# BLACK-BOX MODEL EPISTEMIC UNCERTAINTY AT EARLY DESIGN STAGE. AN AIRCRAFT POWER-PLANT INTEGRATION CASE STUDY

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## Abstract

*Presented is a novel numerical approach for modelling and propagating model epistemic uncertainty in simulation workflows composed of [black-box] deterministic models. This is achieved through the introduction of a Randomisation Treatment. Specifically, the design parameters computed by particular deterministic simulation models are numerically perturbed according to statistical criteria determined on the grounds of prior design experience and/or other considerations. Established uncertainty propagation methods can thus be deployed to estimate the resulting statistical behaviour of the simulation outputs. The proposed approach is demonstrated via an example of industrial relevance based on an aircraft power-plant integration case study. The latter provides a wider context where a reduction of epistemic uncertainty is achieved due to the closer design collaboration between the airframe and engine manufacturers, and in particular, due to the faster introduction of up-to-date and more comprehensive design information. It is demonstrated that the proposed approach, combined with appropriate collaboration processes leads to more robust solutions.*

## 1 Introduction

Early design studies of complex products such as aircraft are generally aimed at rapid definition (synthesis) and evaluation (analysis) of highly idealized concepts rather than accurate and detailed representations of the artefact. These studies, however, are performed with significant levels of inherent uncertainty in the modelling and simulation tools/models involved at this stage. Uncertainty itself is undesired as it diminishes the confidence on the computed results, which can be compensated either by overdesign or by investment in further design iterations.

There are different types of uncertainty and a common practice in the engineering and other fields has been to distinguish between two major classes: aleatory and epistemic uncertainty [4][10]. The former is representative of the physical variability that is inherent in the (design) parameters describing the system of concern or its environment. Epistemic uncertainty on the other hand is related to the lack of knowledge or information. In modelling and simulation it generally arises from the mathematical simplifications/assumptions underlying the development of simulation models, the inclusion of incomplete experimental data, or the incorporation of subjective design judgement and experience. Unlike aleatory uncertainty, it is possible to reduce epistemic uncertainty through the addition of further knowledge if/when this is available, for example, under the form of calibration coefficients or higher-fidelity simulation models.

A methodological perspective and an extensive (epistemic) uncertainty classification can be found in Padulo and Guenov [8]. In this work we concentrate on a particular subset of epistemic uncertainty, namely the one associated with computational models describing a physical system such as aircraft at early design stages. The objective is to develop an approach for modelling and propagating such uncertainty through the computational systems so that it can be taken into account for (design) decision making.

The rest of the paper is organised as follows. Essential background information, definitions and formulations related to uncertainty management are presented in the next section. The methodology for model epistemic uncertainty management under the restrictions outlined above is presented in section 3. The application of the proposed method in the context of an industrial case study concerning power-plant integration is described in section 4. Finally conclusions are drawn and future work is outlined in section 5.

## 2 Uncertainty Management

Uncertainty management includes modelling, propagating and assessing the impact of uncertainty sources embedded in the simulation models [9]. A typical uncertainty management schema is divided into three steps [5][6]: 1) *uncertainty quantification*, in which the sources of uncertainty are formally identified, qualified and quantified; 2) *uncertainty propagation*, where an estimation of the statistical behaviour of the simulation outputs is computed through an adequate propagation of the modelled uncertainties in simulation workflows; 3) *robust assessment*, which allows to assess the robustness of the simulation results with respect to any considered uncertainty. A typical uncertainty management schema is depicted in Fig. 1, whereas an overview of each step is presented in the following sub-sections.

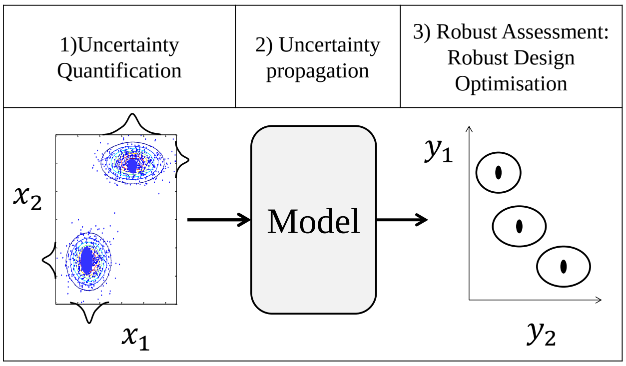


**Fig. 1** Example of a typical uncertainty management schema.

### 2.1 Uncertainty Quantification

Notwithstanding previous research efforts, the quantification of epistemic uncertainty is still an on-going issue in both academia and industry [4][7]. Its characterisation by probabilistic approaches in general is not straightforward because of the difficulty in inferring any statistical information due to the intrinsic lack of knowledge. Prior proposed methods generally require polynomial approximations of the model output [7], which in practice might be inconceivable for large multidisciplinary models, or would allow to consider only a limited number of modelling techniques (e.g., symmetric or uniform distributions) [9][10]. The approach proposed in section 3 resorts to a numerical perturbation of requested simulation outputs on the basis of statistical criteria defined by the designer according to prior experience and/or assumptions. Conceptually similar to modelling aleatory uncertainty, it enables to represent such type of epistemic uncertainty via any desirable probability density function (PDF), which can thus be propagated in computational workflows via established algorithms.

### 2.2 Uncertainty Propagation

Once the uncertainty affecting simulation parameters and/or models has been adequately quantified, it needs to be propagated through the considered computational workflow in order to estimate the statistical behaviour of the simulation outputs. Suitable algorithms need to be selected to achieve this while guaranteeing the satisfaction of a number of simulation requirements (e.g., requested numerical accuracy, computational time, simulation solvers, etc.). In this context, interoperability becomes an important consideration, especially in collaborative enterprise simulations, where models implemented in different environments need to be integrated together to conduct multidisciplinary and multi-fidelity studies. To this end, and accounting also for the importance of intellectual property protection, we utilise the concept of a black-box model. Such a model can be, for example, a compiled code with known inputs, outputs and functional performance, but whose internal implementation is not necessarily accessible. Among other suitable algorithms (e.g., Monte Carlo Simulation, Moment Method, Polynomial Chaos, Stochastic Expansion, etc.), the Univariate Reduced Quadrature (URQ) method [11][12] has been adopted. The URQ method is inspired on the sigma-point techniques and provides an estimate of the simulation outputs mean and variance to a higher accuracy when contrasted with alternative methods of comparable cost. Among its other qualities, it is able to handle several probability density functions (symmetric and non-symmetric distributions) which allows to model diverse types of uncertainty. Moreover, the URQ method relies on a deterministic approximation of the statistical behaviour of the simulation outputs, which makes it compatible with well-known algorithms for conducting different simulation activities (e.g., design optimisation, design of experiments, sensitivity analysis, etc.). A more detailed description of the URQ method is presented in section 3.

## 2.3 Robust Assessment

The third step of uncertainty management in modelling and simulation activities is aimed at exploiting uncertainty quantification and uncertainty propagation for assessing the robustness of the simulation results. To achieve this, the problem to be investigated needs to be formally stated in such a way that it is representative of and consistent with the considered design requirements, assumptions, sought scope, and deployed numerical methods. Considered in this paper is the application of uncertainty management schemas for conducting optimisation studies under uncertainty, which can be generically formulated as follows:

$$\begin{aligned} &\min_{\mu_{\mathbf{x}} \in \Re^n} \mathbf{F}[f_j(\mathbf{x},\mathbf{p})], \\ &\text{subject to: } P(g_i(\mathbf{x},\mathbf{p}) \leq 0) \geq P_{g_i}, \\ &\text{with: } P(x_{lb_k} \leq \mu_{x_k} \leq x_{ub_k}) \geq P_{x_k}. \end{aligned} \tag{1}$$

where:

- $j = 1,2,...,J$, $i = 1,2,...,I$, $k = 1,2,...,n$;
- $\mathbf{p}$ is a vector of design parameters;
- $\mathbf{x}$ is the vector of $n$ design variables with mean $\mu_{\mathbf{x}}$ and affected by uncertainty;
- $\mathbf{F}$ are suitable functions that express the dependence of the probability density function of the $J$ objectives $f_j$ on the input multivariate distribution;
- $P_{g_i}$ and $P_{x_k}$ are the desired probabilities of satisfying the $i^{th}$ constraint $g_i$ and the variable bounds defined for the $k^{th}$ design variable $x_k$, respectively.

The approach adopted here is to express the functions $\mathbf{F}$ via a weighted sum function [12], which leads to the reformulation of Equation (1) as follows:

$$\begin{aligned} &\min_{\mu_{\mathbf{x}} \in \Re^n} F_j = \mu_{f_j} + k_{f_j}\sigma_{f_j}, \\ &\text{subject to: } G_j = \mu_{g_i} + k_{g_i}\sigma_{g_i} \leq 0, \\ &\text{with: } \mathbf{x_{lb}} + \mathbf{k_x}\sigma_{\mathbf{x}} < \mu_{\mathbf{x}} < \mathbf{x_{ub}} - \mathbf{k_x}\sigma_{\mathbf{x}}. \end{aligned} \tag{2}$$

where $F_j$ and $G_i$ are utility (loss) functions that account simultaneously for the mean and variance of the objective $f_j$ and $g_i$ constraint functions, which need to be estimated by propagating adequately the uncertainty affecting particular design parameters or simulation models. Parameters $\mathbf{k_x}$ depend on the $n$ desired probabilities $P_{x_k}$ and the statistical properties assumed for modelling the uncertainty affecting the design parameters. The coefficients $k_{f_j}$ and $k_{g_i}$ can be established on the basis of the desired probabilities $P_{f_j}$ and $P_{g_i}$ , along with the adopted robust design criterion among the two possible choices of

**Table 1** Applicability of the proposed approach.

| **Source of epistemic uncertainty** | **Randomisation Treatment Applicable** | **Randomisation Treatment not applicable** |
|---|---|---|
| *Requirements definition* | Insufficient elicitation (e.g., constraint boundary set arbitrary) | Incomplete elicitation (e.g., missing requirement) |
| *Design assumptions* | Calibration/correction of parametric equations, refinement of operating scenarios/environments | Reduction in number of relevant simulation variables |
| *Phenomenon parameterisation* | Limited scope of the mathematical model (e.g., limited number of operating conditions) | Limited Understanding/Misinterpretation of the phenomenon |
| *Computational accuracy* | Known/estimated accuracy of the selected algorithm/solver | Unknown or unquantifiable computational errors |

quantile and tail conditional expectation (TCE) metrics. Further details and a practical explanation of the two criteria can be found in Padulo and Guenov [12].

## 3 Modelling and Propagation of Epistemic Uncertainty

Described in this section is the proposed approach for handling the model epistemic uncertainty affecting specific design parameters of simulations conducted at preliminary design stage. The key concept is based on modelling such type of uncertainty via a numerical perturbation with statistical properties defined according to design experience and/or assumptions. This is achieved through a numerical method that is described in the next sub-section and will be hereafter referred in this paper as *Randomisation Treatment*. The integration of the proposed treatment with the URQ uncertainty propagation method is described in sub-section 3.2.

### 3.1 Randomisation treatment

Low-fidelity models are deployed at early design stage to rapidly generate a high-level product description and to compare different design alternatives by conducting various fast simulation studies. In turn, it is necessary to adequately acknowledge and manage the epistemic uncertainty stemming from the lack of knowledge inherent in early design, which can be attributed to a number of different sources, such as:

- *Requirements definition*. An incorrect or incomplete elicitation and documentation of design requirements could preclude the adequate development of simulation models to support the design of a product which fully satisfies relevant customer needs and expectations.

- *Design assumptions*. Different assumptions are normally taken into consideration throughout the development of the models to simplify the physics behind the phenomena to be simulated. Examples of common practice are the reduction of the number of relevant simulation variables, the calibration/correction of parametric equations with experimental data, generalisation of operating scenarios and environments (e.g., initial conditions, boundary conditions, etc.).

- *Phenomenon parameterisation*. A limited understanding or misinterpretation of the simulated phenomenon can prevent developing a mathematical model that appropriately captures its behaviour for each operating scenario under investigation.

- *Computational accuracy*. The selection of particular algorithms in the implementation of models impacts on the accuracy of simulation results. The latter in general is also dependant on the choice of solvers along with their corresponding setup parameters for executing the individual models and simulation workflows.

It has to be emphasised at this stage that the proposed approach can handle only subsets of the above epistemic uncertainties. Presented in Table 1 are example scenarios for which the proposed approach can and cannot be applied, respectively.

The *Randomisation Treatment* mentioned above is a numerical perturbation used during the uncertainty quantification stage. Thus outputs deterministically computed through simulation models affected by epistemic uncertainty are stochastically parameterised via *Random Variables* with predefined statistical properties. More precisely, each *Random Variable (RV)* associated with a particular model output will have a unit mean, together with minimum and maximum variations defined according to prior design experience and/or assumptions. In this way, the definition of each *Random Variable* can be formally stated via adequate probability density functions (PDFs). This process is conceptualised in Equation (3) and Fig. 2 by considering a generic output $y$ and its corresponding $RV_y$,

$$y_{stochastic} = RV_y \cdot y_{deterministic}(\mathbf{x_y}) \qquad (3)$$

where $\mathbf{x_y}$ is the vector of design variables on which $y$ is dependent.

It is important to note that the application of the *Randomisation Treatment* allows to model and encapsulate in its output the epistemic uncertainty associated with specific simulation models. In other words, it will transform specific deterministic simulation outputs onto stochastic variables with precise statistical properties depending on the probability density functions of their corresponding *Random Variables*. Such uncertainty

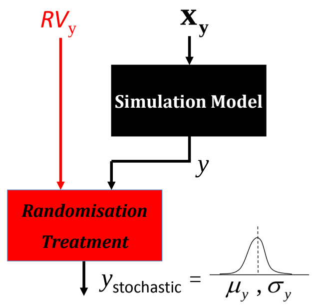


**Fig. 2** *Randomisation Treatment* deployment on a simulation model.

can thus be propagated through simulation workflows via conventional numerical methods. Depicted in Fig. 3 is an example of a *Randomisation Treatment* application on an illustrative simulation workflow.

Described in the next sub-section is the integration of the proposed approach with the Univariate Reduced Quadrature (URQ) method for uncertainty propagation [12].

### 3.2 Epistemic Uncertainty Propagation via the URQ method

A number of numerical methods can be considered for propagating model epistemic uncertainty and handled via the aforementioned *Randomisation Treatment*. As mentioned in Section 2, the Univariate Reduced Quadrature (URQ) method [12] has been chosen for uncertainty propagation. It allows to estimate the statistical properties (mean $\mu$ and standard deviation $\sigma$) of selected simulation outputs. This is generically illustrated in Fig. 3, where the stochastic behaviours attributed to $y_1$ and $y_2$ through the *Randomisation Treatment* need to be propagated downstream in the workflow to approximate the resulting mean and variance of $y_{out}$. Such an approach can be generalised to manage the epistemic uncertainty affecting $r$ simulation parameters through the definition of an equivalent number of *Random Variables*. An accurate estimation of the statistical behaviour of the simulation outputs can thus be computed with the URQ method via a sampling stencil of $2r+1$ workflow evaluations. It is important to note that the computational effort grows linearly with $r$. This is particularly

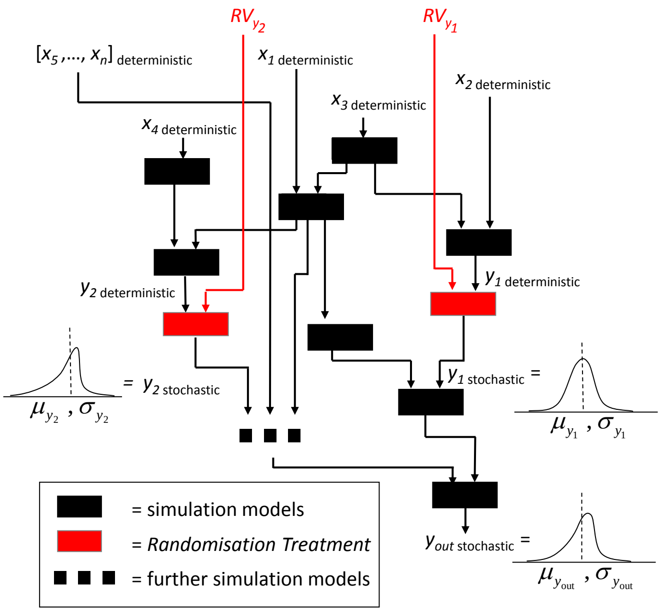


**Fig. 3** Conceptual example of the *Randomisation Treatment* deployment in a generic simulation workflow.

efficient compared to other strategies that require thousands of evaluations, such as the traditional Monte Carlo Simulation method, as conceptually illustrated in Fig. 4.

Illustrated in Fig. 4 is also the ability of the URQ method to handle a number of different symmetric and non-symmetric PDFs (e.g., Gaussian, Triangular, Gamma, Beta, etc.). For example, consider two models that allow the computation of the simulation parameters $y_1$ and $y_2$, as depicted in Fig. 3. The epistemic uncertainty affecting such models is modelled through the formulation of the corresponding *Random Variables* $RV_1$ and $RV_2$. The first is formulated as a non-symmetrical triangular distribution with a given minimum and maximum variation from a nominal value, denoted by $a$ and $b$ respectively. The second is formulated as a Gaussian distribution with a precise standard deviation. The propagation of uncertainty via the URQ method would thus require five evaluations of the workflow to compute an estimation of the mean and standard deviation of the simulation output $y_{out}$ corresponding to any evaluated design point $\mathbf{x}$. The exact location of the evaluation stencil points is defined according to the scalar distances $h_1^{\pm}$ and

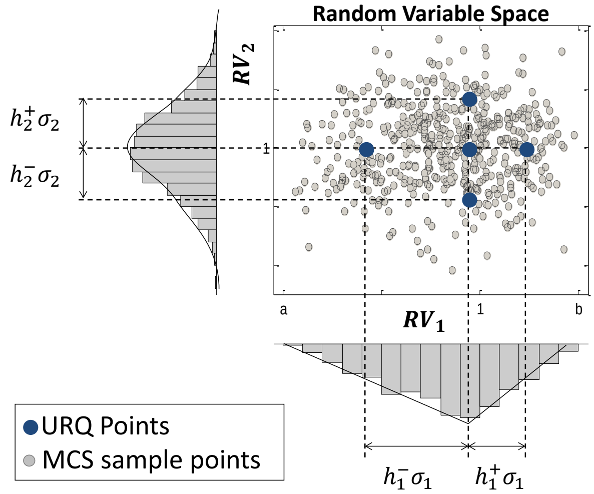


**Fig. 4** Conceptual comparison of the sampling evaluations required by MCS and URQ on a generic random variable space $RV_1$ $RV_2$.

$h_2^{\pm}$, which are functions of the first four statistical moments of the random variables. A complete description of the URQ method, is presented in Padulo et al. [11].

## 4 Application Example: an Aircraft Power-Plant Integration Case Study

The proposed model epistemic uncertainty management method is demonstrated in this section via an application of industrial relevance. It is based on an aircraft power-plant integration scenario in which scheduled design iterations are progressively conducted as up-to-date design information becomes available from the engine manufacturer. In particular, the introduction of surrogate models in simulation studies is adopted in robust design optimisation (RDO) studies aimed at managing the convergence of engine requirements.

### 4.1 Case study description

The case study is based on the *"Preliminary Multi-Disciplinary Power-Plant Design"* test case defined in the EU FP7 CRESCENDO project [1][2]. It sets the scope for robust multidisciplinary optimisation studies for the preliminary design of aircraft power-plant. A particular attention was given to the interaction between air-

**Table 2** Relevant case study nomenclature.

| | |
|---|---|
| $FNslst$ | Sea-level engine thrust [N] |
| $Awing$ | Wing area [$m^2$] |
| $ar$ | Wing aspect ratio |
| $BPR$ | By-pass ratio |
| $sfc$ | Specific fuel consumption [lb/lbd/hr] |
| $LoD$ | Lift over drag coefficient |
| $MWE$ | Maximum weight empty [kg] |
| $tofl$ | Take-off field length [m] |
| $vapp$ | Approach speed [kt] |
| $TTC$ | Time to climb [min] |
| $OEI$ | One engine inoperative |
| $MTOW$ | Maximum Take-off Weight [kg] |
| $Fuel$ | Total fuel for mission [kg] |

**Table 3** Aircraft design optimisation formulation.

| **Objective** |
|---|
| minimise $MTOW$ |
| **Constraints** |
| $tofl$-standard $\leq$ 1700 m |
| $tofl$-hot&high $\leq$ 3200 m |
| $vapp \leq$ 137 kt |
| Climb Ceiling $\geq$ 33000 ft |
| Cruise Ceiling $\geq$ 35000 ft |
| Buffeting Ceiling $\geq$ 37000 ft |
| $OEI$ Ceiling $\geq$ 16000 ft |
| $TTC \leq$ 25 min |
| **Design Variables** |
| $FNslst$ = [70000, 160000] N |
| $Awing$ = [100,200] $m^2$ |
| $ar$ =[7, 12] |
| $BPR$ = [6, 10] |

frame and engine manufacturers. This involved finding the best compromise between the benefits (e.g., thrust and power) and drawbacks (e.g., weight, fuel consumption, etc.) associated with different engine concepts. The relevant nomenclature and the design optimisation formulation are given in Table 2 and Table 3, respectively.

The evaluation of alternative design solutions conducted throughout the optimisation process was computed by a set of simple models dedicated to aircraft conceptual design named SIMCAD, which was provided by one of the industrial partners. SIMCAD allows to evaluate conventional aircraft configurations according to arbitrary Top Level Aircraft Requirements (TLARs) by running a nested mass-mission loop, required to achieve the convergence of $MTOW$ and $Fuel$.

The addressed scenario focuses on the design iterations triggered by the progressive release of more detailed design information by the engine manufacturer. From a business perspective, a secure exchange of design data and models between airframe and engine manufacturers is therefore necessary while at the same time guaranteeing the protection of their corresponding intellectual property (IP). One of the solutions investigated to achieve this is through the use of surrogate and black-box simulation models. Specifically, the models in the abovementioned aircraft sizing code SIMCAD were handled as black-boxes, whereas a more accurate model of the engine was provided as a kriging surrogate model [3]. The investigation of the convergence of engine requirements was thus carried out by considering two design iterations. The first iteration was conducted by means of a simulation architecture entirely based on SIMCAD. The second iteration included the incorporation of the engine surrogate model which replaced specific simulation models within SIMCAD, thus acting as a means for reducing the epistemic uncertainty affecting the computation of particular simulation parameters. The considered set of uncertain parameters and their associated *Random Variables* is presented in Table 4, where each uncertainty is represented as a percentage variation from a nominal value (i.e., the value computed for its corresponding parameter). Such variation is assumed on the grounds of prior experience and is reduced from the first to the second iteration due to the refinement of simulation accuracy introduced by the engine surrogate model.

## 4.2 Robust design optimisation

A weighted sum strategy based on Equation (2) along with the proposed approach for the management of epistemic uncertainty were adopted to tackle the optimisation problem presented in Table 3. All the uncertainties affecting the sim-

**Table 4** Set of simulation parameters computed by models affected by epistemic uncertainty. Note that in the second design iteration the parameters $EngineWeight$, $sfc$ and $FanDiameter$ are assumed not to be affected by uncertainty.

| Simulation parameters | Uncertainty of parameter computation in Design Iteration 1 | Uncertainty of parameter computation in Design Iteration 2 | Corresponding Random Variable |
|---|---|---|---|
| $MaxTrustT/O$ | [-10% , +10%] | [-5%, +5%] | $RV_{Kmto}$ |
| $MaxThrustClimb$ | [-10% , +10%] | [-5%, +5%] | $RV_{Kmcl}$ |
| $MaxThrustCruise$ | [-10% , +10%] | [-5%, +5%] | $RV_{Kmcr}$ |
| $EngineWeight$ | [-5% , +5%] | - | $RV_{Mpwp}$ |
| $sfc$ | [-5% , +5%] | - | $RV_{sfc}$ |
| $FanDiameter$ | [-1% , +1%] | - | $RV_{dnac}$ |
| $LoD$ | [-1% , +0.5%] | [-0.5%, +0.5%] | $RV_{LoD}$ |
| $MWE$ | [-2% , +2%] | [-1%, +1%] | $RV_{MWE}$ |

ulation parameters in Table 4 were modelled via Gaussian distributions, except for $LoD$ in the first design iteration, for which a triangular distribution was considered. This is because the latter reflects better the designer's assumption for asymmetric $LoD$ variation from the nominal value. A summary of the uncertainty modelling via *Random Variables* adopted for the two design iterations presented in this paper is provided in Table 5.

The statistical properties of the objective and constraints were estimated through the *quantile* metric [12], without considering any particular assumption on their respective distribution, and requesting a 75% satisfaction probability.

The execution of the aforementioned optimisation problem represents a clear example of simulation studies requiring a hierarchical application of relevant numerical treatments on a simulation workflow assembled from a given set of computational models. This has been addressed via specific functionalities provided in the innovative model-based design tool AirCADia [13], which enables an interactive formulation and execution of black-box simulation studies without the need for a laborious and specialised lower level programming. Illustrated in Fig. 5 is a schematic representation of the considered simulation architecture. It is possible to note how the optimisation treatment (loop) encapsulates the uncertainty propagation treatment, which in turn is applied onto the simulation workflow after achieving the convergence of $MTOW$ and $Fuel$.

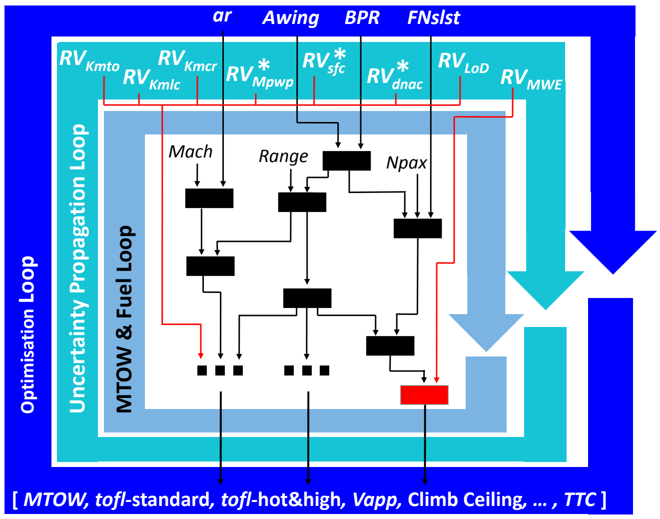


**Fig. 5** Illustration of the considered simulation architecture. Identified with an asterisk are the three *Random Variables* assumed not to be affected by uncertainty in the second design iteration.

The results obtained in AirCADia via the URQ method coupled with an embedded evolutionary algorithm are presented in tabular and graphical form in Table 6, Table 7 and Fig. 6, respectively.

The power-plant robust design optimisation problem described above represents an example of simulation studies, based on the integration of

**Table 5** Set of *Random Variables* modelled as epistemic uncertainty.

| Random Variable | Uncertainty Modelling in Design Iteration 1 | Uncertainty Modelling in Design Iteration 2 |
|---|---|---|
| $RV_{Kmto}$ | Gaussian with: $\mu = 1; \sigma = 0.1/3$ | Gaussian with: $\mu = 1; \sigma = 0.05/3$ |
| $RV_{Kmcl}$ | Gaussian with: $\mu = 1; \sigma = 0.1/3$ | Gaussian with: $\mu = 1; \sigma = 0.05/3$ |
| $RV_{Kmcr}$ | Gaussian with: $\mu = 1; \sigma = 0.1/3$ | Gaussian with: $\mu = 1; \sigma = 0.05/3$ |
| $RV_{Mpwp}$ | Gaussian with: $\mu = 1; \sigma = 0.05/3$ | - - |
| $RV_{sfc}$ | Gaussian with: $\mu = 1; \sigma = 0.05/3$ | - - |
| $RV_{dnac}$ | Gaussian with: $\mu = 1; \sigma = 0.01/3$ | - - |
| $RV_{LoD}$ | Triangular with: a=0.99, $\mu = 1, b = 1.005$ | Gaussian with: $\mu = 1; \sigma = 0.005/3$ |
| $RV_{MWE}$ | Gaussian with: $\mu = 1; \sigma = 0.02/3$ | Gaussian with: $\mu = 1; \sigma = 0.01/3$ |

**Table 7** Objective and constraint values of the optimal points obtained for the robust design optimisation formulation in Table 3.

| Performance | Design Iteration 1 | | Design Iteration 2 | |
|---|---|---|---|---|
| | Mean | Standard Deviation | Mean | Standard Deviation |
| $MTOW$ [kg] | 90136.5341 | 675.4364 | 87289.473 | 204.964 |
| $MaxThrustT/O$ $[lbf]$ @M 0.25; ISA+15; 0ft. | 28482.5293 | 949.4176 | 26980.308 | 449.671 |
| $MaxThrustClimb$ $[lbf]$ @ M 0.76; ISA; 35kft. | 7358.9591 | 245.2986 | 6970.834 | 116.180 |
| $MaxThrustCruise$ $[lbf]$ @ M 0.76; ISA; 35kft. | 6847.9203 | 228.264 | 6486.748 | 108.112 |
| $MWE$ $[kg]$ | 45214.7322 | 394.7925 | 44201.022 | 170.056 |

**Table 6** Design variable values of the optimal points obtained for the robust design optimisation formulation in Table 3.

| | Design Iteration 1 | Design Iteration 2 |
|---|---|---|
| $ar$ | 11.786 | 11.252 |
| $Awing$[m$^2$] | 164.7776 | 161.8699 |
| $BPR$ | 9.5432 | 6.4771 |
| $FNslst$[N] | 154440.7238 | 146295.2379 |

mixed-fidelity models. The results show reduction of the uncertainty associated with simulation outputs due to the incorporation of more accurate higher-fidelity models such as the engine surrogate model introduced in the second design iteration. It is important to note that the reduction of the uncertainty and the improvement of the solution were possible not least because of the results computed in the first design iteration. More specifically, the latter allowed to identify the region in the design space which contained the desired set of optimal solutions. Thus the higher-fidelity (surrogate) engine model was developed from a dataset obtained from more accurate engine simulations conducted around such optimal design points. The insight and knowledge gained from the first design iteration were exploited to limit the epistemic uncertainty affecting the engine simulation model deployed in the second design iteration. The convergence to a more certain set of engine requirements was hence achieved by conducting a second robust design optimisation study based on the refined engine model.

## 5 Summary and Conclusions

Presented in this paper is a novel numerical approach for management of black-box model epistemic uncertainty. The proposed approach is seen as most beneficial to the early stages of com-

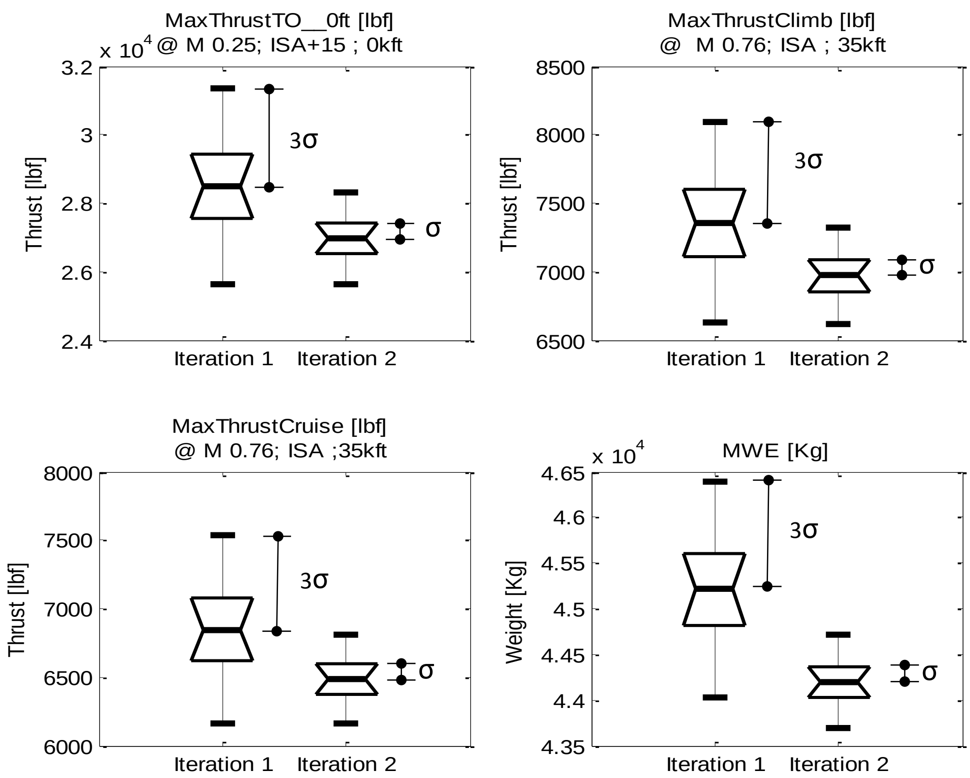


**Fig. 6** Convergence of model epistemic uncertainty associated with engine requirements by means of box plots. The horizontal line in the middle of each box represents the mean value of each engine parameter per iteration, whereas the top and bottom lines of each box identify the range corresponding to a variation of such value within one standard deviation. A variation of three standard deviations is also depicted via the horizontal lines at the extremes of the vertical lines extending above and below each box.

plex systems design, when preliminary simulation studies are based on low-fidelity models and conducted on the grounds of various assumptions and incomplete knowledge. A *Randomisation Treatment* has been proposed to quantify and formally state a class of epistemic uncertainty affecting the computation of simulation parameters. This is numerically obtained through the definition of associated *Random Variables* with precise statistical properties. In the proposed approach the uncertainty propagation is enabled via the integration of the URQ method, which allows the efficient estimation of the simulation output robustness in terms of mean and standard deviation. The application of the proposed approach for the reduction of epistemic uncertainty in an example of industrial relevance demonstrated its value in the process of maturing and converging (engine and airframe) design requirements in a collaborative design environment which respects partners' intellectual property. Future work will concentrate on enabling the modelling of dependencies among random variables, as well as on the extension of the proposed methods to other classes of epistemic uncertainty.

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