MULTI-STEP TRAJECTORY OPTIMIZATION FOR ATM BASED ON APPROXIMATED OPTIMAL PATH

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Abstract

This paper proposes a multi-step approach to optimize the commercial aircraft trajectory in the presence of current Air Traffic Management system. A set of waypoints are first determined by approximated trajectories and a shortest path algorithm. Then, a numerical optimal control method is used to compute the optimal trajectory passing through the waypoints. The convergence rate is analyzed to validate the approximation. Numerical results show that a sequence of waypoints, and optimized trajectories with computational efficiency.

1 Introduction

The increasing demand of public consumers for air traffic services over the past decades has led to the development of Air Traffic Management (ATM) system. To modernize the ATM system, the Single European Sky ATM Research and abbreviated as SESAR [1], the Next Generation Air Transportation System (NextGen) [2], and Seamless Asian Sky (SAS) have been initiated in Europe, United States, and Asia/Pacific region respectively. The aim of these programs is to improve the safety, capacity, efficiency and cost-effectiveness, and to reduce environmental impact of aviation. One of the performances to achieve the goal of these programs is the Trajectory Based Operation (TBO) concept that dynamically, flexibly and efficiently adjusts flight paths in 3-dimensional space (longitude, latitude and altitude) and the time using known positions and intents. Flight plans include information related to the intended flight of an aircraft filed by airliners with the Air Traffic Controller (ATC) authority. In flight planning for the current system, it is essential to divide aircraft trajectory into several phase decided by the optimal sequence of waypoints which is called a flight route. The waypoints are stated in the Aeronautical Information Regulation and Control (AIRAC) cycle [3].

This paper deals with trajectory optimization which considers both continuous (aircraft dynamics, constraints, etc.) and discrete (a flight routing) system for flight planning in the presence of the current Air Traffic Management (ATM) system.

Numerical trajectory optimization methods have been studied for several decades [4]. There are many researches for commercial aircraft trajectory optimization in order to improve ATM system based on the numerical trajectory optimization methods formulated as an optimal control problem [5]–[17]. In [5]–[13], the minimum time and/or fuel consumption trajectory optimization problem for commercial aircraft is studied by using numerical optimal control problem which does not include discrete system. In [14]–[17], they include both continuous and discrete system for the state of aircraft and decision making process of sequencing by using mixed-integer optimizations. Those approaches have disadvantages of absence of decision making process for the sequence of waypoints in [5]–[13] and the high computational load and difficulty of formulation in [14]–[17], respectively.

The proposed approach is to flight planning in the presence of current ATM system based on both the numerical optimal control method and the shortest path algorithm. The approach can consider both the continuous aircraft dynamics constraints and the discrete sequence of waypoints. The advantages of this approach are reducing the computational load and the complexity of formulation compared with the mixed-integer optimization. In addition, the approach has a potential possibility of online modifications of the flight routes to be described in Sec.4.2.

Flight planning problems can be dealt with a multi-step optimization based on both the numerical optimal control method and the shortest path algorithm. In this study, the multi-step optimization is performed as following steps: 1) calculate coarsely optimized weights by using a global pseudospectral method between each vertex used in the shortest path algorithm; 2) find shortest path based on the weights; 3) refine the flight trajectory based on the sequence of waypoints decided by 2).

This paper is composed as follows. Problem modeling for flight planning is the content of Sec.2. The multi-step optimization for flight planning including the numerical optimal control method and the shortest path algorithm is described. This context is provided in Sec.3. In Sec.4, an application of the multi-step optimization to an en route trajectory and results of the application are discussed. Sec.5, finally, contains the conclusions and future works.

2 Problem Modeling

In this section, we present the flight planning problem. This section involves the horizontal aircraft dynamics, path constraints of the dynamics, the meteorological model, the airspace structure and the performance index of the problem.

2.1 Dynamic Constraints

We consider horizontal motion of the aircraft over a spherical Earth, the differential-algebraic equations of motion are as follows [18]:

$$\begin{aligned}
\dot{\theta}(t) &= \frac{V(t)\sin\psi(t) + W_\theta(\theta(t),\lambda(t))}{r} \\
\dot{\lambda}(t) &= \frac{V(t)\cos\psi(t) + W_\lambda(\theta(t),\lambda(t))}{r\cos\theta(t)} \\
\dot{V}(t) &= \frac{T(t) - D(V(t),C_L(t))}{m(t)} \\
\dot{\psi}(t) &= \frac{L(V(t),m(t))\sin\phi(t)}{m(t)V(t)} \\
\dot{m}(t) &= -T(t)\eta(V(t))
\end{aligned} \tag{1}$$

In the set of Eq.(1), the state vector is $x(t) = (\theta(t),\lambda(t),V(t),\psi(t),m(t))$, where θ,λ,V,ψ, and m denote longitude, latitude, the true air speed, the heading angle, and the aircraft mass, respectively. The control vector is $u(t) = (T(t),\phi(t))$, where T and ϕ are the engine thrust and bank angle, respectively. W_θ and W_λ denote the components of the wind vector. The specific fuel consumption corresponds to η. D and L are aerodynamic drag and lift, respectively. The aerodynamic lift and drag are driven by dimensionless coefficient of lift C_L and drag C_D. Lift $L = C_L S\hat{q}$ and drag $D = C_D S\hat{q}$, where S is the reference wing surface area, and $\hat{q} = 1/2\,\rho V^2$ is the dynamic pressure. g and r are the acceleration of gravity and Earth radius, which is assumed as constant, respectively.

2.2 Path Constraints

We use the Base of Aircraft Data (BADA) 3.12, which provides models for fuel consumption, thrust, aerodynamic force, performance limitations, etc. The aircraft motions are constrained by performance limits. Performance constraints are considered to define the domain of state and control variables.

$$\begin{aligned}
&m_{\min} \le m(t) \le m_{\max}, \quad T_{\min} \le T(t) \le T_{\max}, \\
&C_{V_{\min}} V_{\text{stall}} \le V(t) \le V_{\max}, \quad 0 \le C_L(t) \le C_{L\max}, \\
&\phi_{\min,\text{civ}} \le \phi(t) \le \phi_{\max,\text{civ}}, \quad \dot{V}(t) \le \dot{a}_{1,\max,\text{civ}}
\end{aligned} \tag{2}$$

More details can be found in the BADA database manual [19].

2.3 Meteorological Model

Wind forecast data are provided by the National Oceanic and Atmospheric Administration to take into account the influence of wind. The wind forecast data are fitted into analytic functions which are 4rh order multiple linear regression [20].

$$\begin{aligned} W_x &= \beta_{00}^x + \beta_{10}^x x_e + \beta_{01}^x y_e + \beta_{20}^x x_e^2 + \beta_{11}^x x_e x_y + \cdots \\ &\quad + \beta_{13}^x x_e y_e^3 + \beta_{04}^x y_e^4 \\ W_y &= \beta_{00}^y + \beta_{10}^y x_e + \beta_{01}^y y_e + \beta_{20}^y x_e^2 + \beta_{11}^y x_e x_y + \cdots \\ &\quad + \beta_{13}^y x_e y_e^3 + \beta_{04}^y y_e^4 \end{aligned} \tag{3}$$

2.4 Airspace Structure

The trajectory optimization has no meaning until specifying a set of waypoints the aircraft is going to fly (called briefly routes). Therefore, the airspace is structured to ensure the safety and the sequence of commercial aircraft operations. The waypoints are published in a basic manual for called the Aeronautical Information Publication (AIP) [3]. AIP is updated by regular version on a fixed cycle which is known as the Aeronautical Information Regulation And Control (AIRAC) cycle [3]. Flight plans have to include the routes specified in the AIRAC.

We utilize the AIRAC cycle and model applied in [14]. In the paper, the airspace is modeled as a graph, and it is supposed that the aircraft must traverse the route. The graph is a complete multipartite graph structure, whose vertex set is divided into several independent subsets called partite sets. The graph is composed of a sequence of the partite sets that include the initial waypoint and the final waypoint. The aircraft have to pass through one of the waypoints in every partite set of the graph. For example, there are three partite sets except the initial and the final waypoint, and each partite set holds seven waypoints, then, aircraft have to traverse five waypoints from the initial waypoint to the final waypoint as shown in Fig. **1**.

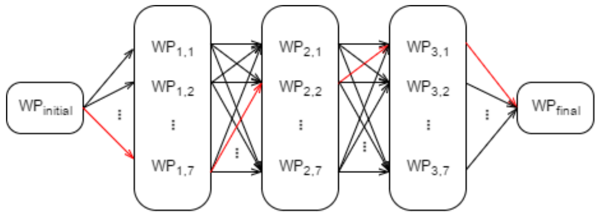

Fig. 1. A graph example (red line is the specified route)

2.5 Performance Index

For flight planning, we utilize the Direct Operating Cost (DOC), which is decided by airliners and aircraft owners, as an objective function. The DOC is composed of the fuel and time cost. The performance index can be written as follows:

$$J(\mathrm{DOC}) = \int_{t_0}^{t_f} \left[FF(x,u) + \mathrm{CI} \right] dt \tag{4}$$

where FF is fuel flow. t_0 and t_f are the initial and terminal time, respectively. The CI , abbreviation of Cost Index, is the ratio of time-related cost and the cost of fuel. When the CI is equal to zero, it generates the fuel optimal trajectory, in the contrast, as the CI is increasing, time performance index tends to be increasing [21].

3 Multi-Step Flight Planning Optimization

The trajectory optimization involves both continuous and discrete system: aircraft dynamics and waypoints. Instead of solving mixed-integer optimization, we propose a multi-step optimization approach as follows:

Step1) Calculating coarsely optimized weights between each vertex of the graph by using a global pseudospectral method.
Step2) Finding a set of waypoints aircraft must traverse by using a shortest path algorithm.
Step3) Refining the coarsely optimized trajectory in step1) and step2) by using a multi-phase pseudospectral method.

In this section, detailed methods of each step are presented. Then, the multi-step approach is validated by analyzing convergence rate and computational expense.

3.1 Radau Pseudospectral Method

We use a Radau Pseudospectral Method (RPM) for step 1) and step 3), while each purpose is different. For step 1), a global RPM is applied with a small number of collocation points for computational efficiency. Then the trajectory is refined by a multi-phase RPM in step 3).

The general multi-phase optimal control problems are formulated in the Bolza form as follows. Given a set of P phase (where $p \in [1,...,P]$, if $p=1$, the problem is a single phase optimal control problem), minimize the cost functional [22].

$$J = \Phi(\mathbf{x}^{(1)}(t_0), t_0^{(1)}, \mathbf{x}^{(P)}(t_f), t_f^{(P)}) + \sum_{p=1}^{P} \left[\int_{t_0^{(p)}}^{t_f^{(p)}} \mathcal{L}^{(p)}(\mathbf{x}^{(p)}(t), \mathbf{u}^{(p)}(t), t) dt \right] \quad (5)$$

subject to the constraints

$$\begin{aligned} &\frac{d\mathbf{x}^{(p)}}{dt} = \mathbf{f}^{(p)}(\mathbf{x}^{(p)}, \mathbf{u}^{(p)}), && t \in [t_o, t_f] \\ &\mathbf{C}^{(p)}(\mathbf{x}^{(p)}(t), \mathbf{u}^{(p)}(t)) \le 0, && t \in [t_o, t_f] \quad (6) \\ &\varphi^{(p)}(\mathbf{x}^{(p)}(t_0), t_0, \mathbf{x}^{(p)}(t_f), t_f^{(p)}) = 0 \end{aligned}$$

and the linkage constraints which are connectin g between phase. $\mathbf{x}^{(p)}(t) \in \mathbb{R}^{n_x^{(p)}}$, $\mathbf{u}^{(p)}(t) \in \mathbb{R}^{n_u^{(p)}}$, and $t \in \mathbb{R}$ are, respectively, the state, control, an d time in phase $p \in [1,...,P]$.

In the RPM, the infinite-dimensional optimal control problem is converted into a finite-dimensional NLP problem [23] using the Lagrange polynomial approximation at a set of discrete Legendre Gauss Radau (LGR) collocation points. In each phase, the state can be approximated by the following polynomial:

$$\mathbf{x}^{(p)}(\tau) \approx \mathbf{X}^{(p)}(\tau) = \sum_{j=0}^{N_p} \mathbf{X}_j^{(p)} \mathbf{L}_j^{(p)}(\tau) \quad (7)$$

where $\tau \in [-1,+1]$ is the transformed domain via the affine transformation, and $\mathbf{L}_j^{(p)}(\tau)$ are defined as follows:

$$\mathbf{L}_j^{(p)}(\tau) = \prod_{i=0, i \ne j}^{N_p} \frac{\tau - \tau_i^{(p)}}{\tau_j^{(p)} - \tau_i^{(p)}}, \qquad j = 0,1,\ldots,N_p \quad (8)$$

where $\tau_1^{(p)},\ldots,\tau_{N_p}^{(p)}$ are the LGR collocation points in the p^{th} mesh interval and the final point $\tau_{N_p}^{(p)}$ is a non-collocation point. The control can be also approximated similarly with the state.

The Lagrange polynomials Eq.(8) have the property:

$$\mathbf{L}_j^{(p)}\left(\tau_i^{(p)}\right) = \begin{cases} 1, & i = j \\ 0, & i \ne j \end{cases} \quad (9)$$

Differentiating the expression in Eq.(7) with respect to τ produces:

$$\begin{aligned} \frac{d\mathbf{X}^{(p)}(\tau)}{d\tau} &\equiv \dot{\mathbf{X}}^{(p)}(\tau) = \sum_{j=0}^{N_p} \mathbf{X}_j^{(p)} \dot{\mathbf{L}}_j^{(p)}(\tau) \\ &= \sum_{j=0}^{N_p} \mathbf{X}_j^{(p)} \mathbf{D}_{ij}^{(p)}(\tau) \end{aligned} \quad (10)$$

$$\mathbf{D}_{ij}^{(p)} = \dot{\mathbf{L}}_j^{(p)}(\tau_i^{(p)}), \quad i = 1,\ldots,N_p, \quad j = 0,1,\ldots,N_p \quad (11)$$

The dynamic constraints that are transcribed into algebraic constraints via the differential approximation matrix are as follows:

$$\sum_{j=0}^{N_p} \mathbf{D}_{ij}^{(p)} \mathbf{X}_j^{(k)} = \mathbf{f}(\mathbf{X}_i^{(p)}, \mathbf{U}_i^{(p)}, \tau_i^{(k)}; t_{p-1}, t_p), \quad i = 1,\ldots,N_p \quad (12)$$

Furthermore, the inequality path constraints can be evaluated at the N_p LGR points in each mesh interval as follows:

$$\mathbf{C}(\mathbf{X}_i^{(p)}, \mathbf{U}_i^{(p)}, \tau_i^{(p)}; t_{k-1}, t_k) \le 0, \quad i = 1,\ldots,N_k \quad (13)$$

Finally, the boundary condition can be

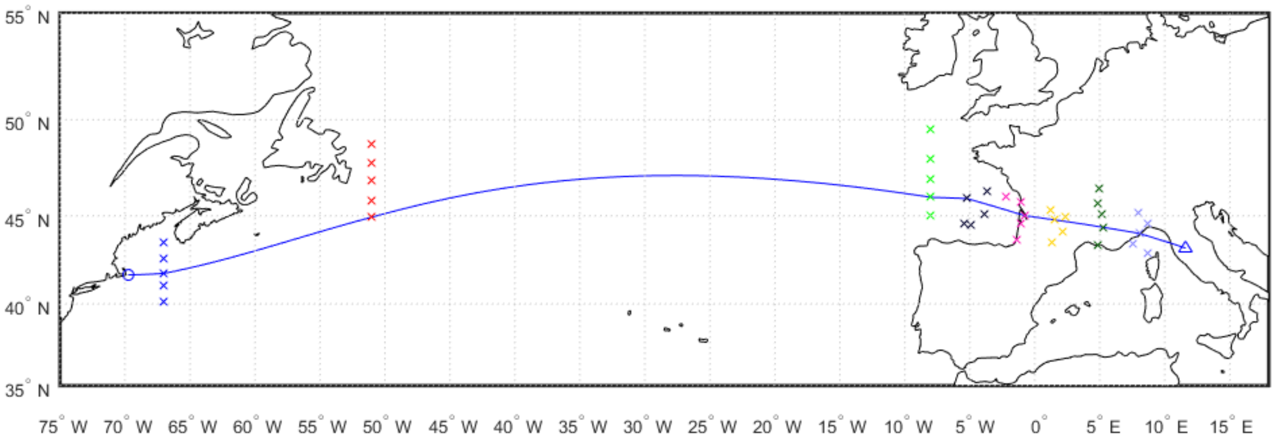

Fig. 2. A set of 8×5 waypoints (blue line is an optimal sequence of waypoints)

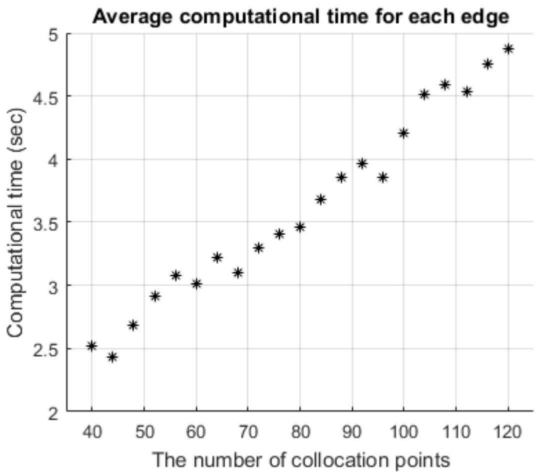

Fig. 3. Computational time comparison as the number of collocation points

rewritten at the N_k LGR points in each mesh intervals as:

$$\varphi(\mathbf{X}_0^{(1)}, t_0, \mathbf{X}_{N_p}^{(P)}, t_P) = 0 \tag{14}$$

The continuous-time cost functional can be constituted by the multi-interval at LGR points, resulting in:

$$J \cong \Phi(\mathbf{X}_0^{(1)}, t_0^{(1)}, \mathbf{X}_{N_p}^{(P)}, t_f^{(P)}) + \sum_{p=1}^{P} \sum_{j=0}^{N_p} \omega_j^{(p)} \mathbf{L}^{(p)}(\mathbf{X}_j^{(p)}, \mathbf{U}_j^{(p)}, \tau_j^{(p)}, t_{p-1}, t_p) \tag{15}$$

where $\omega_j^{(p)}$ represent the LGR weights. The NLP problem that arises from the RPM is then used to minimize the objective functional of Eq.(15) subject to the algebraic constraints of Eqs.(10)-(14). Then the existing nonlinear programming solvers can be applied to the NLP problem. More details are in [23].

$WP_{n,m}$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$n=1$	(44.11°, -67.00°)	(40.11°, -67.00°)	(41.78°, -67.00°)	(42.63°, -67.00°)	(43.56°, -67.00°)
$n=2$	(44.93°, 51.00°)	(45.83°, -51.00°)	(46.87°, -51.00°)	(47.82°, -51.00°)	(48.76°, -51.00°)
$n=3$	(45.00°, -8.00°)	(46.00°, -8.00°)	(47.00°, -8.00°)	(48.00°, -8.00°)	(49.50°, -8.00°)
$n=4$	(44.50°, -4.94°)	(44.61°, -5.40°)	(45.08°, -3.86°)	(45.93°, -5.22°)	(46.32°, -3.69°)
$n=5$	(43.69°, -1.41°)	(44.55°, -1.12°)	(45.01°, -0.78°)	(45.73°, -1.06°)	(46.05°, -2.25°)
$n=6$	(43.54°, 1.36°)	(44.12°, 2.16°)	(44.78°, 1.47°)	(44.95°, 2.36°)	(45.33°, 1.23°)
$n=7$	(43.38°, 4.84°)	(44.37°, 5.26°)	(45.10°, 5.16°)	(45.66°, 4.89°)	(46.50°, 4.95°)
$n=8$	(42.89°, 8.67°)	(43.45°, 7.59°)	(44.03°, 8.03°)	(44.59°, 8.66°)	(45.15°, 7.99°)
- $WP_{initial}$ = (41.69°, -69.74°), WP_{final} = (43.21°, 11.60°)					

Table 1 A set of 8×5 waypoints

3.2 Dijkstra Algorithm

Dijkstra algorithm is used as a shortest path algorithm for step 2). The algorithm can solve the problem which is a static weighted graph $\mathcal{G}(\mathcal{V}, \mathcal{A})$, given a starts vertex to a final vertex in polynomial time. Each waypoint is considered as vertices (or nodes) and edges (or arcs) represent possible transitions between the vertices [24].

Most of the shortest path algorithms adopt distance based weights, therefore, using the weights are highly limited and difficult to handle complex and nonlinear system. In the contrast, as aforementioned in Sec.3.1, using the weights calculated by the global RPM have advantages of usage of various performance indices, although the approach has weakness of high computational load compared with distance based weights.

3.3 Analysis

Compared with using mixed-integer optimization, approximation in the multi-step approach is expected to reduce the computational load. It is shown in Fig. 3 that the computational time increases with the number of collocation points.

While having computational efficiency, the error between the coarsely optimized and the exactly optimal trajectory occurs due to the approximation. However, the error is proved to be bounded with convergence rate [25]. Assuming coercivity and smoothness condition, a convergence rate for the global RPM exponentially increases as the number of collocation points increases as follows:

$$R \le cN^{2-\eta} \tag{16}$$

where R is the maximum error between the approximated and the exact trajectory, c is a constant independent of N, and η (≥ 3)is the degree of differentiability. More details about the convergence rate is in [25].

The computational time and the convergence rate conflict each other. Considering their trade-off, we can decide the number of collocation points.

	Initial condition	Terminal condition
r(m)	11582.4	11582.4
θ(deg)	$\mathrm{WP}_{n,m}$	$\mathrm{WP}_{n',m'}$
λ(deg)		
V(m/s)	Free	Free
γ(deg)	-	-
ψ(deg)	Free	Free
m(kg)	77,000	Free
C_L(−)	Free	Free
ϕ(deg)	Free	Free
T(N)	Free	Free

Table 2 Initial and terminal conditions of each edge between $\mathrm{WP}_{n,m}$ and $\mathrm{WP}_{n',m'}$ for calculating weights

4 Case Study

In this section, we apply the multi-step optimization approach into a realistic scenario from [14]. The problem of an A320 aircraft flying the en route phase from New York to Rome with fixed altitude is presented. A set of 8×5 waypoints is selected from the AIRAC cycle published in June 2012. The wind forecast of 21 May 2016 has been used. The en route phase is divided into nine phases including the initial and final waypoint based on AIP, and each partite set includes five waypoints as shown in Fig. 2 and each waypoint position is given in Table 1.

4.1 Flight Planning

Given the scenario, we describe the flight planning optimization according to the multi-

	Initial condition	Middle waypoints conditions	Terminal condition
r(m)	11582.4	11582.4	11582.4
θ(deg)	$\mathrm{WP}_{\mathrm{Initial}}$	$\mathrm{WP}_{1,m1},\cdots,\mathrm{WP}_{8,m8}$	$\mathrm{WP}_{\mathrm{Terminal}}$
λ(deg)			
V(m/s)	Free	Free	Free
γ(deg)	-	-	-
ψ(deg)	Free	Free	Free
m(kg)	77,000	Free	Free
C_L(−)	Free	Free	Free
ϕ(deg)	Free	Free	Free
T(N)	Free	Free	Free

Table 3 Initial and terminal conditions for refining the trajectory

$\mathrm{WP}_{n,m}$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$n=1$			•		
$n=2$	•				
$n=3$		•			
$n=4$				•	
$n=5$			•		
$n=6$			•		
$n=7$		•			
$n=8$			•		

Table 4 Optimal sequence of waypoints

step optimization described in Sec.3.

Step1) The weights between each vertex of the graph are calculated by using the global RPM. In Table 2, the initial and terminal conditions of each edge between $\mathrm{WP}_{n,m}$ and $\mathrm{WP}_{n',m'}$ are described, and the performance index is the terminal time. The number of collocation points is 40 for calculating weights on each edge. The computation time for calculating all the weights is 465.7052 seconds on a Windows 7 OS 3.40 GHz desktop computer with 16 GB RAM.

Step2) Dijkstra algorithm is applied based on the weights calculated in step1). The shortest path result is blue line in Fig. 2 and the optimal sequence of waypoints are indicated in Table 4. The performance index is 20,815 seconds which is the total flight time from the initial waypoint to the terminal waypoint. The computation time for finding the shortest path is less than 0.1 seconds on the same environment.

Step3) Now we have the set of waypoints the aircraft must traverse. Using the set of waypoints, multi-phase hp-adaptive RPM is used to refine the coarsely optimized trajectory. The performance index is same with step1). In Table 4, equality constraints of each waypoint are described. The result is depicted in Fig. 2. The computation time to optimize the multi-phase trajectory optimization is 210.3992 seconds on the same environment. The total computational time is 676.2044 seconds to optimize the flight planning for the case.

4.2 Result and Discussion

The optimal sequence of waypoints is depicted Fig. 2 and corresponding states and controls are

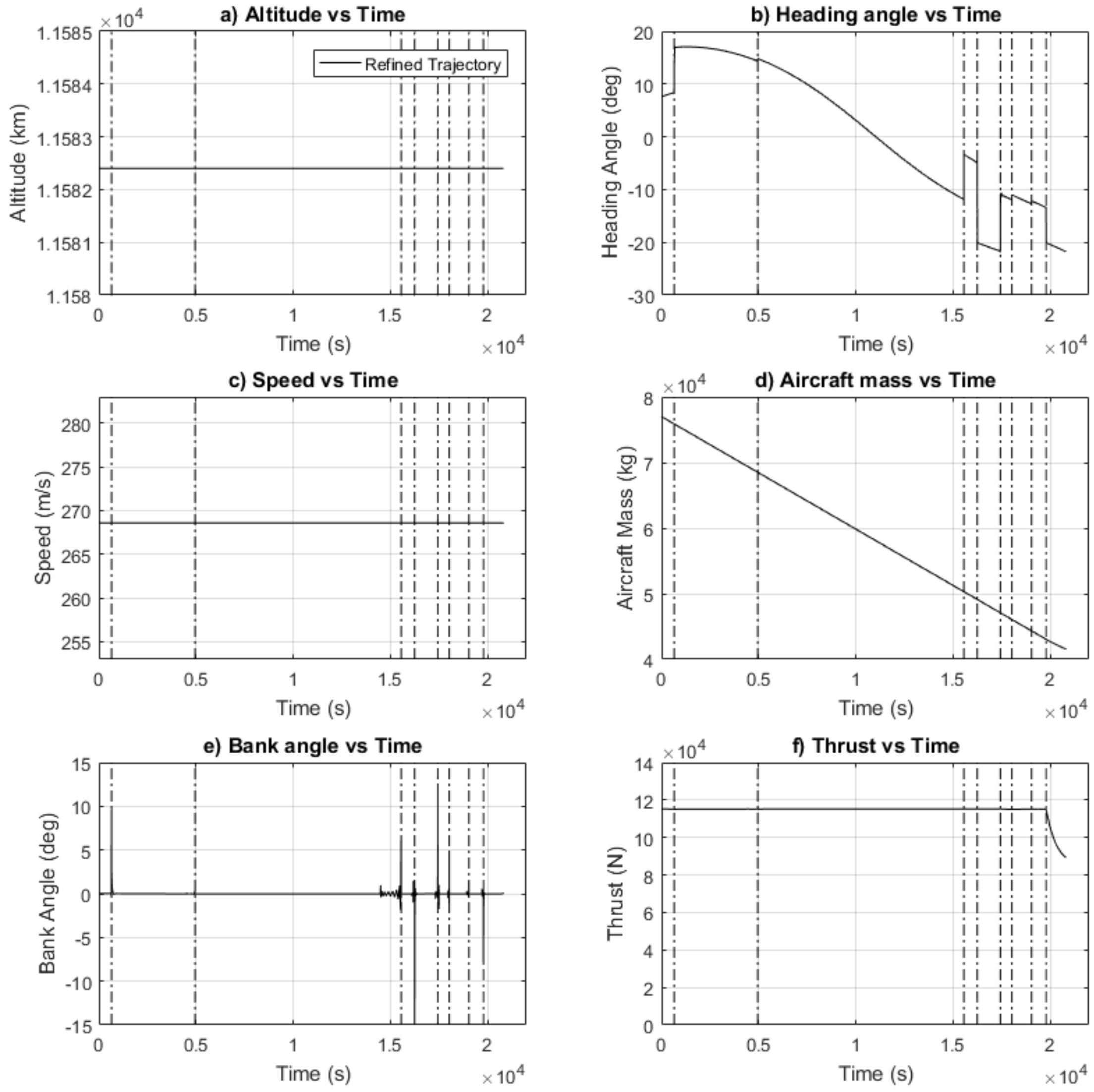

Fig. 4. State and control variables of the refined trajectory

in Fig. 4. In Fig.1, ○, △, and × are the initial waypoint, the terminal waypoint, and 8 partite sets of waypoints, respectively. In Fig. 4, the vertical dash-dotted lines are waypoint transit time.

The altitude of the aircraft is constant, as we consider horizontal model of the en route phase in the scenario. The speed trajectory is constant during the whole phase due to the performance index of the scenario. The heading angle and bank angle are maneuvering when passing through the waypoints.

Flight planning is done in offline as a part of strategic and/or tactical planning. Once the flight plan is submitted, the plan will not change during the flight. However, there are possibilities that the aircraft cannot traverse the routes, such as, sudden deteriorating weather conditions. In such a case, flight route modifications are carried out by negotiation between ATCs and pilots without mathematical optimization. Thus, reducing the computational load is a key factor of a mathematical based flight planning to modify the routing during the flight. This is corresponding to the proposed approach. The computational efficiency of the multi-step approach is better than mixed-integer optimization. In addition, the approach can be adopted to modify the routes in case of unexpected situations including adverse weather. The problem formulation to modify the routes will be easier and faster than the initial flight planning due to the reduced waypoints.

5 Conclusion and Future Work

In this paper, we propose a multi-step approach to optimize the flight planning problem using both the RPMs and Dijkstra algorithm. The efficiency and the validation of the approach are shown in this paper by planning the realistic flight scenario and by analyzing the convergence rate of the RPM. In addition, the approach has a great potentiality in flight planning modifications during the flight.

As a future work, it is recommended to consider emission models, and a 3-dimensional model instead of the horizontal model. An online flight planning modification framework will be considered. Finally, to the best of our belief, there is no practical approach to find an optimal trajectory for flight planning due to the complexity and nonlinearity of the problem. Therefore, we need to find methods for assessment of the proposed approach.

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