# ICA(S) 2016 $30^{\text {min }}$ Congress of the International Council of the Aeronautical Sciences DCC, Dajjoon, Korea ; Seplember 25-30, 2016 <br> DYNAMIC MODELING AND STABILITY ANALYSIS OF A HIGH ALTITUDE AIRSHIP 

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#### Abstract

Use of Lighter-Than-Air (LTAs) vehicles for the telecommunication and surveillance purposes has attained a great deal of interest in recent years. This platform is very attractive because of its long endurance and cost effectiveness. The shape of airship envelope has huge impact on its performance. Literature suggests that shape of the airship should be such that it should have significant maneuvering capabilities and stability in the stratosphere. This paper presents open loop and closed loop stability analysis of Gertler shape of stratospheric airship which has minimum circumferential stresses and minimum drag characteristics. The complete 6-DOF nonlinear mathematical model of Gertler shape has been developed for the analysis of stability. As airship is a buoyant vehicle, added mass effect is taken into account while deriving the equations of motion. Stability analysis is carried out using the linearized model at a desired operating condition using both time domain and frequency domain techniques. Different motions of the airship with its control characteristics is explained at the end of paper. This analysis is to be used to develop Guidance, Navigation and Control (GNC) strategy for the stratospheric airship.


## 1. Introduction

Stratospheric airships are LTA systems emerging as potential replacement for satellites missioned to carry out low earth surveillance and telecommunication exercise. There are several critical issues before such a technology can be realized, foremost among them being 1)
development of materials for retaining lighter than air available gases for longer duration of time, 2) optimization of airship profile (shape and size) for better performance, and 3) development of Guidance Navigation and Control (GNC) capabilities from launch to station-keeping operations. Performance based design of an airship has implications on its stability characteristics which further has bearing on its control characteristics. Analysis of these characteristics requires development of a comprehensive mathematical model of airship including important features related to shape and size parameters as a crucial step.

Shape of airship is maintained by differential pressure between lifting gas and environment. The shape of airship should be such that it should experience minimum hoop stress [1] and minimum drag in atmosphere [2]. Liao et al. [1] noted that each shape has its own advantages and disadvantages depending on airship application. As propulsive efficiency is directly proportional to aerodynamic drag of airship hull, small reduction in drag results in tangible savings in fuel. Therefore, during the aerodynamic design of airship, it is very crucial to arrive at a minimum drag configuration. This aspect of airship design was studied by Rana et al. [3] using aerodynamic model of stratospheric airship. It appears that selection of exact shape for stratospheric airship is still an open problem for researchers. Stability analysis of a comprehensive mathematical model of the stratospheric airship is expected to provide answers to some of the questions above which forms the subject matter of the work reported in this paper.

The mathematical model developed in this paper is based on the work done by Rana et al. [3] and Mueller et al. [4]. Aerodynamics model of the airship used in this analysis is developed in-house [3] in MATLAB ${ }^{\circledR}$ using geometrical aerodynamic parameter method. This computer simulation method of development of aerodynamics model is discussed by Ashraf et al. [5]. Linear model for stability analysis of airship in this work is based on small perturbation theory as outlined in Khoury [6] and Cook [7].

This paper is organised as follows. Section 2 describes baseline design specifications with respect to airship shape. In Section 3 the six dof nonlinear mathematical model as well as the linearized state space model of airship have been developed. It also includes the discussion of linear open loop and closed loop stability analysis along with results. Section 4 represents the various motion of airship in response to controls and section 5 concludes the overall work along with future scope of this work.

## 2. Base Line Design

### 2.1 Design Parameters

The selection of design parameters are based on rigorous study of targeted mission requirements. The basic design parameters of stratospheric airship are listed in table 1. The desired altitude for the stratospheric airship is selected as 21 km where mean wind speed is expected $50 \%$ less than compared to its sea level value. Along with this, there are several other advantages at this altitude which are outlined in [3]. The total calculated mass of the stratospheric airship configuration is $23146(\mathrm{~kg})$ which includes payloads, power management, gases, fins, ballonet, hull, propulsion systems etc. The complete analysis of mass estimation is given in [3] which is developed at flight dynamics lab. The target airship is 217.2 meter in length, 54.3 meter in diameter and total volume of the airship hull is $327160\left(\mathrm{~m}^{3}\right)$. Helium is selected as a lifting gas because of safe operation and fact that it has better lifting capacity next to hydrogen which is flammable. Desired endurance is targeted for at least 6 month as model is designed for surveillance purposes. Flexible solar array techniques will be adopted for power generation
during station keeping phase. Gertler shape is selected based on the analysis given in the next subsection.

## Table 1. Design Parameter

| Design altitude | $21(\mathrm{~km})$ |
| :---: | :---: |
| Nominal velocity | $20(\mathrm{~m} / \mathrm{s})$ |
| Total mass | $23146(\mathrm{~kg})$ |
| Volume | $327160\left(\mathrm{~m}^{3}\right)$ |
| Max. Dia | $54.3(\mathrm{~m})$ |
| Length | $217.2(\mathrm{~m})$ |
| Lifting gas | Helium |
| Endurance | 6 months |
| Fabric density | $300(\mathrm{gsm})$ |
| Shape | Gertler |

### 2.2 Shape Selection

Shape selection of hull can be considered as an apex of airship model design because it directly affects the propulsive efficiency. Shape should be such that it should experience minimum drag in atmosphere. Comparison of various shape with drag values are shown in figure1. Result shows that Gertler shape has minimum drag coefficient compared to other shapes [3].


Fig. 1 Altitude v/s drag for different shapes [3].

## 3. Mathematical Model

### 3.1 Axis Assumption

Axis reference for the airship is defined by right hand orthogonal axis system like aircraft. The difference in axes reference of airship with an aircraft model is that, the airship equations of motion are developed with respect to a body axes reference frame with the origin at the centre of volume (figure 2 ) instead of the centre of gravity.

The centre of volume has been chosen because this center is assumed to be constant during flight unlike the center of gravity. This leads to additional mass and inertia terms in the equations of motion.


Fig. 2 Body axis reference frame [9].

### 3.2 Non-linear Equations of Motion

Basic six dof nonlinear mathematical model is described briefly as main idea of this paper is to carry out the stability analysis of the developed model. The developed mathematical model is written in the airship frame. The orientation of body frame w.r.t Earth frame is obtained through Euler angles. Airship linear velocity is given by $u, v, w$ and angular velocity is given by $p, q, r$. The equation of motion of airship can be represented as,
$\mathbf{M} \underline{\dot{X}}=\underline{\boldsymbol{F}}_{d}(u, v, w, p, q, r)+\underline{\boldsymbol{A}}(u, v, w, p, q, r)+\underline{\boldsymbol{G}}+\underline{\boldsymbol{P}}$
Where $\mathbf{M}$ is a $6 \times 6$ mass matrix contains mass and inertia terms due to added mass or virtual mass effect which is given by following matrix.

M
$=\left(\begin{array}{cccccc}m_{x} & 0 & 0 & 0 & m a_{z}-X_{\dot{q}} & 0 \\ 0 & m_{y} & 0 & -m a_{z}-Y_{\dot{p}} & 0 & m a_{x}-Y_{\dot{r}} \\ 0 & 0 & m_{z} & 0 & -m a_{x}-Z_{\dot{q}} & 0 \\ 0 & -m a_{z}-L_{\dot{v}} & 0 & J_{x} & 0 & 0 \\ m a_{x z}-M_{\dot{u}} & 0 & -m a_{x}-M_{\dot{w}} & 0 & J_{y} & 0 \\ 0 & m a_{x}-N_{\dot{v}} & 0 & -J_{x z} & 0 & J_{z}\end{array}\right)$
Where,
$m_{x}=m-X_{\dot{u}}, \quad m_{y}=m-Y_{\dot{v}}, \quad m_{z}=m-Z_{\dot{w}}$
$J_{x}=I_{x}-L_{\dot{p}}, \quad J_{y}=I_{y}-M_{\dot{q}}, \quad J_{z}=I_{z}-N_{\dot{r}}$
$J_{x z}=I_{x z}+N_{\dot{p}}=I_{x z}+L_{\dot{r}}$
$\underline{\boldsymbol{X}}=[u, v, w, p, q, r, \theta, \phi]^{T}=$ State vector
$\underline{\boldsymbol{F}}_{\boldsymbol{d}}$ is $6 \times 1$ column matrix contains terms associated with translation motion and rotational motion which is given by,
$\underline{\boldsymbol{F}}_{\boldsymbol{d}}=\left[\begin{array}{llllll}f_{1} & f_{2} & f_{3} & f_{4} & f_{5} & f_{6}\end{array}\right]^{T}$
Where dynamics terms are expressed as,
$f_{1}=-m_{z} q w+m_{y} r v+m\left\{a_{x}\left(q^{2}+r^{2}\right)-\right.$ $\left.a_{z} r p\right\}$
$f_{2}=-m_{x} u r+m_{z} p w+m\left\{-a_{x} p q-a_{z} r q\right\}$
$f_{3}=-m_{y} v p+m_{x} q u+m\left\{-a_{x} r p+a_{z}\left[q^{2}+\right.\right.$ $\left.\left.p^{2}\right\}\right]$
$f_{4}=-\left(J_{z}-J_{y}\right) r q+J_{x z} p q+m a_{z}(u r-p w)$
$f_{5}=-\left(J_{x}-J_{z}\right) p r+J_{x z}\left(r^{2}-p^{2}\right)+$ $m\left\{a_{x}(v p-u q)-a_{z}(w q-r v)\right\}$
$f_{6}=-\left(J_{y}-J_{x}\right) p q-J_{x z} q r+m\left\{-a_{x}(u r-\right.$
$w p)\}$
$\boldsymbol{A}$ is $6 \times 1$ column matrix consists of terms associated with aerodynamics force and moments and given by,
$A^{w}=Q S_{r e f}\left[\begin{array}{llllll}-C_{D} & C_{Y} & -C_{L} & C_{l} & C_{m} & C_{n}\end{array}\right]^{T}$
Where $A^{w}$ is in wind axis which can be converted in to body axis using transformation,

$$
\boldsymbol{A}=\left[\begin{array}{ccc}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha
\end{array}\right] A^{w}
$$

$\underline{\boldsymbol{G}}$ is $6 \times 1$ column matrix contains the terms associated with buoyancy and gravitational force and moments which is expressed as,
$\underline{\boldsymbol{G}}=\left[\begin{array}{llllll}g_{1} & g_{2} & g_{3} & g_{4} & g_{5} & g_{6}\end{array}\right]^{T}$
Where,

$$
\begin{aligned}
g_{1}= & -(m g-B) \sin \theta \\
g_{2}= & (m g-B) \sin \phi \cos \theta \\
g_{3}= & (m g-B) \cos \phi \cos \theta \\
g_{4}= & -\left(m g a_{z}+B b_{z}\right) \sin \phi \cos \theta \\
g_{5}= & -\left(m g a_{z}+B b_{z}\right) \sin \theta \\
& -\left(m g a_{x}+B b_{x}\right) \cos \phi \cos \theta \\
g_{6}= & \left(m g a_{x}+B b_{x}\right) \sin \phi \cos \theta
\end{aligned}
$$

$\underline{\boldsymbol{P}}$ is $6 \times 1$ column matrix containing the terms related to the propulsive force and moments and expressed as,
$\underline{\boldsymbol{P}}=\left[\begin{array}{llllll}X_{\text {prop }} & Y_{\text {prop }} & Z_{\text {prop }} & L_{\text {prop }} & M_{\text {prop }} & N_{\text {prop }}\end{array}\right]^{T}(7)$
Where,

$$
\begin{aligned}
X_{\text {prop }}= & T_{s} \cos \mu_{s}+T_{p} \cos \mu_{p} \\
Y_{\text {prop }}= & 0 \\
Z_{\text {prop }}= & -T_{s} \sin \mu_{s}-T_{p} \sin \mu_{p} \\
L_{\text {prop }}= & -\left(T_{s} \sin \mu_{s}-T_{p} \sin \mu_{p}\right) d_{y} \\
M_{\text {prop }}= & T_{p}\left(d_{z} \cos \mu_{p}-d_{x} \sin \mu_{p}\right) \\
& +T_{s}\left(d_{z} \cos \mu_{s}-d_{x} \sin \mu_{s}\right) \\
N_{\text {prop }}= & \left(-T_{s} \cos \mu_{s}+T_{p} \cos \mu_{p}\right) d_{y}
\end{aligned}
$$

The terms related to kinematics equations are given by,
$\dot{\phi}=p+q \sin \phi \tan \theta+r \cos \phi \tan \theta$
$\dot{\theta}=q \cos \phi-r \sin \phi$

### 3.3 Linearized Model

The complete set of nonlinear equations are developed to simulate the behavior of an airship motion at different flight conditions. Equation (1) is linearized about straight and level flight condition with trim point,

$$
[u, v, w, p, q, r, \phi, \theta]=[10 \mathrm{~m} / \mathrm{s}, 0,0,0,0,0,0,0]
$$

This linearized model of six dof non-linear mathematical model is obtained using small perturbation theory outlined in [7]. The linearized equations of motion are decoupled into two different dynamics namely longitudinal dynamics and lateral directional dynamics for the analysis. Resulting state space form can be represented as

$$
\begin{align*}
& \boldsymbol{M} \Delta \underline{\dot{\boldsymbol{X}}}=a \Delta \underline{\boldsymbol{X}}+b \Delta \underline{U}  \tag{10}\\
& \Delta \underline{\dot{\boldsymbol{X}}}=A_{s} \Delta \underline{\boldsymbol{X}}+B_{c} \Delta \underline{U} \tag{11}
\end{align*}
$$

Where,
$\underline{U}$ is a vector of control parameters
$A_{s}=M^{-1} a=\left(\frac{\partial F}{\partial X}\right)_{\left(X^{*}, U^{*}\right)}$ is state matrix
$B_{c}=\boldsymbol{M}^{-1} b=\left(\frac{\partial F}{\partial U}\right)_{\left(X^{*}, U^{*}\right)}$ is control matrix

### 3.3.1 Open Loop Stability Analysis

The state space model obtained in equation (11) is very important for the stability analysis of airship model. Open loop stability analysis is straight forward and completely depends on open loop system transfer functions and their response.

The open loop stability analysis is carried out to check the effectiveness of various control surfaces on the system. The open loop transfer functions along with time response analysis of longitudinal and lateral directional dynamics are given below.

## Longitudinal Dynamics

The longitudinal dynamics is represented by the states $u, w, q, \theta$ and $\delta_{e}$ as a control input. The state space form for the longitudinal dynamics is given by,
$\underline{\dot{X}}_{l}=A_{l} \underline{X_{l}}+B_{l} \underline{U_{l}}$
Where, $A_{l}$ state matrix of longitudinal state $B_{l}$ control matrix of longitudinal state. Laplace transform technique is used to find the transfer function of each state variables w.r.t elevator as a control parameter. The complete set of transfer functions of longitudinal dynamics are shown in table 2. It is clear from the transfer functions that all states of longitudinal dynamics are stable in open loop configuration. It is also important to note that roots are very close to the imaginary axis which suggests that sluggish response is expected in the time simulation.

Table 2. Open Loop Transfer Function

| $\frac{\boldsymbol{o} / \boldsymbol{p}}{\boldsymbol{i} / \boldsymbol{p}}$ | Transfer Function |
| :---: | :---: |
| $\frac{u(s)}{\delta_{e}}$ | $\frac{0.0022 k(s+0.06931)\left(s^{2}+0.4852 s+0.1492\right)}{(s+0.04833)(s+0.004352)\left(s^{2}+0.06012 s+0.01331\right)}$ |
| $\frac{w(s)}{\delta_{e}}$ | $\frac{-0.1066 k(s+0.0004367)\left(s^{2}+0.114 s+0.0155\right)}{(s+0.04833)(s+0.004352)\left(s^{2}+0.06012 s+0.01331\right)}$ |
| $\frac{q(s)}{\delta_{e}}$ | $\frac{-0.001228 k s(s+0.0875)(s+0.0004435)}{(s+0.04833)(s+0.004352)\left(s^{2}+0.06012 s+0.01331\right)}$ |
| $\frac{\theta(s)}{\delta_{e}}$ | $\frac{-0.00122 k(s+0.0875)(s+0.004435)}{(s+0.04833)(s+0.004352)\left(s^{2}+0.06012 s+0.01331\right)}$ |

The open loop response of longitudinal dynamics with 0.2 rad elevator step input is shown in figure 3(a). The Stratospheric Airship (SA) response is compared with standard YEZ-2A airship [2] for the validation purposes which is developed based on wind tunnel data. The positive command of elevator cause a nose down response of the airship. Therefore there is reduction of pitch angle and forward velocity from their equilibrium value. It should be noted here that the equilibrium value of forward velocity is $10 \mathrm{~m} / \mathrm{s}$ while pitch angle is at zero degree.






Fig. 3 (a) Time response of longitudinal states

## Lateral Directional Dynamics

The lateral directional dynamics consists $v, p, r, \phi$ as state variables and $\delta_{r}$ as control input. The state space form for the lateral directional dynamics is given by,
$\dot{X}_{l t}=A_{l t} \underline{X}_{l t}+B_{l t} \underline{U}_{l t}$
Laplace transformation is applied to equation (13) to get the transfer function of lateral states with respect to rudder input. All the eigen values are laying in the left half of s plane hence Lateral directional states are stable in the open loop configuration. Again, the roots are very close to imaginary axis which suggests that time response of lateral direction dynamics are sluggish. The lateral response to rudder step input of 0.2 rad is shown in the figure 3(b). This is very large input in aerodynamic sense but response magnitude is very small as shown in figure 3(b). Also, the settling time taken by the lateral directional states
is very large and hence it is clear that rudder control power is low. This is due to the large inertia of the airship hull. The response of side velocity $(v)$ to the positive rudder input is initially acting in opposite direction as shown in figure 3 (b). This is because, the transfer function of side velocity $(v)$ contains one zero in the right half of the s-plane which exhibits the nonminimum phase characteristics.

Table 3. Open Loop Transfer Function

| $\frac{\boldsymbol{o} / \boldsymbol{p}}{\boldsymbol{i} / \boldsymbol{p}}$ | Transfer Function |
| :---: | :---: |
| $\frac{v(s)}{\delta_{r}}$ | $\frac{-0.00031 k(s-2.014)\left(s^{2}+0.227 s+0.3969\right)}{(s+0.07923)(s+0.008027)\left(s^{2}+0.02244 s+0.2657\right)}$ |
| $\frac{p(s)}{\delta_{r}}$ | $\frac{-0.0000346 k s(s+1.236)(s+0.01386)}{(s+0.07923)(s+0.008027)\left(s^{2}+0.02244 s+0.2657\right)}$ |
| $\frac{r(s)}{\delta_{r}}$ | $\frac{-0.00025 k(s+0.008691)\left(s^{2}+0.2353 s+0.2674\right)}{(s+0.07923)(s+0.008027)\left(s^{2}+0.02244 s+0.2657\right)}$ |
| $\frac{\phi(s)}{\delta_{r}}$ | $\frac{-0.0000346 k(s+1.236)(s+0.01386)}{(s+0.07923)(s+0.008027)\left(s^{2}+0.02244 s+0.2657\right)}$ |



Fig. 3 (b) Time response of lateral direction states

### 3.3.2 Closed Loop Stability Analysis

Closed loop stability analysis is carried out in this sub-section to check the behavior of each state in closed loop configuration. The closed loop stability depends on roots of characteristics equation of closed loop transfer function. However control system provides powerful graphical techniques to analyze the closed loop stability without finding closed loop transfer function [8]. The closed loop stability analysis using two different techniques are explained below.

## Time Domain Technique

Time domain technique is used to find the range of gain value $k$ within which closed loop system is stable. The root locus diagram is one of the traditional time domain technique and very helpful to find the closed loop stability range.

Therefore, root locus diagram is drawn for each transfer function for the analysis of closed loop system stability. Root locus diagrams for longitudinal dynamics and lateral directional dynamics are shown in figure 4.

## Frequency Domain Technique

The frequency domain technique called bode plot is used to find the stability margins of each states in a closed loop configuration. Stability margins indicates how far system is away from stable critical boundary $(-1+j 0)$. These stability margins are expressed in the terms of Gain Margin (GM) and Phase Margin (PM). The values of $G M$ and $P M$ are obtained using Bode plots. The Bode diagram of longitudinal and lateral directional dynamics are shown in figure 5 (a) and figure 5 (b) respectively.


Fig. 4 Root locus diagram of (a) Longitudinal dynamics (b) Lateral directional dynamics


Fig. 5(a) Bode diagram of longitudinal dynamics


Fig. 5(b) Bode diagram of lateral directional dynamics

The summary of open loop and closed loop stability analysis of the stratospheric airship model is given in table 4. It shows that airship model is absolute stable in open loop configuration with sluggish behavior while it has some gain constraint limitation in closed loop configuration. Therefore model is said to be conditionally stable in the closed loop configuration. It also shows the range of values of gain $k$ for witch system is stable in closed loop configuration.

## Table 4. Summary of Stability Analysis

| $\mathbf{T / F}$ | O/L <br> stable? | $\boldsymbol{G M}$ <br> $(\mathbf{d B})$ | $\boldsymbol{P M}$ | Gain limitation for <br> closed loop stability |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{u(s)}{\delta_{e}}$ | Yes | -18.1 | $-95^{\circ}$ | $0<k \leq 0.1$ |
| $\frac{w(s)}{\delta_{e}}$ | Yes | -8.22 | $-95.1^{\circ}$ | $0<k \leq 0.38$ |
| $\frac{q(s)}{\delta_{e}}$ | Yes | 32.7 | - | $0<k \leq 42.5$ |
| $\frac{\theta(s)}{\delta_{e}}$ | Yes | 15.4 | - | $0<k \leq 5.87$ |
| $\frac{v(s)}{\delta_{r}}$ | Yes | 32.6 | $127^{\circ}$ | $0<k \leq 42.4$ |
| $p(s)$ | Yes | 53.7 | - | $0<k \leq 482$ |
| $\frac{\delta_{r}}{\delta_{r}}$ |  |  |  |  |
| $\frac{r(s)}{\delta_{r}}$ | Yes | 49.3 | - | $0<k \leq 290$ |
| $\frac{\phi(s)}{\delta_{r}}$ | Yes | 49.1 | - | $0<k \leq 284$ |

## 4. Mode Analysis

Mode analysis of the airship is very crucial analysis to characterize the various motions of the airship in atmosphere [9]. Longitudinal dynamics represents three modes namely Surge Mode(SM), Heave Mode (HM) and Pendulum Mode (PM). Similarly lateral directional dynamics represents three modes namely Yaw Subsidence Mode (YSM), Sideslip Subsidence Mode (SSM) and Roll Oscillation Mode (ROM). The various modes and its important characteristics are listed in tables 5 and 6. Various modes of the Stratospheric Airship (SA)
are compared with standard YEZ-2A airship for validation purposes [7]. The controllability of motion can be examined by kalman's controllability test which is given by,

$$
\begin{align*}
Q_{1 c} & =\left[B_{l}\left|A_{l} B_{l}\right| A_{l}^{2} B_{l} \mid A_{l}^{3} B_{l}\right]  \tag{14}\\
Q_{2 c} & =\left[B_{l t}\left|A_{l t} B_{l t}\right| A_{l t}^{2} B_{l t} \mid A_{l t}^{3} B_{l t}\right] \tag{15}
\end{align*}
$$

The rank of $Q_{1 c}$ and $Q_{2 c}$ matrix is 4 which shows that longitudinal and lateral plane motions are completely controllable.

Table 5. Longitudinal Mode Characteristics

| Characteristics | LONGITUDINAL MODES |  |  |  |
| :---: | :--- | :--- | :--- | :--- |
|  | SM |  | HM | PM |
| Type of motion |  | Forward | Vertical | Oscillatory |
| Characterized by |  | $X_{u}$ | $Z_{w}$ | $M_{q}$ and $M_{\theta}$ |
| Eigen <br> values | SA | -0.0043 | -0.0483 | $-0.030 \pm 0.111 i$ |
|  | YEZ-2A | -0.0087 | -0.0683 | $-0.169 \pm 0.210 i$ |
| $T$ (sec) | SA | 229.77 | 20.69 | 32.99 |
|  | YEZ-2A | 114.94 | 14.64 | 6 |
| Zeta | SA | - | - | 0.26 |
|  | YEZ-2A |  |  | 0.62 |
| $w_{n}$ | SA | - | - | $0.11 \mathrm{rad} / \mathrm{sec}$ |
|  | YEZ-2A |  | $0.27 \mathrm{rad} / \mathrm{sec}$ |  |
| $T_{1 / 2}(\mathrm{sec})$ | SA | 160.2 | 14.2 | 9.98 |
|  | YEZ-2A | 78.7 | 10.1 | 5.11 |
| $\mathrm{~T}_{\mathrm{r}}(\mathrm{sec})$ | SA | 505.5 | 45.5 | 11.10 |
|  | YEZ-2A | 253 | 32.2 | 7.12 |

Table 6. Lateral Mode Characteristics

| Characteristics | LATERAL MODES |  |  |  |
| :---: | :--- | :--- | :--- | :--- |
|  | YSM | SSM | ROM |  |
| Type of motion |  | Yaw | Sideslip | Roll |
| Characterized by |  | $Y_{v}, N_{r}$ | $Y_{v,} Y_{\phi}, L_{v}, L_{\phi}$ | $L_{p}, L_{\phi}, L_{v}, Y_{\phi}$ |
| Eigen <br> value | SA | -0.0792 | -0.0080 | $-0.011 \pm 0.51 i$ |
|  | YEZ-2A | -0.3688 | -0.0522 | $-0.012 \pm 0.73 i$ |
| $T$ (sec) | SA | 12.62 | 125.1 | 90.90 |
|  | YEZ-2A | 2.71 | 19.15 | 83.33 |
| Zeta | SA | - | - | 0.0217 |
|  | YEZ-2A |  |  | 0.0167 |
| $w_{n}$ | SA | - | - | $0.51 \mathrm{rad} / \mathrm{sec}$ |
|  | YEZ-2A |  | $0.73 \mathrm{rad} / \mathrm{sec}$ |  |
| $T_{1 / 2}(\mathrm{sec})$ | SA | 8.74 | 86.3 | 1.9 |
|  | YEZ-2A | 1.86 | 13.3 | 1.38 |
| $T_{r}(\mathrm{sec})$ | SA | 27.7 | 274 | 2.06 |
|  | YEZ-2A | 5.96 | 42.1 | 1.44 |

## 5. Conclusions

Design strategy and complete stability analysis of stratospheric airship is carried out in this paper. Analysis of variation of drag with altitude for different shapes is studied and shape having minimum drag is selected for mission. The
design parameters of airship is presented and based on these parameters, six dof equations of motion is derived to forms the mathematical model. Nonlinear mathematical model is linearized about cruise condition with velocity $\operatorname{trim} 10 \mathrm{~m} / \mathrm{s}$ to develop linearized state space model. This linearized model provides very useful framework to determine open loop transfer functions and dynamic stability. The determination of the closed loop dynamic stability is carried out with various stability techniques in MATLAB ${ }^{\circledR}$. Stability margins called Gain Margin (GM) and Phase Margin $(P M)$ are obtained from frequency response to ensure that how far system response is away from critical unstable point $(-1+j 0)$. Analysis of each states is carried out separately in time/frequency domain to check the behavior in closed loop configuration and it is observed that all the states are conditionally stable in closed loop configuration. The different modes of airship dynamics with performance characteristics are presented at the end of paper. All the results obtained in this study will be used for the design of Navigation, Guidance and Control algorithm (GNC) in the next stage of stratospheric airship development program.

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## References

[1] Liao, L. and Pasternak I. "A review of airship structural research and development", Progress in Aerospace Sciences 45, no. 4 (2009): 83-96.
[2] Gomes, S. B. V. "An investigation into the flight dynamics of airships with application to the YEZ-2A", Ph.D. thesis, College of Aeronautics, Cranfield University, (1990).
[3] Rana, R., Kumar, A., Sinha, N.K., Pal, A. and Sati, S.C. "Configuration Analysis of Stratospheric Airship", Symposium on Applied Aerodynamics and Design of Aerospace Vehicles, VSSC, Thiruvananthapuram, India, 2015.
[4] Mueller, J. B., Michael A. P. and Zhao Y. "Development of an Aerodynamic Model and Control Law Design for a High Altitude Airship",

AIAA $3^{\text {rd }}$ Unmanned Ultimate Technical Conference, Workshop and Exhibits, AIAA, Chicago, IL, 2004.
[5] Ashraf, M. Z. and Choudhry, M. A. "Dynamic modeling of the airship with Matlab using geometrical aerodynamic parameters", Aerospace Science and Technology 25, no. 1 (2013): 56-64.
[6] Khoury, G. A. "Airship technology", Cambridge university press, second edition, USA, 2012.
[7] Cook, M. V. "The linearized small perturbation equations of motion for an airship", Cranfield report, UK, (1990).
[8] Roskam, J. "Airplane flight dynamics and automatic flight controls", DAR Corporation, 1995.
[9] Lee, H.C. "Analysis of the static and dynamic stability properties of the unmanned airship", KSAS International Journal 2, no. 2 (2001): 82-94.

## Appendix

## Notation

| $\boldsymbol{A}$ | $6 \times 1$ aerodynamic vector |
| :--- | :--- |
| $A_{s}$ | State matrix |
| $A_{l}$ | State matrix of longitudinal plane |
| $A_{l t}$ | State matrix of lateral plane |
| $a_{x}, a_{y}, a_{z}$ | CG coordinates in the body axis system |
| $a, b$ | Linearized matrix |
| $B$ | Buoyancy force |
| $B_{c}$ | Control matrix |
| $B_{l}$ | Control matrix of longitudinal plane |
| $B_{l t}$ | Control matrix of lateral plane |
| $b_{x}, b_{y}, b_{z}$ | CB coordinates in body axis system |
| $C B$ | Centre of Buoyancy |
| $C G$ | Centre of Gravity |
| $C V$ | Centre of Volume |
| $C_{L}, C_{D}, C_{Y}$ | Coefficient of aerodynamic force |
| $C_{l}, C_{m}, C_{n}$ | Coefficient of moments |
| $d_{x}, d_{y}, d_{z}$ | Position of propulsion system |
| $\boldsymbol{F}_{\boldsymbol{d}}$ | Dynamics vector |
| $\boldsymbol{G}$ | 6x1 gravitational vector |
| $g$ | Gravitational acceleration |
| $G M$ | Gain Margin |
| $G N C$ | Guidance Navigation and Control |
| HM | Heave Mode |
| $I_{x}, I_{y}, I_{z}, I_{x z}$ | Moment of inertia |
| $J_{x}, J_{y}, J_{z}, J_{x z}$ | Component of apparent inertia |
| $M_{u}, M_{w}, M_{q}, M_{\theta}$ | Pitching moment derivatievs |
| $\boldsymbol{M}$ | $6 \times 6$ mass matrix |
| $m$ | Airship total mass |
| $N_{v}, N_{P}, N_{r}$ | Yawing moment derivative |
| $T$ | Time constant |
| $\boldsymbol{P}$ | $6 \times 1$ propulsion vector |
| PM | Pendulum mode |
| $P M$ | Phase Margin |
| $p, q_{1}, r$ | Angular velocities |
| $Q$ | Dynamic pressure |
| $Q_{1 c}$ | Controllability matrix-longitudinal plane |
|  |  |


| $Q_{2 c}$ | Controllability matrix of lateral plane |
| :--- | :--- |
| ROM | Roll Oscillation Mode |
| $S_{r e f}$ | Reference surface area |
| SM | Surge Mode |
| SSM | Sideslip Subsidence Mode |
| $T_{s}$ | Thrust of starboard side engine |
| $T_{p}$ | Thrust of port side engine |
| $T$ | Time constant |
| $T_{1 / 2}$ | Half time period |
| $T_{r}$ | Rise time |
| $T_{s}, T_{p}$ | Thrust of star board and port side engine |
| $u, v, w$ | Linear velocities |
| $w_{n}$ | Natural frequency |
| $X_{\dot{q}}, Z_{\dot{q}}, M_{\dot{u}}, M_{\dot{w}}$ | Derivative expressing virtual mass |
| $X_{u}, X_{w} X_{q}$ | Longitudinal derivatives along X |
| $Y_{\dot{v}}, Y_{\dot{p}}, Y_{\dot{r}}, L_{\dot{v}}, L_{\dot{p}}$ | Derivatives expressing virtual mass |
| $Y v, Y_{p}, Y_{r}, Y_{\phi}$ | Lateral derivatives along Y |
| YSM | Yaw Subsidence Mode |
| $\mu_{s}, \mu_{p}$ | Angle of rotation for thrust vector |
| $\phi, \theta, \psi$ | Euler angle |
| $\delta_{e}$ | Elevator deflection |
| $\delta_{r}$ | Rudder deflection |
| $\alpha$ | Angle of attack |
| $\beta$ | Sideslip angle |

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