ICAS 2016
30th Congress of the International Council
of the Aeronautical Sciences
DCC, Daejeon, Korea ; September 25-30, 2016

SYSTEM IDENTIFICATION OF A SCALE HELICOPTER IN HOVERING FLIGHT

Felipe Cezar Reis, Eduardo Morgado Belo
Engineering School of São Carlos, University of São Paulo
13566-590 São Carlos, São Paulo, Brazil

Keywords*: system identification; scale helicopter; hovering flight*

Abstract

The present work aims to study and obtain the linear dynamic model of a scale helicopter from data of hovering flights. These flight tests aim to excite an actual scale model of a radio-controlled helicopter instrumented with a set of sensors and systems for managing the activities in flight, being the input and output signals stored in an onboard portable Universal Serial Bus flash memory device. The linear model is determined by the study of the aerodynamic forces and moments generated in the main and tail rotors and by physical characteristics of the aircraft, being ignored the aerodynamic influence of the fuselage. The parameters of the linear dynamic model in hovering flight is achieved iteratively with the use of a gray box type identification algorithm. The inputs of this algorithm are the data from the flight tests, the state space model structure and a set of initial parameters from the simulation of a nonlinear model, which provides the initial boundary condition and limits for numerical solution. Being this an experiment, the results are verified and validated. Thus, the dynamic model, identified from a given flight test data, is simulated with input signals from a different test. Then the output signals from the simulation are compared with actual ones and if the error between them is less than or equal to the accuracy of the sensors, the identified model is considered as a valid one.

1 Introduction

Unmanned aerial vehicles and their variants are being largely used in both civil and military missions. In the civil segments, we can mention imaging, photography, video, agriculture, construction, mining, railways, security, mapping, surveying, research, conservation, inspections of electricity transportation lines, communications, parcel delivery, humanitarian efforts and more, as some missions executed by UAVs. Within these segments, the highlights are the air monitoring by cameras filming and/or photographing the ground. However, some of these aircraft are not able to perform such images acquisition since they cannot stay still over a certain point, in space, during the time necessary for such a mission or even operate on inappropriate runways. To overcome this problem three types of aircrafts can be used: airships, multi-rotors and helicopters; the latter has great advantage over the others regarding maneuverability, range and carrying capacity, however, it is operationally more complex and higher costly in terms of acquisition and operation.

The development and the operation of a helicopter that can be used in the mentioned segments requires knowledge of its flight mechanics or in other words its mathematical model. This can be accomplished by flight tests with embedded systems to produce data to be used to generate dynamic models that represent the physical behavior of the aircraft in a given operating condition. Besides the importance to the development of systems that will manage, control, guide and navigate the helicopter, the dynamic models are useful to study changes that alter the flight mechanics, these may be of structural, aerodynamic or propulsion types. Thus, the effects on the performance of the aircraft may be predicted saving money and reducing the number of prototypes to be tested.

So, this paper shows a way to obtain the dynamic model of a scale helicopter that served as a platform to test hovering flights and can be used, in future studies, for the development of control systems, guidance and navigation, among other activities.

Fig. 1. The adapted scale helicopter for tests.

2 Mathematical model for helicopter

A dynamic system is mathematically modeled by a set of differential equations that accurately, or at least reasonably well, represents the dynamics of the system. One system can be represented in many different ways, and therefore may have various mathematical models, depending on the approach being considered.

In flight dynamics, the mathematical model of a flying vehicle and its subsystems is the starting point for computational analysis or simulation to understand its movements. An aircraft in flight is a very complex dynamic system. It can be considered as a set of rigid and elastic bodies mainly under aerodynamic forces and moments, resulting in correlated rigid and elastic movements.

The process of obtaining the mathematical model that expresses the dynamic behavior of a physical system, requires that one has to have some amount of knowledge about the system itself. Depending on the way the mathematical modeling process is accomplished, the models obtained can be classified into black box or white box ones. The black-box model corresponds to a system of which there is no a priori information available and the white-box one corresponds to a system where, on contrary, all necessary information is available. Talking in terms of physical systems, one can say that most of them are neither black-box neither white-box models, so they are called as grey-box ones, for which one can establish mathematical relationships between input and output, but there is no knowledge of the physical parameters that relate the input to output. The helicopter is an example of these latter models.

Considering the helicopter motion equations described in linear form as:

$$\dot{x} = F(x,u,t) \tag{1}$$

$$x = \{u, w, q, \theta, v, p, \phi, r, \psi\} \tag{2}$$

$$u = \{\theta_0, \theta_{1s}, \theta_{1c}, \theta_{0T}\} \tag{3}$$

where the variables are the standard ones used in the literature.

Thus, the equations that describe the forces and moments on the helicopter are given as:

Forces:

$$\dot{u} = -(wq - vr) + \frac{X}{M_a} - g\sin\theta \tag{4}$$

$$\dot{v} = -(ur - wp) + \frac{Y}{M_a} + g\cos\theta\sin\phi \tag{5}$$

$$\dot{w} = -(vp - uq) + \frac{Z}{M_a} + g\cos\theta\cos\phi \tag{6}$$

Moments:

$$I_{xx}\dot{p} = (I_{yy} - I_{zz})qr + I_{xz}(\dot{r} + pq) + L \tag{7}$$

$$I_{yy}\dot{q} = (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + M \tag{8}$$

$$I_{zz}\dot{r} = (I_{xx} - I_{yy})pq + I_{xz}(\dot{p} - qr) + N \tag{9}$$

And representing them in the state space form:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{10}$$

$$\boldsymbol{y} = \mathbf{Cx} + \mathbf{Du} \tag{11}$$

where:

$$B = \begin{bmatrix} X_{\theta_0} & X_{\theta_{1s}} & X_{\theta_{1c}} & X_{\theta_{0T}} \\ Z_{\theta_0} & Z_{\theta_{1s}} & Z_{\theta_{1c}} & Z_{\theta_{0T}} \\ M_{\theta_0} & M_{\theta_{1s}} & M_{\theta_{1c}} & M_{\theta_{0T}} \\ 0 & 0 & 0 & 0 \\ Y_{\theta_0} & Y_{\theta_{1s}} & Y_{\theta_{1c}} & Y_{\theta_{0T}} \\ L'_{\theta_0} & L'_{\theta_{1s}} & L'_{\theta_{1c}} & L'_{\theta_{0T}} \\ 0 & 0 & 0 & 0 \\ N'_{\theta_0} & N'_{\theta_{1s}} & N'_{\theta_{1c}} & N'_{\theta_{0T}} \end{bmatrix} \tag{12}$$

$$A = \begin{bmatrix} X_u & X_w - Q_e & X_q - W_e & -g\cos\Theta_e & X_v + R_e & X_p & 0 & X_r + V_e \\ Z_u + Q_e & Z_w & Z_q + U_e & -g\cos\Phi_e\sin\Theta_e & Z_v - P_e & Z_p - V_e & -g\sin\Phi_e\cos\Theta_e & Z_r \\ M_u & M_w & M_q & 0 & M_v & M_p - \frac{2P_eI_{xz}}{I_{yy}} - \frac{R_e(I_{xx}-I_{zz})}{I_{yy}} & 0 & M_r + \frac{2R_eI_{xz}}{I_{yy}} - \frac{P_e(I_{xx}-I_{zz})}{I_{yy}} \\ 0 & 0 & \cos\Phi_e & 0 & 0 & 0 & -\Omega_a\cos\Theta_e & -\sin\Phi_e \\ Y_u - R_e & Y_w + P_e & Y_q & -g\sin\Phi_e\sin\Theta_e & Y_v & Y_p + W_e & -g\cos\Phi_e\sin\Theta_e & Y_r - U_e \\ L'_u & L'_w & L'_q + k_1P_e - k_2R_e & 0 & L'_v & L'_p + k_1Q_e & 0 & L'_r - k_2Q_e \\ 0 & 0 & \sin\Phi_e\tan\Theta_e & \Omega_a\sec\Theta_e & 0 & 1 & 0 & \cos\Phi_e\tan\theta_e \\ N'_u & N'_w & N'_q - k_1R_e - k_2P_e & 0 & N'_v & N'_p + k_3Q_e & 0 & N'_r - k_1Q_e \end{bmatrix} \quad (13)$$

In addition:

$$L'_p = \frac{I_{zz}}{I_{xx}I_{zz}-I_{xz}^2}L_p + \frac{I_{xz}}{I_{xx}I_{zz}-I_{xz}^2}N_p \quad (14)$$

$$N'_r = \frac{I_{zz}}{I_{xx}I_{zz}-I_{xz}^2}L_r + \frac{I_{xz}}{I_{xx}I_{zz}-I_{xz}^2}N_r \quad (15)$$

For the moments of inertia, one assumes that:

$$k_1 = \frac{I_{xz}(I_{zz}+I_{xx}-I_{yy})}{I_{xx}I_{zz}-I_{xz}^2} \quad (16)$$

$$k_2 = \frac{I_{zz}(I_{zz}-I_{yy})+I_{xz}^2}{I_{xx}I_{zz}-I_{xz}^2} \quad (17)$$

$$k_3 = \frac{I_{xx}(I_{yy}-I_{xx})-I_{xz}^2}{I_{xx}I_{zz}-I_{xz}^2} \quad (18)$$

3 System identification

System identification is an area of knowledge that studies alternative techniques of mathematical modeling. One of the features of this technique is that little or no prior knowledge of the system is required [1]. For the system identification to be carried out in order to accurately represent a dynamic behavior, one needs to follow some basic steps, as follows:

1) Dynamic tests and data acquisition: since identification aims to get models from data, it is imperative to execute correctly the experiments so that the acquired data are consistent with the dynamic behavior of the helicopter. The biggest concerns of this stage are the excitation signals, that cannot exceed the operating capacity of the helicopter, its safe flight regime and consequently its physical integrity;
2) Mathematical representation to be used: as already said, various mathematical representations can express a dynamic behavior; this can be done through transfer functions, state space, frequency response functions, among others. Two models, in the form of state space, will be used for mathematical representation, being one non-linear and another linear;
3) The model structure: in the case of linear models, the choice of its structure is limited basically to the choice of the number of poles and zeros, and the determination of the pure delay time; in the case of non-linear models, an approach is used varying the parameters of the test; these parameters define a different set of dynamic models which are adapted according to the real system;
4) Parameters estimation: this step is characterized by the choice of the identification algorithm to be used; here, the Estimated Error Prediction Model (PEM), which is part of the systems identification toolbox present in MATLAB® and described ahead in a simplified manner, is utilized;
5) Model validation: having obtained a family of models, it is necessary to determine whether or not they incorporate the features of interest of the original system. Moreover, it is necessary to compare the models with each other and decide if there is any better candidate than the others. This step is certainly very subjective, and the validation result strongly depends on the intended application of the model and the amount of information available of the original system.

The basic structure of the identification algorithm/subroutine used here is as follows:

1) Declaration of the variables;
2) Import of test data: radio control inputs and outputs of the sensors;
3) Filtering of the acceleration signals;
4) Determination of speed;
5) Conversion of the input signals to adjust the model inputs to the PX4;
6) Declaration of test data;
7) Modeling the system in C language;

8) Generate the structure of the parameters present in the system model;
9) Generate the structure of the initial estimated parameters;
10) Generate the structure of the gray box type non-linear model;
11) Compare the generated model data with the test data;
12) Start of iteration;
    12.a) execute PEM;
    12.b) execute N4SID;
    12.c) execute comparison;
    12.d) execute adjustment;
13) End if converges;

PX4 is a multi integrated circuit board developed by pixhawk.org, an independent, open-hardware project. It was design to be used in different flying platforms, such as, drones, airplanes and helicopters and, basically, is made up of two boards, the PX4FMU and the PX4IO.

PEM (Prediction Error Method), in MATLAB®, is a routine for estimating error of linear or nonlinear models. It uses numerical optimization to minimize the cost functions, one weighting vector of the prediction error, defined by scalar outputs as in the following equation:

$$V_H(G,H) = \sum_{t=1}^{N} e^2(t)$$

where $e(t)$ is the error obtained by the difference between the actual measured output of the system and the output from the predicted model; $V_H(G,H)$ is a scalar; G and H are the transfer functions from u and e to y respectively and N is the number of input data samples used. For a generic linear model, this error is defined by the following equation:

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)]$$

where q is the forward shift operator (ex: $qu(t) = u(t+1)$); $y(t)$ and $u(t)$ are the output and input variables at time t respectively.

N4SID is a MATLAB® routine for estimating state-space model.

The identification algorithm needs a routine in C language to describe the state space representation to be generated by the gray box type non-linear modeling function. The reason for the use of C is to use pointers which allow an optimization of the identification algorithm. The iteration process was done starting with initial conditions known from the test data and a rough estimate of the parameters to be identified. The estimate is said rough since the used results were derived from a non-linear model adjusted according to aerodynamics and geometry of the helicopter. Obviously, the data for the iteration start are not accurate due to parameters such as downwash which is difficult to obtain through test, but that can be obtained through a CFD simulation.

4 Planning of the experiment

Planning of the experiment involves determining what physical variables will be observed and measured during the flight tests, and how their data should be stored throughout the experiment and also how they should be treated at the end or during data collection. In addition, the conditions and the physical environment in which the experiment is carried out are important for analysis of the acquired data. Due to disturbances affecting the experiment, the acquired data will not be precise and accurate. Thus, the objective of the experiment is to maximize the quality with which information is obtained, but we must be aware of the limitations that the experiment can support. Some of these limitations are:

1) The inputs and outputs range limits;
2) The resolution limits of the data acquisition embedded sensors;
3) The embedded hardware limitations in terms of ability to store or transmit data via telemetry;
4) The frequencies limits of the acquisition sensors;
5) The time available for execution of each maneuver and / or experiment as a whole;
6) The mechanical limits that the system can support;
7) The limits on control inputs that can affect the stability of the aircraft and cause damage.

The helicopter mathematical model, to be obtained, depends on following variables that must be fed to the identification algorithm:

1) The angular velocities (p, q, r);
2) Attitudes or Euler angles (φ, θ, ψ);
3) Acceleration (a_x, a_y, a_z);
4) The speeds (v_x, v_y, v_z);
5) The input commands ($\theta_0, \theta_{1S}, \theta_{1C}, \theta_{0T}$);

The selection of the physical platform for the experiment took into consideration the above-mentioned restrictions and some other factors, namely:

1) Acquisition cost and operation;
2) Quality, robustness and security;
3) Flexibility to make modifications;
4) Performance expressed in terms of flight autonomy, rate of climb, stability, load capacity and cargo volume;
5) Availability of components for repair and replacement.

Considering the above restrictions, a radio controlled helicopter was elected. Among the different brands available in the market and the various possible power configurations, a helicopter powered by internal combustion engine, used in aerobatics, was chosen. The electric engine has been dropped off due to low autonomy and the excess of weight of the batteries. The chosen helicopter was the T-Rex 700, manufactured by Align Corp., Ltd.

Table 1. Specifications of the helicopter

Specifications	Value	Unit
Power	2.76 @ 14000 RPM	[HP]
Range	80	[min]
Tank capacity	1300	[ml]
Empty weight	4.5	[kg]
Payload	3.5 à 7.5	[kg]
Max velocity	120	[km]

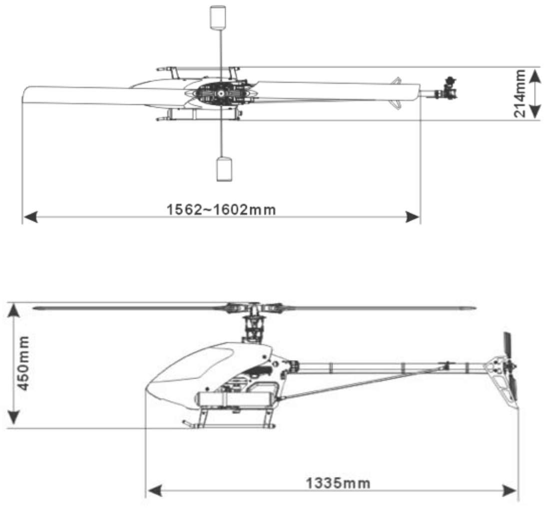

Fig. 2. Dimensions of the helicopter.

Source: www.align.com.tw/helicopter-en/

5 Flight tests

Being a RC helicopter, the pilot is external to the aircraft and maneuvers it exclusively considering its field of view. This implies that the execution of a straight flight is compromised because the pilot is not able to maintain the constant flight parameters. Therefore, the experiment was aimed at only to hovering flight where the pilot's proximity to the aircraft is sufficient for full implementation of the maneuvers.

The maneuvers were designed to excite the lateral and longitudinal modes of the helicopter following a script, not necessarily in this order:

1) Take off and keep the helicopter still: this prevents exciting modes early during the trial starting ;
2) Act the collective and acceleration to change the altitude, ranging from mild to more abrupt inputs, and return to the hovering flight condition;
3) Successive rolls on both sides and at different amplitudes and periods;
4) Excitations in roll and pitch independently: this step is the most risky for a helicopter since the recovery maneuver may be not be agile enough to avoid an uncontrolled fall;
5) Combined excitations of rolling and pitching: more complex than the previous one for recovery and changing the flight regime;

The helicopter has the difficulty imposed by the presence of several modes of vibration that are not part of the flight dynamics, but its structure and rotors. The presence of these modes is not desirable since they mask the actual values. Elimination of these modes is complicated because there is no way to previously identify what is the contribution of each party without performing specific aeroelasticity studies. Engine noise is another serious interference to the acquisition system since it may cause saturation of the sensors and the presence of more undesirable modes. To work around this problem, cushions were used to eliminate part of this vibration as well as filters to remove the motor vibrations frequency band of the acquired signals. Another disturbance is the electromagnetic interference from the ignition system (CDI) of the engine, but easily eliminated with proper positioning of the sensors. The intensity of all these interferences is sufficient to affect the actuators, causing them to be positioned in a manner inconsistent with the command signals.

6 Results

The maneuvers of the flight test were carried out to excite the model in all its degrees of freedom by taking care to avoid the transition from hovering flight to the longitudinal or lateral ones. The execution of the maneuvers is effected with the help of experienced pilots oriented so that each command was executed to maintain the flight envelope. However, despite the expertises, it took some time to adapt to the intensity of the commands necessary for the experiment.

Adjusting the course of each control surface and acceleration ramps was essential for the flight test to be carried out correctly, this process is known as trimming. With the aircraft trimmed, early trials were done, and the data were analyzed to understand the capacity of the commands to execute certain maneuver.

An important aspect is that the aircraft has a range of more than a hour of flight while the necessary tests windows are no more than 20 seconds, letting the pilot time to adjust the performance of the maneuvers, ie, the autonomy is consistent to the number of attempts to run the experiment without the need to interrupt the flight for refilling the tank. This produces a considerable amount of data for later choosing which data surplus should be discarded as irrelevant to the experiment, leaving only the windows in which it is assumed that the identification will give good results.

The flight tests data and the subsequent identification procedure resulted in the obtainment of the numerical parameters of the mathematical model matrices A and B (equations 12 and 13).

The figures presented ahead represent real responses of the helicopter and the simulated ones (from an identified model) due to the same input signals.

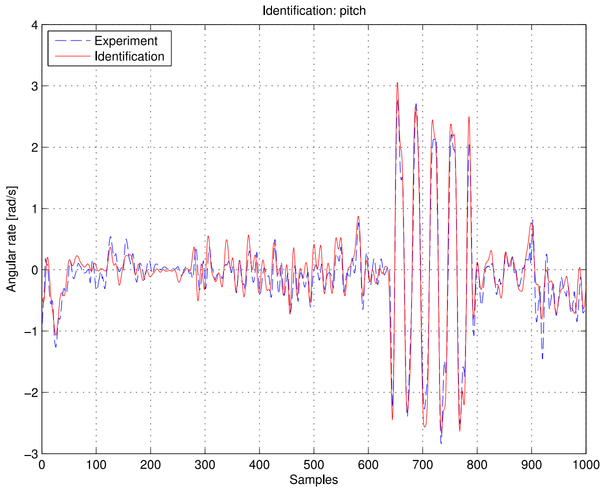

Fig. 3. Identification of pitch rate

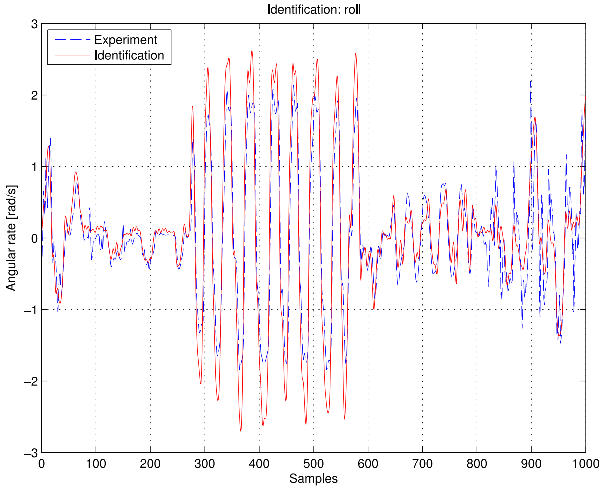

Fig. 4. Identification of roll rate

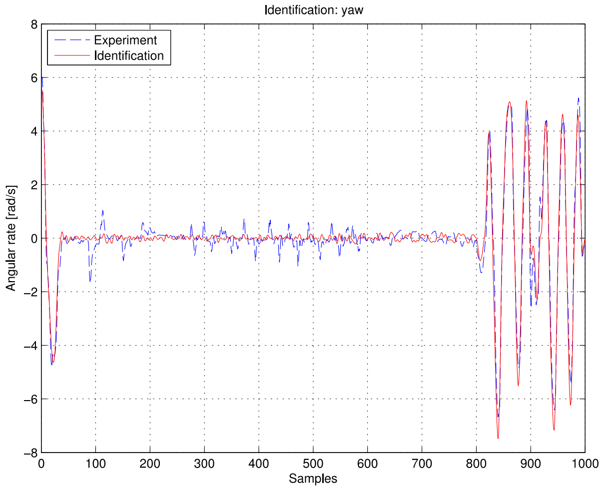

Fig. 5. Identification of yaw rate

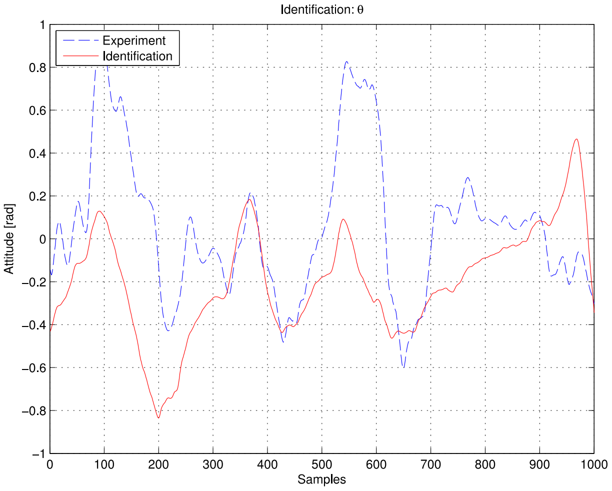

Fig. 6. Identification of pitch attitude

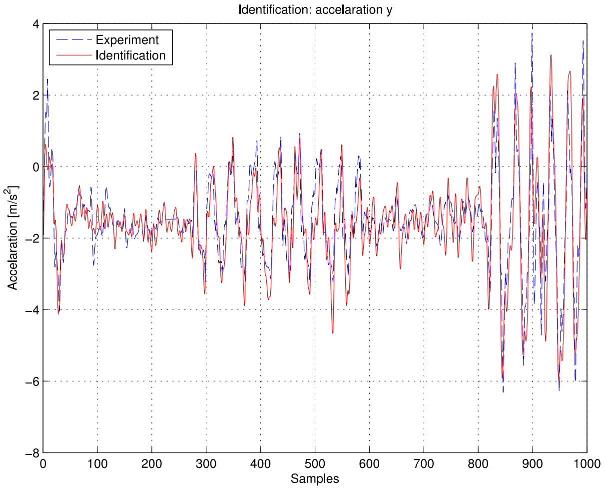

Fig. 9. Identification of y direction acceleration

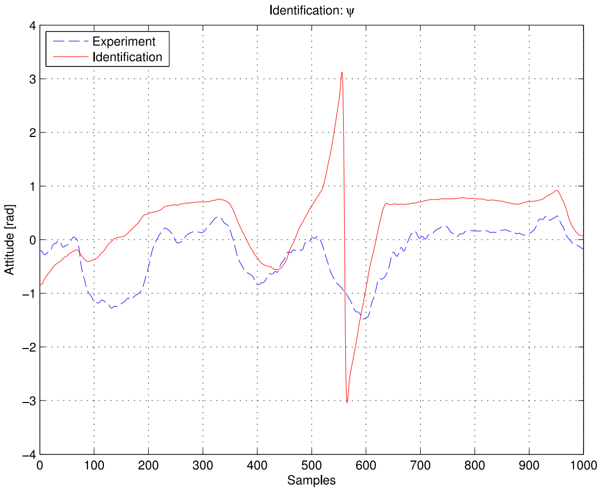

Fig. 7. Identification of yaw attitude.

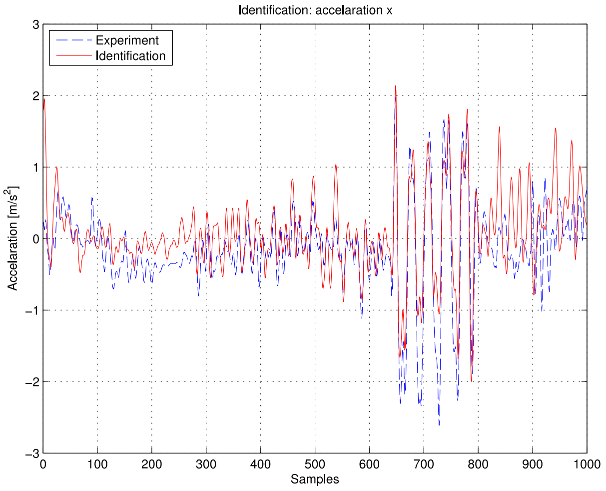

Fig. 10. Identification of x direction acceleration

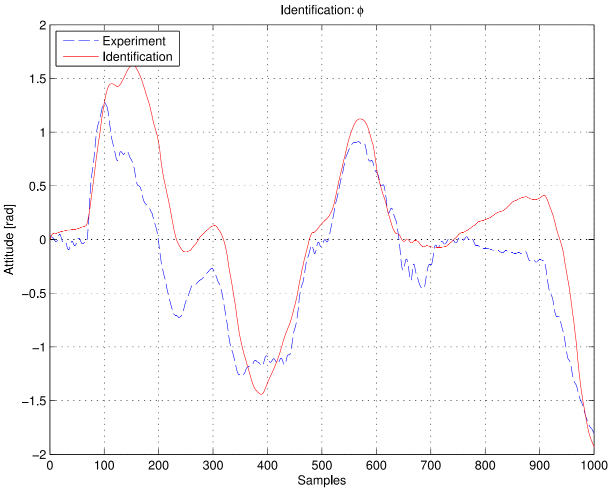

Fig. 8. Identification of roll attitude

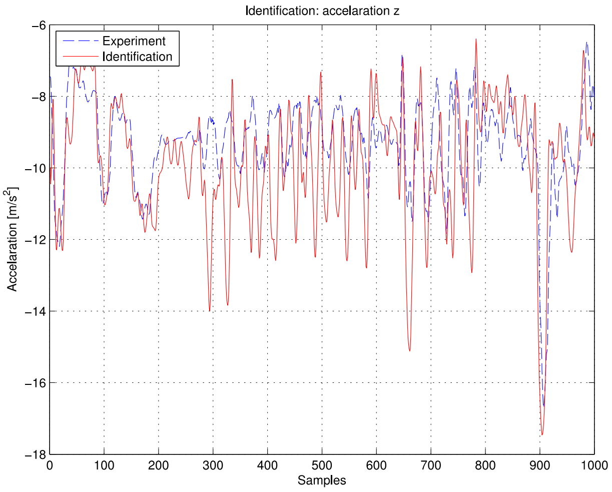

Fig. 11. Identification of z direction acceleration

7 Conclusions

The multi-disciplinary aspect of this work involving the development of the helicopter experimental setup assembled with elements of high degree of difficulty, the execution of maneuvers of reasonably complexity, the need of specific materials, the study of mathematical identification of dynamic systems, the implementation of electronic circuits and systems, etc., required immersion in different subjects of study allowing the construction of a major generalist research profile.

With the work done was possible to obtain an approximation of the dynamic behavior of an aircraft with rotating wings. Improving the accuracy of the results depends on using more sophisticated and calibrated sensors and longer test flights as well as more intense and complex identification routines. However, the procedures for obtaining the parameters remains the same.

8 Bibliography

[1] Aguirre, L. *Introdução à identificação de sistemas.* 3a edição. UFMG. 2007.

[2] Bhandari, S. and Colgren, R. Six-Dof dynamic modeling and flight testing of a UAV helicopter. *AIAA Modeling and Simulation Technologies Conf. and Exhibit.* São Francisco, California. 2005.

[3] Blakelock, J. H. *Automatic control of aircraft and missiles*. 2nd edition. New York. Wiley. 1991

[4] Branwell, A. R. *Helicopter dynamics*. 2nd edition. AIAA. 2001.

[5] Breganon, R. *Controle arfagem e guinada de um sistema de hélices paralelas.* Dissertação. EESC-USP. 2009.

[6] Cai, G. and Chen, B. M. Design and implementation of a hardware-in-the-loop simulation system for small-scale UAV helicopters. *Journal of Mechatronics*. Vol 19, pp 1057-1066. Elsevier. 2009.

[7] Corrêa, M. V. and Aguirre, L. A. Identificação não-linear caixa-cinza: uma revisão e novos resultados. *Rev. Controle & Automação.* Vol. 15, no 2. 2004.

[8] Etkin, B. e Reid, L. D. *Dynamics of Flight Stability and Control.* 3rd edition. New Jersey. John Wiley & Sons Inc. 1996.

[9] Filippone, A. *Flight performance of fixed and rotary wing aircraft*. AIAA. 2006.

[10] Klein, V. e Morelli, E. A. *Aircraft system identification: theory and practice*. AIAA. 2006

[11] Ljung, L. *System Identification: theory for the user.* Prentice-Hall, Inc. 1987.

[12] Lopes, D. F. A. *Estimativa da atitude e posição e controle robusto de um helicóptero autônomo*. Dissertação. EESC-USP. 2010

[13] The MathWorks, Inc. *MATLAB® Software.* www.mathworks.com

[14] Mettler, B. *Identification modeling and characteristics of miniature rotorcraft*. Springer. 2010.

[15] Ogata, K. *Engenharia de controle moderna*. 4a edição. São Paulo. Prentice-Hall do Brasil. 2003.

[16] Padfield, G. *Helicopter flight dynamics*, 2nd edition. AIAA. 2007.

[17] Schafroth, D. et al. Modeling and system identification of the muFly micro helicopter. *Journal of Intelligence and Robotic Systems*. 57:27-47. Springer. 2010.

[18] Seddon, J. and Newman, S. *Basic helicopter aerodynamics*. 3rd edition. AIAA. 2011.

[19] Taamallah, S. *Flight dynamics modelling for a small-scale flybarless*. National Aeropace Laboratory NLR. 2011.

[20] Taha, Z e Tang, Y. R. Development of an onboard system for flight data collection of a small-scale UAV helicopter. *Journal of Mechatronics.* Vol 21, pp 132-144. Elsevier. 2011.

[21] Takahashi, M. D. *A flight-dynamic helicopter mathematical model with a single flap-lag-torsion main rotor*. NASA Techinacal Memorandum 102267. 1990.

[22] Tischler, M. B. e Remple, R. K. *Aircraft and rotorcraft system identification*. 2nd edition. AIAA. 2012.

Contact Author Email Address

mail to: fcreis@live.com; belo@sc.usp.br

Copyright Statement