30th Congress of the International Council of the Aeronautical Sciences
DCC, Daejeon, Korea ; September 25-30, 2016

EVOLUTIONARY GAME BASED MULTI-OBJECTIVE OPTIMIZATION FOR MULTI-AGENT NETWORK RESILIENCE

Hae-In Lee*, Hyo-Sang Shin*, Antonios Tsourdos*
***Institute of Aerospace Sciences, SATM, Cranfield University,**
College Road, Cranfield, Bedfordshire, MK43 0AL, United Kingdom

Keywords: *Multi-Objective Optimization, Network Resilience, Core-Periphery Structure, Evolutionary Game Theory, Weighted Sum Method*

Abstract

This research presents multi-objective optimization for network resilience. The scale free core-periphery structure is parameterized to formulate the network properties as objective functions. Optimizing the conflicting network properties, an evolutionary game based approach is used to find the weightings of the weighted sum method. Numerical results show the optimal weightings and network structures depending on the size of network.

1 Introduction

Modern aerospace technologies often incorporate complex structures of networks. The more complicated and intensive the dependency on network connection is, the more the functionality suffers from node failure or communication malfunction. Furthermore, wireless communication networks are vulnerable to the cyber-physical failure. It is an important issue to model the failing vertices as a percolation process and to design a resilient topology in autonomous operations.

There have been several attempts to find robust and efficient networks. Motter et al. [1] have addressed the conflict between robustness and synchronizability, and other properties including global and local efficiency have been detailed later [2]. As an option to improve the overall network properties, Peixoto et al. [3] propose the core-periphery topology, but the detailed topology and consideration of conflicting network properties remain illusive.

This research aims to find the resilient network topology by formulating the problem into multi-objective optimization, where the conflicting properties of network are objective functions and the topology of network is a decision variable. As the full topology is determined by a large number of variables, we propose a single parameter design method assuming scale free core-periphery network.

Solving the formulated multi-objective optimization problem, this paper suggests to use the evolutionary game based multi-objective optimization method [4]. It is based on the weighted-sum method, while the weightings are determined by an evolutionary game. The main advantage is the consideration of solution's survivability in the other criteria without any expert decision or use of aggregation coefficient while providing low computational load.

The main contribution thus lies in suggesting a new perspective on formulating the resilient network and applying a suitable multi-objective optimization method, which has not been applied comprehensively.

This paper is composed as follows: the first part summarizes the definitions and characteristics of scale free core-periphery topology. Then finding the optimal network structure is formulated into multi-objective optimization problem. The evolutionary game based method is explained in section 4, and examined through numerical simulations in section 5. Finally, conclusions and future works are addressed.

2 Scale Free Core-Periphery Network

2.1 Definition

The core-periphery network with n nodes is defined with core nodes and periphery nodes [5]. Defining the fraction of core nodes as f_{core}, core nodes of the number nf_{core} are connected with each other, and peripheral nodes of the rest are connected randomly either to the core or to the periphery.

The number of edges connected to each node is decided by the total degree k. The scale free network follows the power-law distribution, defined with the probability and cumulative density function, $p(k)$ and $P(K)$ respectively:

$$p(k) = \frac{k^{-\gamma}}{\zeta(\gamma;k_{\min}) - \zeta(\gamma;n)} \qquad P(K) = \frac{\zeta(\gamma;k_{\min}) - \zeta(\gamma;K+1)}{\zeta(\gamma;k_{\min}) - \zeta(\gamma;n)} \tag{1}$$

$$\zeta(\gamma;a) \triangleq \sum_{k=a}^{\infty} k^{-\gamma} \simeq \frac{a^{-\gamma+1}}{\gamma-1} \tag{2}$$

where γ is an exponent in the power-law distribution, and $\zeta(\gamma;a)$ is the Hurwitz zeta function for normalization. The long-tail effect of the probability distribution is eliminated by constraining the degree $k_{\min} \le k \le n-1$ [6].

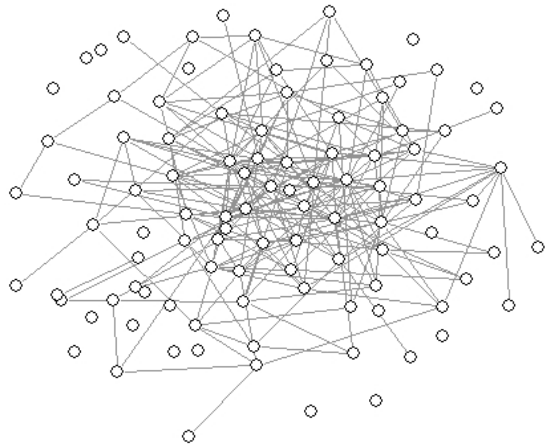
The structure of scale free core-periphery network is thus determined by three parameters: f_{core}, $k_{\min}$ and γ. By the definition of core-periphery nodes, the probability of $k \ge nf_{core} - 1$ is f_{core}, which means the cumulative density function satisfies

$$\frac{\zeta(\gamma;k_{\min}) - \zeta(\gamma;nf_{core})}{\zeta(\gamma;k_{\min}) - \zeta(\gamma;n)} = 1 - f_{core} \tag{3}$$

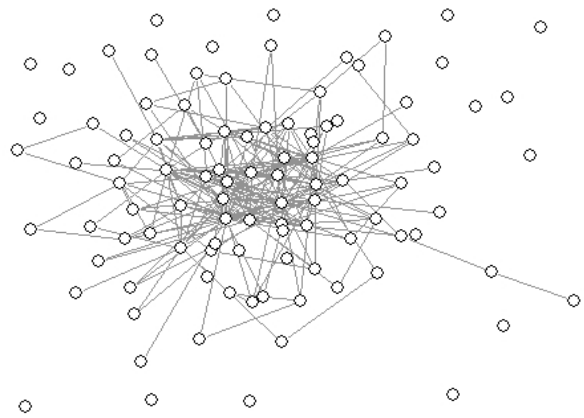
Also, the average degree is fixed for that the unlimited degree enhances all the theoretical properties. The average degree can be computed as

$$\langle k \rangle = \frac{\zeta(\gamma+1;k_{\min}) - \zeta(\gamma+1;n)}{\zeta(\gamma;k_{\min}) - \zeta(\gamma;n)} \tag{4}$$

where <·> denotes the average value. From the definition of the Hurwitz zeta function, existence of the average degree requires $\gamma > 2$.



(a) Homogeneous Network



(b) Heterogeneous Network

Fig. 1. Visualization of Scale Free Networks

Using Eq. (3) and (4), f_{core} and $k_{\min}$ are computed from γ. Therefore, this paper simplifies the design of network into a single-parameter problem with γ. Increase in γ results in large $k_{\min}$, and the network has homogeneous degree over the nodes. On the contrary, decrease in γ yields a heterogeneous network with highly concentrated nodes. The number of core nodes remains similar, but the number of periphery nodes connected directly to the core is larger in heterogeneous networks than in the homogeneous. Both homogeneous and heterogeneous networks are visualized in Fig. 1 using the Pajek program [7]. While $\langle k \rangle$ is fixed to 3, Fig. 1 (a) is plotted with $\gamma = 5$ and (b) with $\gamma = 3$. Although the size of network is large – a hundred – the visualization clearly shows homogeneity and heterogeneity respectively.

2.2 Properties

The network properties are classified into three main categories – robustness, efficiency and synchronizability [1], [2] – which are derived from the percolation process.

The percolation process is crucial in understanding the network properties, especially robustness. The percolation is a phenomenon where some fraction of nodes and edges are removed. The analysis on the percolation phenomenon in the generated network enables to model the cyber-physical failure of the autonomous system and to compute the remaining network functionality. From this analysis, the size of the largest cluster S with respect to the occupational probability ϕ is obtained, where the cluster is defined with the groups of nodes connected to each other and the occupational probability is the portion of remaining nodes [6].

Robustness is the ability of a network to maintain its function under the presence of failure by taking a detour or multi-hop communication. The value is determined by the area of $S-\phi$ plot below the critical size of the largest cluster, S_c.

$$J_1 = \int_0^{S_c} \phi \, dS \tag{5}$$

Efficiency is the measure of how the nodes communicate their information each other. The global efficiency is proportional to the closeness centrality, which is the average length of a geodesic path d_{ij} as,

$$J_2 = \left(\left\langle d_{ij} \right\rangle\right)^{-1} \tag{6}$$

where i and j are the indices of nodes.

Synchronizability depends on the speed of diffusion along the connections, which is determined by the eigenvalues of the Laplacian matrix L,

$$J_3 = \frac{\lambda_n}{\lambda_2}, \qquad \frac{d\psi}{dt} + cL\psi = 0 \tag{7}$$

where ψ is the network state, c is the diffusion rate constant, λ_n is the maximum eigenvalue, and λ_2 is the minimum non-zero eigenvalue.

3 Multi-Objective Optimization Problem Formulations

To find the optimal network structure, an optimization problem is formulated with its objective functions set as the network properties. As the design of network depends on a single parameter, the multi-objective optimization problem is formulated as

$$\begin{aligned} &\max_{\gamma} \quad J_1, J_2, J_3 \\ &\text{subject to } \gamma > 2 \end{aligned} \tag{8}$$

The objective functions may conflict with each other; enhancement in one objective deteriorates at least one of the rests. The weighted sum method aggregates multiple weighted objectives into a single cost function as

$$\begin{aligned} &\max_{\gamma} \quad J = w_1 J_1 + w_2 J_2 + w_3 J_3 \\ &\text{subject to } \gamma > 2 \end{aligned} \tag{9}$$

This method is the most widely used thanks to its simplicity [8], but difficulties arise when determining the weightings, which are resolved in this paper by using the evolutionary game theory.

4 Evolutionary Game Based Multi-Objective Optimization

4.1 Payoff Matrix

The core concept of the evolutionary game based multi-objective optimization method is that the optimization problem is a type of non-cooperative game. Among the conflicting objectives, improving an objective deteriorates at least one of the others. One needs to consider both gain and loss of choosing which objective to be optimized. Optimizing the ratios of gain to loss, called tradeoffs, it is expected that more weighting is applied to the objectives that are sensitive to the choice of which objective to be optimized. It is compared to the concept of equilibrium in the game theory.

Finding the equilibrium in a non-cooperative game requires analyzing the players' utility to formulate a payoff matrix [9]. The decision variables and cost functions act as players, and which objectives to be optimized are the possible strategies. The weightings of a multi-objective optimization problem correspond to the Nash equilibrium of the mixed strategies. Therefore, the payoff matrix is composed such that objective functions comprise the rows and optimal decision variables with respect to each criterion are substituted to the columns. For the problem formulation in Eq. (9), the payoff matrix A is composed as

$$A = \begin{pmatrix} J_1(\gamma_1^*) & J_1(\gamma_2^*) & J_1(\gamma_3^*) \\ J_2(\gamma_1^*) & J_2(\gamma_2^*) & J_2(\gamma_3^*) \\ J_3(\gamma_1^*) & J_3(\gamma_2^*) & J_3(\gamma_3^*) \end{pmatrix} \quad (10)$$

where γ_i^* is the optimal γ for the i-th objective.

The normalization method and the form of cost function affect the characteristics of the payoff matrix. Different scales of each cost function have an effect of varying absolute and relative importance, and thus the cost functions are normalized in each step. The normalized payoff matrix $\bar{A}$ is composed as

$$\bar{A}_{ij} = \frac{J_i(\gamma_j^*) - J_i^-}{J_i^+ - J_i^-} \quad (11)$$

where J_i^+ and J_i^- are the maximum and minimum value of i-th objective respectively.

4.2 Replicator Equation

Using the payoff matrix, the fitness of mixed strategies p_i evolves in each time step through the replicator equation,

$$\dot{p}_i = p_i(e_i A p^T - p A p^T) \quad (12)$$

where $e_i \in \mathbb{R}_{1\times 3}$ is a vector with one at the i-th element and zeros at the other.

The evolutionary stable solution $\bar{p}$ is computed with an augmented matrix as

$$\begin{pmatrix} \bar{p}^T \\ a \end{pmatrix} = \begin{pmatrix} A & -\mathbf{1}_{3\times 1} \\ -\mathbf{1}_{1\times 3} & 0 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0}_{3\times 1} \\ 1 \end{pmatrix} \quad (13)$$

where a is a constant.

A single solution exists when the augmented matrix is invertible. If the problem is singular, infinitely many solutions exist and this paper uses the average solution. The stability of the dynamics is determined by the eigenvalues of the payoff matrix, and guarantee of stability is easily shown in the multi-objective optimization problems [10].

5 Numerical Results

5.1 Simulation Settings

The algorithm of the proposed approach is summarized in Fig. 2. Given the average degree $\langle k \rangle$, objective functions for the optimization problem are derived. Then, optimal degree exponent γ is obtained using the evolutionary game based multi-objective optimization (MOO) method. The optimal γ leads to the optimal structure of the network.

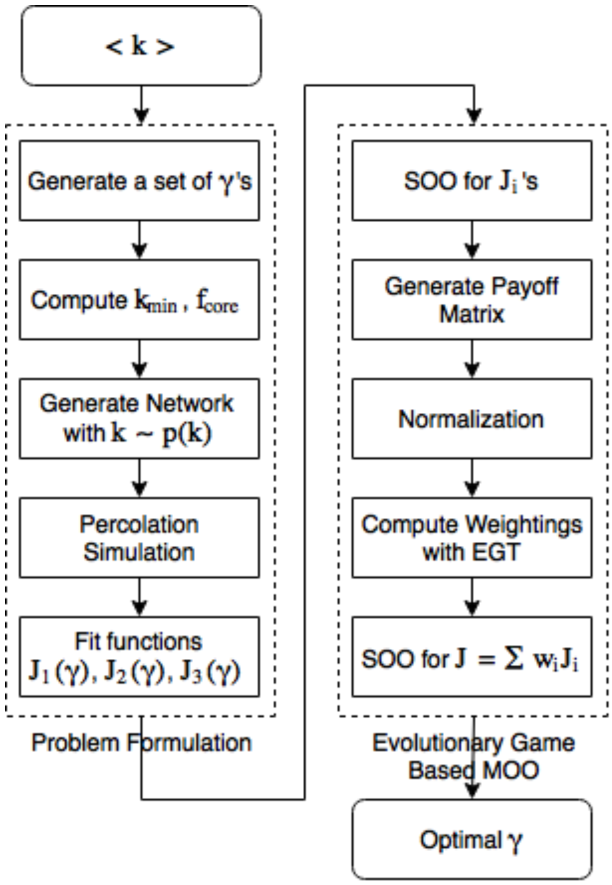


Fig. 2. Algorithm of the Proposed Approach

Table 1. Simulation Parameter Settings

Simulation Parameter		Value
The number of nodes	n	100
Upper bound of exponent	$\gamma_{\max}$	5
Initial average degree	$\langle k \rangle_0$	30
Change in average degree	$\Delta\langle k \rangle$	3

Numerical simulation is conducted with different $\langle k \rangle$'s. Physical limit of multi-agent network such as transmission power and coverage decides $\langle k \rangle$; for instance, given the fixed transmission power, spread of the networked vehicles reduces $\langle k \rangle$. Exploiting the advantage of the propose approach that the weightings are determined dynamically, the simulation is composed with reducing $\langle k \rangle$ as

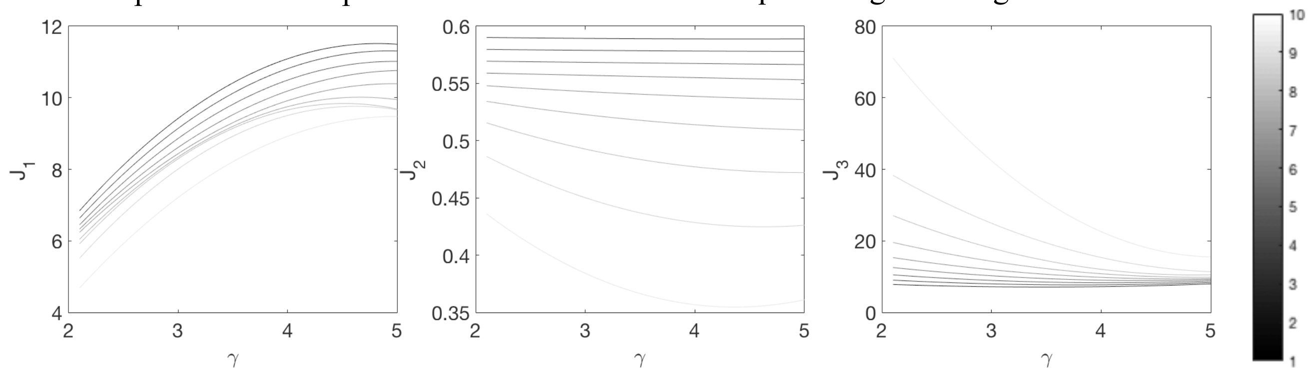
$$\langle k \rangle = \langle k \rangle_0 - \Delta\langle k \rangle t \qquad (14)$$

where t is the simulation time. The values of simulation parameters are specified in Table 1.

The objective functions are obtained with Monte-Carlo simulations with 10 runs generating the different networks and same degree distribution, and then are fitted into second-order polynomial. In the multi-objective optimization part, each single objective optimization is conducted using fmincon from MATLAB. The evolutionary stable solutions are evolved 100 times at each run.

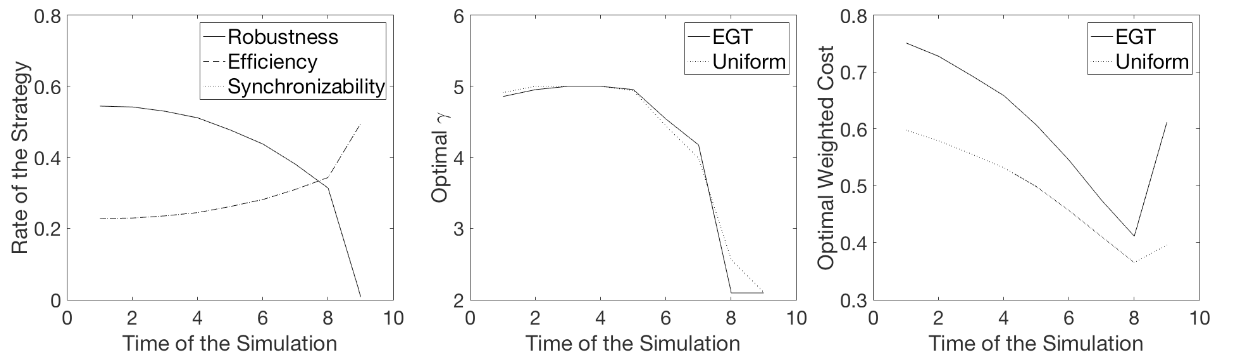
5.2 Simulation Results

The objective functions with respect to γ are shown in Fig. 3. Three figures show different network properties – robustness, efficiency, and synchronizability. The brightness of the lines indicates the simulation time. Whereas the change in robustness stays similar throughout the simulation, efficiency and synchronizability varies in their minimum and maximum values. It can be concluded that efficiency and synchronizability are more sensitive to network structure when the transmission power is small or requires large coverage area.



(a) Robustness (b) Efficiency (c) Synchronizability

Fig. 3. Results of Problem Formulation



(a) EGT based Weightings (b) Optimal Network Structure (c) Optimal Weighted Cost

Fig. 4. Results of Multi-Objective Optimization

Using the obtained objective functions, the evolutionary game based multi-objective optimization method is implemented. The resultant optimal weightings, decision variable, and cost are shown in Fig. 4. In Fig. 4 (a), the weightings on robustness decrease while efficiency and synchronizability increases almost simultaneously with the decrease of $\langle k \rangle$. This corresponds to the fact that efficiency and synchronizability are more sensitive in the later part of simulations. Increase of weightings on sensitive cost functions implies that the evolutionary game base approach is successfully considering the tradeoffs. In Fig. 4 (b), the optimal network structure is implied by optimal γ. It is more advantageous to formulate a homogenous network when the transmission power is not enough or desired coverage is broad, while a heterogenous network is better in the other case.

For validating the performance of evolutionary game based approach, the optimization result using uniform weightings, one third on each objective function, is compared. Fig. 4 (b) shows that similar optimal network structures are obtained in each method, but Fig. 4 (c) shows that the evolutionary game based approach results in the increase of maximum weighted cost.

6 Conclusions

In this paper, design of scale free core-periphery networks is formulated into a single parameter problem, and the network properties are optimized using the evolutionary game based multi-objective optimization method. Numerical simulation is conducted with different average degrees, which are relevant to the size of network by the transmission power or coverage. The results suggest that a homogeneous network is advantageous in a concentrated network with either sufficient transmission power or narrow coverage, whereas a heterogeneous network is of more importance in the other case. The parameter of the network structure is given by the optimal exponent of the degree distribution. This research is expected to suggest a guide for designing a topology in the various fields including multi-agent or sensor network design.

Further studies are focused on mobile network system. Most of the previous works on mobile networks [11], [12] have been dedicated to maximize connectivity, which is similar to synchronizability, not considering the other properties. The results often result in a rendezvous formation without a constraint on coverage. We expect that controlling the multiple agents to form an optimal degree distribution from this research may suggest more robust, efficient and fast synchronizing network.

References

[1] A. E. Motter, C. Zhou, and J. Kurths, "Network synchronization, diffusion, and the paradox of heterogeneity," *Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys.*, vol. 71, no. 1, pp. 1–10, 2005.

[2] B. Wang, H. Tang, C. Guo, Z. Xiu, and T. Zhou, "Optimization of network structure to random failures," *Phys. A Stat. Mech. its Appl.*, vol. 368, no. 2, pp. 607–614, 2006.

[3] T. P. Peixoto and S. Bornholdt, "Evolution of robust network topologies: Emergence of central backbones," *Phys. Rev. Lett.*, vol. 109, no. 11, pp. 1–5, 2012.

[4] C. Leboucher, H. S. Shin, S. Le Menec, A. Tsourdos, A. Kotenkoff, P. Siarry, and R. Chelouah, "Novel Evolutionary Game Based Multi-Objective Optimisation for Dynamic Weapon Target Assignment," *19th World Congr. Int. Fed. Autom. Control*, no. 2010, pp. 3936–3941, 2014.

[5] P. Holme, "Core-periphery organization of complex networks," *Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys.*, vol. 72, no. 4, 2005.

[6] M. E. J. Newman, "Networks. An introduction," *Oxford Univ. Press*, p. 772, 2010.

[7] V. Batagelj and A. Mrvar, "Pajek – program for large network analysis," *Connections*, pp. 47–57, 1999.

[8] K. Deb, "Classical Methods," in *Multi-objective optimization using evolutionary algorithms*, John Wiley & Sons, LTD., 2011, pp. 47–75.

[9] M. Kok, "A Note on the Pay-off Matrix in Multiple Objective Programming," *Eur. J. Oper. Res.*, vol. 26, no. 1, pp. 96–107, 1986.

[10] J. Hofbauer and K. Sigmund, "Evolutionary Game Dynamics," *Bull. Anerican Math. Soc.*, vol. 40, no. 4, pp. 479–519, 2003.

[11] M. M. Zavlanos and G. J. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 812–816, 2007.

[12] M. M. Zavlanos and G. J. Pappas, "Distributed connectivity control of mobile networks," *Proc. IEEE Conf. Decis. Control*, vol. 24, no. 6, pp. 3591–3596, 2008.

Copyright Statement