$
$$=r\sum_{i=1}^{l}N_i\{c_{\alpha i}(\theta_{si}-\theta)+C_R[u_{si}(u_{si}-u)+v_{si}(v_{si}-v)]\};$$
$$\frac{p}{\rho}=\frac{k-1}{k}\left(h-\frac{u^2+v^2}{2}\right).$$

$$\frac{\partial rN_i}{\partial t}+\nabla\cdot rN_i\vec{u}_{si}=-rN_i\sum_{j=i}^{l}k_{ij}\Phi_{ij}Nj;$$
$$\frac{\partial r\rho_{si}}{\partial t}+\nabla\cdot r\rho_{si}\vec{u}_{si}=r\left[N_i\sum_{j=1}^{i}k_{ij}\Phi_{ij}\rho_{sj}-\rho_{si}\sum_{j=i}^{l}k_{ij}\Phi_{ij}N_j\right];$$
$$\frac{\partial r\rho_{si}u_{si}}{\partial t}+\nabla\cdot r\rho_{si}u_{si}\vec{u}_{si}=r\left[N_iC_{Ri}(u-u_{si})+N_i\sum_{j=1}^{l}k_{ij}\rho_{sj}(u_{sj}-u_{si})+\right.$$
$$\left.+\rho_{si}\sum_{j=1}^{l}k_{ij}(1-\Phi_{ij})(u_{sj}-u_{si})N_j-u_{si}\left(\rho_{si}\sum_{j=i}^{l}k_{ij}\Phi_{ij}N_j-N_j\sum_{j=1}^{l}k_{ij}\Phi_{ij}\rho_{si}\right)\right];$$
$$\frac{\partial r\rho_{si}v_{si}}{\partial t}+\nabla\cdot r\rho_{si}v_{si}\vec{u}_{si}=r\left[N_iC_{Ri}(v-v_{si})+N_i\sum_{j=1}^{i}k_{ij}\rho_{sj}(v_{sj}-v_{si})+\right.$$
$$\left.+\rho_{si}\sum_{j=1}^{l}k_{ij}(1-\Phi_{ij})(v_{sj}-v_{si})N_j-v_{si}\left(\rho_{si}\sum_{j=1}^{l}k_{ij}\Phi_{ij}N_j-N_i\sum_{j=1}^{i}k_{ij}\Phi_{ij}\rho_{sj}\right)\right];$$
$$\frac{\partial rc_s\theta_{si}\rho_{si}}{\partial t}+\nabla\cdot r\rho_{si}c_s\theta_{si}\vec{u}_{si}=r\left[N_ic_{\alpha i}(\theta-\theta_{si})+N_i\sum_{i=1}^{j}k_{ij}\rho_{si}(E_j-E_i)+\right.$$
$$\left.+\rho_{si}\sum_{j=1}^{l}k_{ij}(1-\Phi_{ij})N_j(E_j-E_i)-c_s\theta_{si}\left(\rho_{si}\sum_{j=1}^{l}k_{ij}\Phi_{ij}N_j-N_i\sum_{j=1}^{i}k_{ij}\Phi_{ij}\rho_{sj}\right)\right];$$

$$i=1,\dots l, \qquad (13)$$

where $\vec{u}_{si}=(u_{si},v_{si})$ – velocity vector of particles i fractions in axisymmetric coordinate system, $\vec{u}=(u,v)$ – velocity vector of gas, $\nabla=\frac{\partial}{\partial x}+\frac{\partial}{\partial r}$; $E_j-E_i=c_s(\theta_{sj}-\theta_{si})+\frac{|u_{sj}-u_{si}|^2}{2}$, c_s – thermal capacity of particles, θ_{si} and θ_{sj} temperatures of particles, k_{ij} – coefficient of coagulation; Φ_{ij} – coefficient of effectiveness of impacts, N_i – number of particles i phases, C_{Ri} and $c_{\alpha i}$ – coefficients of interaction between particles and gas.

The presented set of equations consists of two subsystems: the subsystem describing movement of gas consists of five equations and designated (12), the subsystem describing movement of particles, also consists of 5 equations and designated (13). In case of the accounting of crushing of particles at the expense of rotation to the presented set of equations, it is necessary to add an equation for a square of the average moment of momentum:

$$\frac{\partial rN_iM_i^2}{\partial t}+\nabla\cdot rN_iM_i^2\vec{u}_{si}+\frac{2B_iM_i^2}{I_i}rN_i=r\left(\sum_{j=1}^{i}k_{ij}N_j\left(M_{1ij}'^2-M_i^2\right)+\right.$$
$$\left.+\sum_{i=j}^{l}k_{ij}(1-\Phi_{ij})\left(M_{2ij}'^2-M_i^2\right)N_j-M_i^2\sum_{j=1}^{i}k_{ij}\Phi_{ij}N_j\right)N_i \qquad (14)$$

$$i=1,\dots,l,$$

where M_i – moments of rotation of particles i and j phases before encounters, M_{1ij}', M_{2ij}' - moments of rotation of the fragments obtained at encounters of particles i and j phases, B_i - moment of force; I_i – inertia moment of a particle i phase.

Subsystems (12) and (13) are connected with each other only through right members, which do not contain derivative of parameters of a problem owing to what the study of characteristic properties of these subsystems can be seen off independently from each other.

Characteristics of subsystems (12) and (13) in the space $\{x,r,t\}$ coincide with trajectories of particles along which so many equations of compatibility are carried out, how many the equations contain in these systems.

The differential equations of a gas phase differ from equations of movement of perfect gas only the right members, and, means, they possess the same characteristic properties, as the equations describing movements of perfect gas. It means that boundary conditions for the subsystem (12) should be set the same way, as at research of movement of perfect gas in an annular nozzle.

Boundary conditions are set as follows. For integration of set of equations on a part of boundary through which input of a multicomponent mixture is carried out, the enthalpy, an entropy and allocation of a velocity of gas are set, on walls of the nozzle the impermeability condition a gas phase $v=r_x'u$ used, on output supersonic boundary of boundary conditions it is not required. By virtue of characteristic properties of equations (12) and (13) boundary conditions are necessary for setting only on a part of boundary through which it is carried out the injection of a multicomponent mixture. It is supposed, that on parameters of gas and particles on input boundary are in balance, and the moment of rotation of particles is equal to zero.

Parameters subsonic and transonic areas of flow are calculated with the help of a relaxation method with use of the scheme of Godunov–Kolgan, in supersonic area for calculation of parameters of a gas phase Ivanov–Kraiko–Mikhailov modified mid-flight scheme is used. Calculation of interaction of particles of a condensed phase with gas was carried out with the help of the exchange members considering exchange of an impulse and energy between considered phases.

In a series of chances of flow, for example, in case of absence in the field of flow of closed trajectories of the particles caused by vortex flows, for the solution of a considered problem expediently to apply the pseudo relaxation method consisting in division of set of equations on two subsystems: for gas and for particles. Subsystem for calculation of parameters of gas decides by a relaxation method, and for the solution of subsystem for calculation of parameters of particles the stationary equations which integration is possible different modes, for example, with the help of implicit difference schemes are used.

At calculation of a supersonic flow as initial data gas parameters on a sound surface are used. Such parameters can be set on a flat annular sound surface (coinciding with the plane of annular throat of a nozzle) with a uniform allocation of velocity vector, on value equal speeds of sonic, in this case on an acoustical surface it is possible to consider parameters invariable at change of width of minimum annular throat and an angle of its inclination to a nozzle axis. Other way of the representation of initial data for calculation of parameters of supersonic area of flow assumes calculation of parameters subsonic and transonic areas of flow with definition of the form of a sound surface and the representation of the distributed parameters on this surface. In this case changes of geometrical parameters of an annular nozzle during optimization results to change of geometry of a subsonic part of the nozzle and annular throat that, in turn, results in necessity of recalculation of parameters subsonic and transonic areas of flow and essentially complicates a procedure of optimization, but increases accuracy of the received solution.

4 Results of optimization of annular nozzle′s geometrical configuration

For search of an extremum of criterion function various methods [7], such as a cyclical alternating-variable descent method, Rozenbrok′s method (versions of a method with discrete step and with minimization on a direction), allowing to carry out multidimensional search without use derivatives and possessing a sufficient velocity of convergence, a steepest descent method using derivatives at definition of directions of search are used.

Optimization of a geometrical configuration of an annular nozzle with the account to subsonic and transonic areas of flow was carried out for an initial configuration of an annular nozzle with following parameters (dimensionless, divided by value R_a):

- relative radius of top point of annular throat $\overline{R}_a = 1$;
- relative area of annular throat $\overline{F}_a = 0.65$;
- isentropic exponent $k = 1.25$;
- length of the nozzle $L_c = R_a$;
- pressure differential in the nozzle $p_O / p_H = 100$.

The profile of a center body was approximated by polynomials of Chebyshev and power-mode polynomials which efficiency for approximating a nozzle contour in a problem of optimization is shown in research efforts [3, 6].

On an optimized a profile dimensional restriction on length L_c and to radius R_a were imposed. Restrictions on ordinate of a final point of a profile of a center body were not imposed, as its position, in the majority of considered cases does not exceed the bounds of the control dimensions of the nozzle.

On fig. 3 results of optimization of geometry of an annular nozzle with the account and without taking into account subsonic and transonic areas of flow are presented.

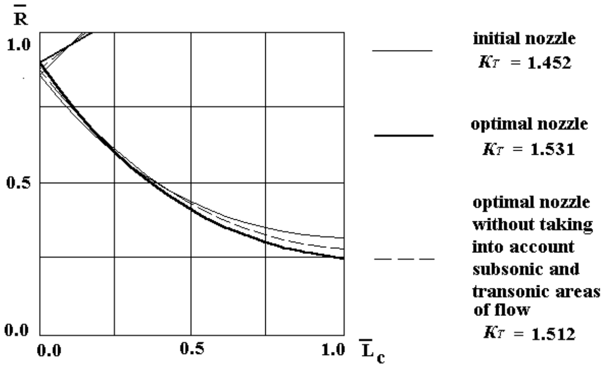

Fig. 3. The configurations of annular nozzles received under different conditions of profiling

The received optimal profile of a center body locates a slightly below, than the profile of a nozzle, received at carrying out direct calculations of a field of flow from a flat sonic surface, angle of inclination of the plane of annular throat was increased. The value K_T of the nozzle received with the account subsonic and transonic areas of flow, appeared above K_T, received without taking into account this area of flow on ~ 1.2 % (K_T=1.531 against K_T=1.512), that makes enough considerable change of propulsive performance characteristics. An interesting feature of received result is that inequality of a flow in throat of the nozzle results in increase of its propulsive performance characteristics.

In a variational problem put in the present research effort during optimization two major factors, operational in a multicomponent medium are taken into account: effect of motion of a condensed phase on gas dynamic structure of a flow and a deposition of particles on streamlined surfaces. Change of geometry of profiled surfaces owing to a deposition (attachment) of particles of a condensed phase on these surfaces was not taken into account. It is necessary to mark, that research of change of geometry of profiled surfaces of the nozzle owing to erosive effect of a part of a condensed phase requires demands realization of special physical and chemical researches, which are beyond the present research. Refusal of the accounting of effect of change of profiles of streamlined surfaces because of erosive effect of particles on surfaces of the nozzle is represented lawful because such effect, as a rule, results in the considerable decrease of propulsive performance characteristics of the nozzle, on an order of magnitude superior a gain in propulsive performance characteristics. Therefore becomes logical to carry out a statement or problem as, providing absence of deposition on walls of the nozzle of fractions of the particles rendering intensive destroying effect on structural materials of the nozzle.

At statement of a variational problem three cases can be considered:

- absence of prohibitions for deposition of particles of a condensed phase on walls of the nozzle;
- prohibition of deposition of particles of a condensed phase on walls of the nozzle;
- prohibition of deposition of some fractions of particles of a condensed phase on walls of the nozzle.

The last case most corresponds to real multicomponent polydisperse flow in an annular nozzle as apparent erosive effect appears the particles exceeding diameters d ~ 0.5÷1 a micron. A condition forbidding complete or partial deposition of particles on a surface of an annular nozzle, it is possible to put only for supersonic area of flow where velocities of particles are great also they, owing to the big kinetic energy, can render erosive effect on walls of the nozzle.

Direct calculations of a field of flow during realization of the further optimization were carried out with the account subsonic and transonic areas of flow.

The first stage of a mathematical modeling that searching of optimal configurations of annular nozzles with two-phase monodisperse and multicomponent polydisperse working mediums in the absence of restrictions on a deposition of particles on nozzle surface. The geometry of an initial approximation of the nozzle for a two-phase monodisperse medium is selected close to geometry of an optimal annular nozzle for calculation of the perfect gas, received above.

On fig. 4 results of optimization of an annular nozzle with the two-phase working medium containing condensed particles in the size d = 5 microns are presented.

The mass part of a condensed phase in a two-phase mixture made $z = 0.4$. The procedure of optimization included the accounting of value of an impulse of a deposition on a center body of an annular nozzle. Thus, it was considered that the particles which have settled on the center body completely transfer it their impulse. The increase of propulsive performance characteristics of the nozzle on ~ 7.2 % (from K_T = 1.590 up to K_T = 1.705) is received as the result of optimization.

Characteristic feature of the received optimal profile is practically invariable parameters (a breadth and angle of inclination) annular throat that is connected to weak effect of presence of a condensed phase on parameters of annular throat. On the contrary, coordinates of a profile of a center body have essential differences, especially, in a final part of a profile that speaks the form of trajectories of movement of particles of the condensed phase starting intensively to deposit in this part of an annular nozzle. There are minimal applications of the specified optimization criterion – maximum value of a thrust coefficient results in geometry of the nozzle, in which losses of thrust because of a deposition of particles.

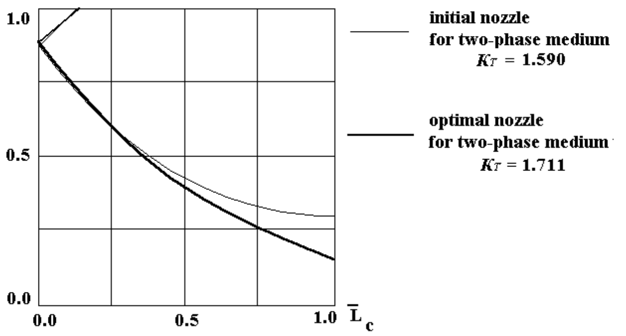

Fig. 4. Initial and optimal nozzles with a two-phase working medium

Results of optimization of an initial profile of an annular nozzle and results of its optimization for perfect gas and for a multicomponent polydisperse medium with a mass part of a condensed phase z = 0.3 are presented on fig. 5.

In this case, parameters of an initial profile of the center body were equal to both considered variational problems.

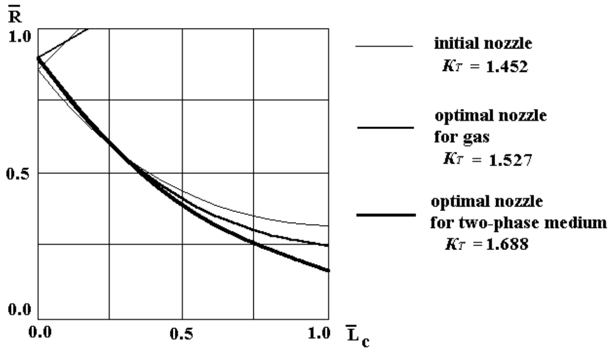

Fig. 5. Comparison optimal nozzles for different working mediums(perfect gas and a multicomponent medium)

Results of a mathematical modeling showed the considerable effect of a condensed phase on an optimized profile of the center body of a nozzle, thus optimal for perfect gas and the multicomponent medium annular nozzles considerably differ a profile of a end segment of a center body. It is interesting to note that at shortening of a center body up to the dimensionless length $L_c \sim 0.25\ R_a$ let's receive practically identical optimal profiles of a center body for perfect gas and a multicomponent medium that speaks absence of deposition of particles on a surface of a center body up to coordinate $L_c \sim 0.25\ R_a$ on an abscissa axis. The determinate feature is a specific singularity of an annular nozzle of external expansion, and allows to apply annular nozzles with a shortcut center body of small length ($L_c < 0.25\, R_a$) at use as a working medium, both gas, and a multicomponent medium. The multicomponent flow in such nozzle is expanded around of a final edge of the external contour which is coming to an end in a plane of annular throat, thus radial a component of a vector of a velocity of particles of a condensed phase, directional towards a nozzle axis, considerably decreases, as the particle has a fancy dense enough gas stream aside, counter axis of the nozzle. In the further expansion of a gas flow aside axis of the nozzle at streamlining of a profiled center body causes movement of particles to an axis of the nozzle (the velocity of particles in a radial direction is insignificant as they are entrain a gas flow having a low density) and, as a result of it, a deposition of particles on a surface of a center body.

The second stage of a mathematical modeling is carrying out search of an optimal configuration of an annular nozzle in the presence of the restrictions forbidding deposition of particles of a polydisperse condensed phase on a surface of a center body [1]. Let′s enter restriction on distance between extreme trajectories of particles of various fractions and nozzle contour as an inequality:

$$y(x) - y_i(x) = \Delta y_i > \mathrm{h} > 0, \qquad (15)$$

where y and y_i – ordinates of a nozzle contour and ordinate of an extreme trajectory of particles i fractions, h – the set constant. For fractions of size less ~ 1 micron (15) can be depressed restriction, without considering it for the specified fractions of particles:

$$y(x) - y_i(x) = 0 \text{ at } d < 1 \text{ micron}, \qquad (16)$$

$$y(x) - y_i(x) = \Delta y_i > \mathrm{h} > 0 \text{ at } d \geq 1 \text{ micron}.$$

Conditions (15) and (16) can be taken into account with the help of penalty functions as follows:

$$\Phi = \left[\sum_l \varphi_i(x) dx \right]^2, \qquad (17)$$

where $\varphi_i(x) = h - \Delta y_i$, at $\Delta y_i < h$ $\varphi_i(x) = 0$, at $\Delta y_i \geq h$.

Using a penalty functions, an initial criterion function for a considered problem takes a form:

$$J = K_T - k\Phi, \qquad (18)$$

where $k > 0$ – coefficient of the penalty.

Application of such method of the accounting of conditions (15) and (16) previously showed the efficiency at creation of optimal configurations of supersonic parts of Laval nozzles with a two-phase working medium. The coefficient of the penalty used in calculations, made $k \sim 10^3 \div 10^4$.

Results of modeling are presented on fig. 6 for a mass part of the condensed phase z = 0.4.

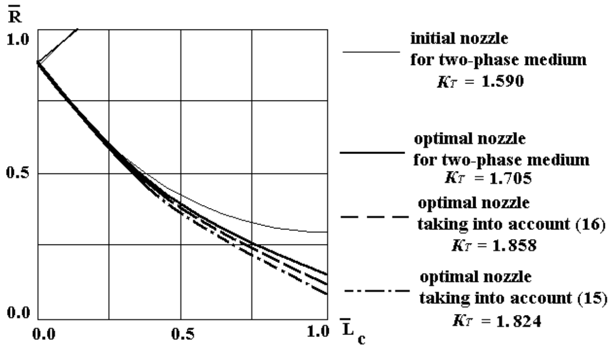

Fig. 6. Annular nozzles, profiling at the accounting of conditions on deposition of particles

Among three optimal configurations of annular nozzles, the best propulsive performance characteristics the nozzle profiled taking into account restrictions (16), not allowing a deposition only those fractions, which have appreciable erosive effect of particles on nozzle walls, possesses. Interesting result of the carried out optimization is the simultaneous combination of two useful properties: maximum propulsive performance characteristics and minimum erosive effect on walls of an annular nozzle. Therefore, restriction (15) can be recommended as the basic by search of optimal geometry of a nozzle.

Besides, the revealed features of flow caused of research of properties of optimal configurations of annular nozzles of other types. Search of an optimal configuration of an annular nozzle of the «mixed» expansion having a center body and an external contour was carried out.

Optimization of a geometrical configuration of the nozzle was carried out for an initial configuration of an annular nozzle with following geometrical and operational factors (dimensionless, divided by value R_a):

- relative radius of the top point of annular throat $\overline{R}_a$ = 1;
- relative area of annular throat =0.65 $\overline{F}_a$;
- length of the nozzle L_c = 1.88 R_a;
- radius of exit section of the nozzle $R_{вых}$ = 1.93 R_a;
- pressure differential in the nozzle p_o/p_H = 100;
- isentropic exponent of gas k = 1.25;

- mass part of the polydisperse condensed phase z = 0.3.

Results of search of an annular nozzle of an optimal configuration are presented on fig. 7.

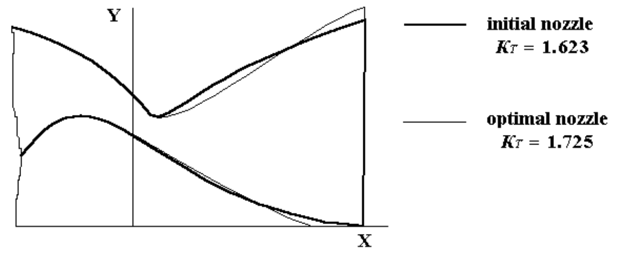

Fig. 7. Various configurations of the annular nozzles of the «mixed» expansion

The received optimal nozzle possesses higher propulsive performance characteristics: the thrust coefficient increased with K_T = 1.623 up to K_T = 1.725, that made ~ 6.3 %. A feature of geometry of the received nozzle is smaller expansion, both a center body, and an external contour in initial area of a supersonic part of the nozzle and the greater expansion of both streamlined surfaces to a nozzle exit.

5 Conclusion

Optimal configurations of annular nozzles with two-phase monodisperse and multicomponent polydisperse working mediums are profiled as a result of the solution of put variational problems and carried out by means of a direct method of optimization of annular nozzles' geometry. Comparison of optimal annular nozzles possesses by higher thrust with two-phase and multicomponent-working mediums with an optimal annular nozzle for perfect gas is carried out. Optimal annular nozzles with two-phase and multicomponent working mediums are profiled under the various conditions restricting deposition of particles of a condensed phase on walls of the nozzle.

Results of mathematical modeling are received with application of high-efficiency calculations using a supercomputer «TORNADO» of South Ural State University with processing power up to 473,6 TFlops.

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