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MATHEMATICAL MODELS AND OPTICAL INVESTIGATION OF TWO PHASE FLOWS IN WIND TUNNELS

Amelyushkin I.A.
***Central Aerohydrodynamic institute**

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Abstract

Mathematical model of aerosol flow with non-spherical ice crystals was proposed. Algorithms were developed in order to determine non-equilibrium aerosol flow with supercolled droplets and ice crystals parameters in wind tunnels via laser sheet images analysis. Numerical algorithms were developed in order to calculate carrying gas parameters fields (velocity, temperature, pressure, etc.) via processing particles' velocity fields. Differences between velocities of particles and gas were taken into account. Examples of application of technologies are presented. A system and numerical algorithms of image processing (inverse problems' solutions) were developed in order to visualize two phase flow and to determine particle and droplet concentration (mass, volumetric and numerical) as well as their size distribution in laser sheet plane near a solid body in aerosol flow. Stability of inverse problems' solutions were proved and investigated. A method of calibration was developed. These methods which are devoted for two phase flow parameters determination were developed to take into account some essential physical effects. Mathematical models of scattered light propagation in multiphase flow were developed.

1. Introduction

Multiphase flows play significant role in technology [1,2], nature and life of human beings, in particular, in icing wind tunnels [2,3]. Although in most cases particles are assumed to have spherical shape in aircraft icing and icing wind tunnels, as a matter of fact, shapes of particles are far not spherical ones [4,5]. In present paper mathematical model of two-phase flow with non-spherical particles is proposed in terms of continuum mechanics equations. Particles-based methods of flow diagnostics enable to restore flow velocity vector fields in 3D space and its evolution in time as well as some derivatives of this field[6–8]. Increase of information which is taken from wind tunnel experiments enable to reduce costs of these experiments and improve precision of the results[8–11]. In present work methods of flow parameters optical diagnostics were developed.

2. Mechanics of aerosol flow with non-spherical ice crystal

In spite of the variety of drag coefficient models of single non-spherical particles motion (e.g. [4,5,12]) in a fluid numerical simulation of two phase flows with billions of non-spherical particles or ice crystals in a frame of continuous fluid is still problematic. In present paper non-spherical particles' behavior is described in terms of pseudodiffusion coefficient D_{NS}. Direction and magnitude of a force which acts on non-spherical particle in moving gas or liquid depend on particle's orientation. Let us determine the average force vector, which acts on non-spherical particle:

$$\boldsymbol{F}_{A}(\mathrm{Re},\mathrm{M})=\frac{\iiint \boldsymbol{F}(\mathrm{Re},\mathrm{M},\varphi,\theta,\psi)\,d\varphi\,d\theta\,d\psi}{\iiint d\varphi\,d\theta\,d\psi}.$$

This average force depends mainly on Reynolds and Mach numbers (Re and M correspondingly) of the flow near the particle. Root-mean-square deviation of this force also depends mainly on

Mach and Reynolds number of the particle in a flow. As a result, the following equation could be written:

$$\delta F_{NS}=\sqrt{\frac{\iiint\left|\boldsymbol{F}(\mathrm{Re},\mathrm{M},\varphi,\theta,\psi)-\boldsymbol{F}_{\mathrm{A}}\right|^{2}\mathrm{d}\varphi\mathrm{d}\theta\mathrm{d}\psi}{\iiint\mathrm{d}\varphi\mathrm{d}\theta\mathrm{d}\psi}}.$$

Having root-mean-square deviation of the force which acts on non-spherical particle the non-spherical particles' scattering (pseudodiffusion) coefficient could be obtained: $D_{\mathrm{NS}}=\sqrt{\frac{\delta F_{\mathrm{NS}}}{\rho_p}}$.

Here ρ_p – is a density of particle's material. Scattering of non-spherical particles leads to the appearance (in two-phase flow motion equations) of additional mass flow:

$\boldsymbol{q}_m^{\mathrm{DNP}}=D_{\mathrm{NS}}\left(\rho_p^V+\rho\right)\bar{\nabla}\left(\rho_p^V/\left(\rho_p^V+\rho\right)\right)$.

Neglecting phase transitions and collisions between particles equations aerosol flow motion will be following ones:

1. Mass conservation equations:

$\frac{\partial\rho_p^V}{\partial t}+\left(\bar{\nabla}\cdot\rho_p^V\boldsymbol{V}_p\right)=\boldsymbol{q}_m^{\mathrm{DNP}}$ – for non-spherical particles' mass concentration ρ_p^V. Here $\boldsymbol{V}_p$ – vector of particles' velocity, t – time.

$\frac{\partial\rho}{\partial t}+\left(\bar{\nabla}\cdot\rho\boldsymbol{V}\right)=-\boldsymbol{q}_m^{\mathrm{DNP}}$ – for gas density ρ, here $\boldsymbol{V}$ – gas velocity.

2. Pulse conservation equations:

$\rho_p^V\frac{\mathrm{d}\boldsymbol{V}_p}{\mathrm{d}t}=\frac{\rho_p^V}{m_p}\boldsymbol{F}_p+\bar{\nabla}\boldsymbol{q}_m^{\mathrm{DNP}}\boldsymbol{V}_p$ – for particles.

$\rho\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}=\bar{\nabla}\cdot\hat{\boldsymbol{P}}-\frac{\rho_p^V}{m_p}\boldsymbol{F}_p-\bar{\nabla}\boldsymbol{q}_m^{\mathrm{DNP}}\cdot\boldsymbol{V}$ – for gas.

Here $m_p=\frac{4}{3}\pi a_p^3\rho_p$ – mass of a single particle, where a_p – is a radius of a sphere whose volume equals a volume of non-spherical particle, $\boldsymbol{F}_p$ – force, which acts on particles from gas. Various kinds of mathematical models for this force are given in [14,15]. According to simple physical estimations and numerical calculations for particle's motion in a gas the dominant force is drag force $\boldsymbol{F}_p=\frac{\pi a_p^2\rho}{2}C_D\left|\boldsymbol{V}-\boldsymbol{V}_p\right|\left(\boldsymbol{V}-\boldsymbol{V}_p\right)$, C_D – drag coefficient. The first component in the right side of pulse equation for gas is viscous strain tensor: $\hat{\boldsymbol{P}}=2\mu\hat{\boldsymbol{S}}+b\hat{\boldsymbol{E}}$, where strain tensor rate:

$\hat{\boldsymbol{S}}=\frac{1}{2}\bar{\nabla}\boldsymbol{V}+\frac{1}{2}\left(\bar{\nabla}\boldsymbol{V}\right)^{\mathrm{T}}$, $\hat{\boldsymbol{E}}$ – unity matrix, μ – dynamic gas viscosity, $b=-P-\left(\frac{2\mu}{3}-\frac{\zeta}{3}\right)\bar{\nabla}\cdot\boldsymbol{V}$, P – gas pressure, ζ – volume viscosity.

3. Energy balance equations:

$\rho_p^V\frac{\mathrm{d}e_p}{\mathrm{d}t}=\frac{\rho_p^V}{m_p}\boldsymbol{F}_p\cdot\boldsymbol{V}_p+\bar{\nabla}\boldsymbol{q}_m^{\mathrm{DNP}}\frac{\boldsymbol{V}_p^2}{2}+\frac{\rho_p^V}{m_p}Q_p$ – for energy density $e_p=U_0+C_0T_p+\frac{\left|\boldsymbol{V}_p\right|^2}{2}$ of non-spherical particles;

$\rho\frac{\mathrm{d}e}{\mathrm{d}t}=\bar{\nabla}\hat{\boldsymbol{P}}\cdot\boldsymbol{V}-\bar{\nabla}\boldsymbol{q}_m^{\mathrm{DNP}}\frac{\boldsymbol{V}^2}{2}+\bar{\nabla}\left(\lambda\bar{\nabla}T\right)-\frac{\rho_p^V}{m_p}\boldsymbol{F}_p\cdot\boldsymbol{V}_p-\frac{\rho_p^V}{m_p}Q_p$ – for gas energy density $e=\frac{C_V}{\mu_m}T_p+\frac{\left|\boldsymbol{V}_p\right|^2}{2}$.

U_0 – internal energy of particles, C_0 – specific heat capacity of particles' material, λ – is a gas heat conductivity coefficient, T_p – temperature of particles, T – gas temperature. $Q_p=2\pi a_p\lambda\left(T_p-T\right)\mathrm{Nu}(\mathrm{Re},\mathrm{Pr})$ – heat flux from gas to moving particles, where

$\mathrm{Nu}=2+\frac{1}{2}\mathrm{Re}_p^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$ – Nusselt number, $\mathrm{Re}=2a_p\rho|\boldsymbol{V}-\boldsymbol{V}_p|/\mu$, $\mathrm{Pr}=\mu C_p/\mu_m\lambda$ – Prandl number.

A described above system of equations could be used for simulation two phase flows with non-spherical particles or ice crystals.

3. Flow parameters reconstruction via droplets' and crystals' motion analysis

Measurement of particles' velocity in a laser sheet plane may give information about gas parameters[6–9] with a required precision. The simplest way of gas velocity field (which may significantly differ from particles velocity field) correction could be connected with well-known

Stokes's drag coefficient model (C_D = 24/Re) which could be applied for small differences between particle and gas velocities, and as a result small Reynolds numbers Re. In this case because of small influence of temperature on particle motion in the gas in many practical problems there is no need to calculate density (because its influence on particles' motion is negligible when Re is small) field and the correction procedure may be expressed by the following equation (which takes in account unsteadiness in comparison with[15]:

$$\boldsymbol{V} = \boldsymbol{V}_p + \left[\frac{\partial \boldsymbol{V}_p}{\partial t} + \left(\boldsymbol{V}_p \cdot \bar{\nabla}\right)\boldsymbol{V}_p\right]\tau_R .$$

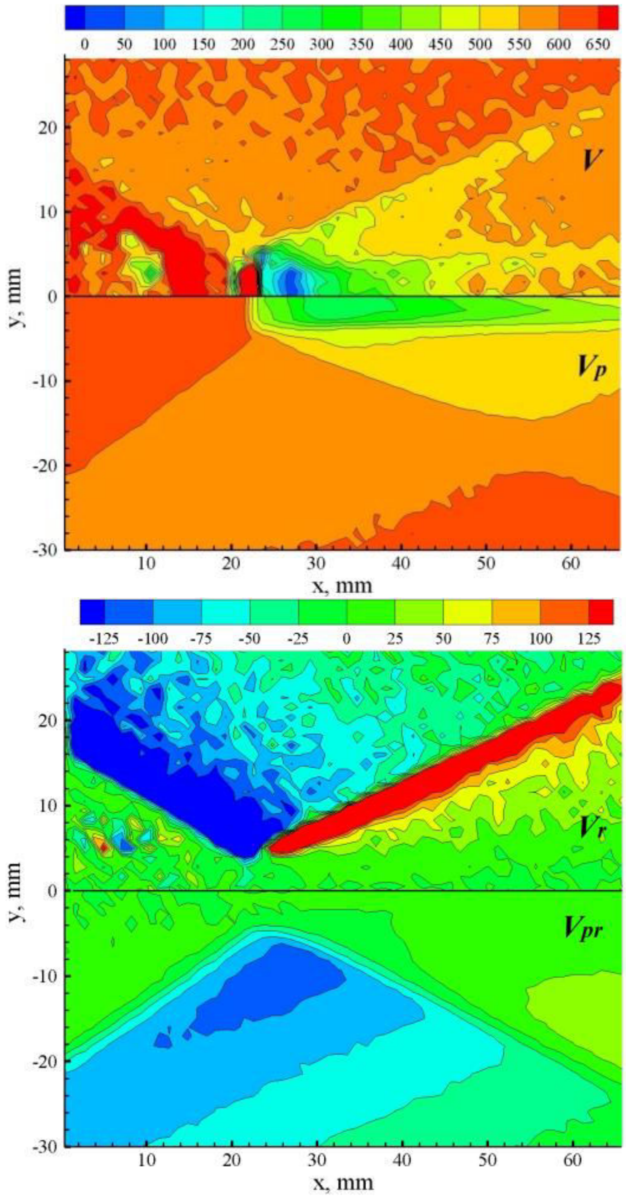

Fig. 1 Particles velocity distribution [16,17] and gas velocity distribution which was obtained via particles velocity vector field analysis.

Here $\tau_R = \dfrac{2\rho_p a_p^2}{9\mu}$ – particles' relaxation time.

In a stationary flow temperature field could be obtained from gas velocity field, neglecting viscous effects from the following expression:

$$T = \frac{\mu_m}{R}\,\frac{\boldsymbol{V}\cdot\left[(\boldsymbol{V}\cdot\bar{\nabla})\boldsymbol{V} - \dfrac{\gamma-1}{2\gamma}\bar{\nabla}|\boldsymbol{V}|^2\right]}{\bar{\nabla}\boldsymbol{V}} .$$

Here μ_m – gas molar mass, R – universal gas constant, γ – adiabatic index. The derivation of the last formula is given in [8] as well as expressions for pressure, and density distributions.

In order to make a correction for the difference between particles' velocity and gas velocity more complex models (in comparison with Stokes drag coefficient model) may be used (e.g. [13,14,18]). Although Stokes drag coefficient model couldn't be applied correctly for big Reynolds numbers of flow around particles, (e.g. for high velocity gradient and low density flows[2, 15,16], e.g. supersonic flows in shock wind tunnel and for flows with massive droplets or particles, e.g. in icing wind tunnel[2,3,8,9]) it is not necessary to take into account compressibility of the flow. In present work evolution of gas density, pressure and temperature were taken into account. Thus gas parameters (gas velocity, density, temperature and pressure) were restored on wind tunnel axis (Fig. 2). The description of wind tunnel experiment is given in [17,18]. The method was described and tested in [8,9]. Experimental data which was used to verify algorithms was taken from [16]. Fig. 2 illustrated that distributions of gas flow parameters could be restored via particles' velocity field analysis and additional information about gas parameters in settling chamber – which is far away from wind tunnel working section.

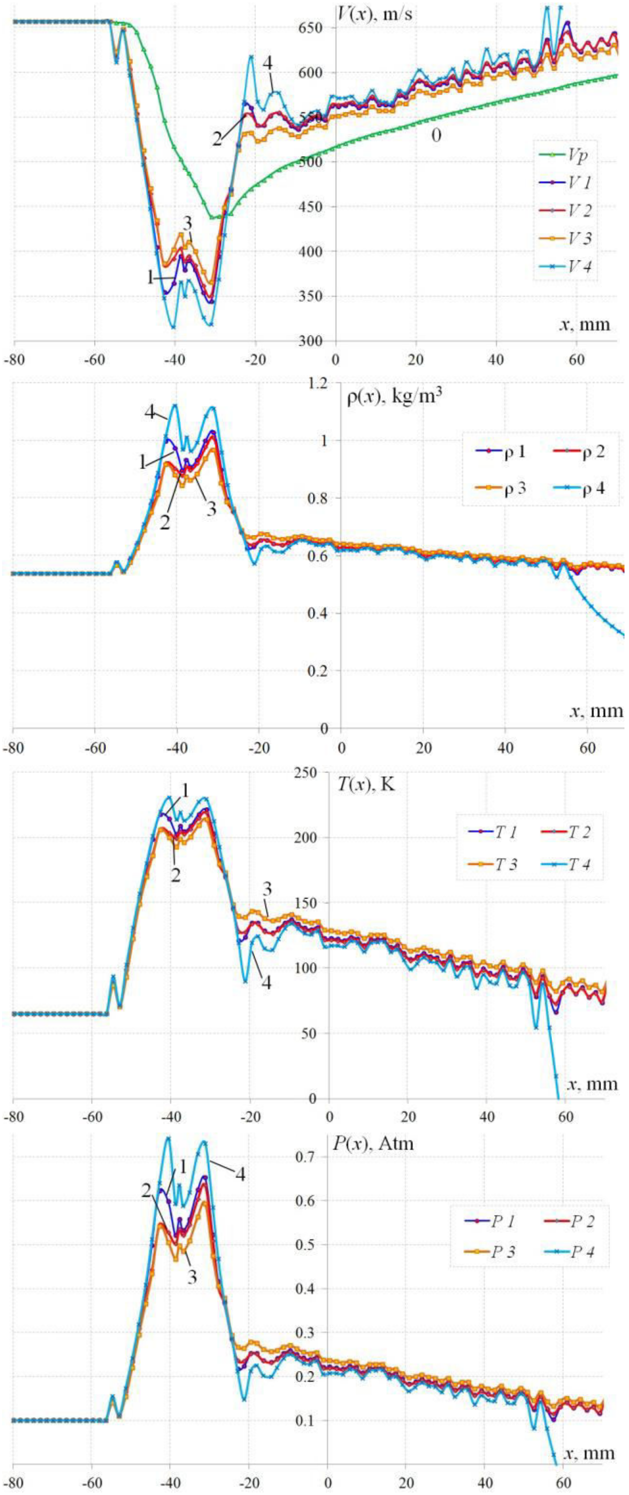

Fig. 2: Examples of gas parameters' distributions along wind tunnel symmetry line which were obtained via particles' velocity field analysis. From top to the bottom velocity (particles' velocity is green), density, temperature and pressure. Curves 1÷4 correspond to different models of particles' motion which were used for particles' image analysis.

4. Panoramic nephelometry of polydisperse flow

In aircraft icing wind tunnel investigation a problem of uniform distribution of supercooled droplets and ice crystals along the section of wind tunnel becomes essential. The method of particles' concentration and sizes distribution is described in [8–10]. The plane-parallel laser sheet was formed and used in order to simplify image processing and apply methods of inverse problems' solution in image analysis[8–10].

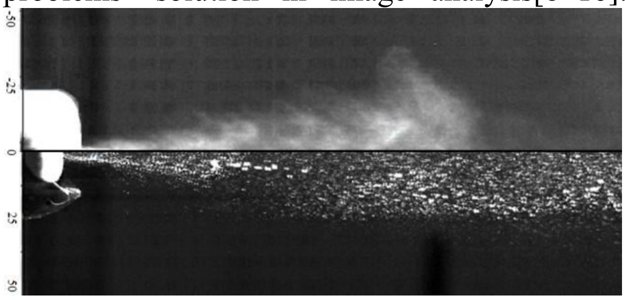

Fig. 3: Comparison of two-phase jet atomizers

Example of 2D diagnostics of particles' concentration distribution is given in Fig. 3. In comparison with [19] algorithms which are described in [8–10] allow to take into account extinction of propagating light (through two-phase flow) and to determine particles' size distribution in laser sheet plane.

5. Calibration of measurement system

Nephelometrical measurement system could be calibrated via comparison between gas intensity of light on the laser sheet image and particles concentration which is obtained via weighting them and measurement their sizes through the instrumentality of electron microscope. Way of calibration of the measurement system is described in [8,10].

6. Conclusion

Methods of optical diagnostics of two-phase flows parameters (including supercooled droplets and non-spherical ice crystals concentration and sizes distribution) were developed and used for wind tunnel experiments Mathematical model for aerosol flow with non-spherical particles was proposed.

7. Acknowledgement

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References

[1] Miller A.B., Potapov Yu.F. and Stasenko A.L. Experimental and theoretical investigation of aircraft icing in the case of crystal and mixed-phase flow. *29th Congress of the International Council of the aeronautical sciences*, St-Peterburg, Russia, Paper No 0576, 2014.

[2] Vasilevskii E.B. and Osiptsov A.N. Experimental and numerical study of heat transfer on a blunt body in dusty hypersonic flow. *AIAA Paper*, Vol. 99, No3563, pp 1-11, 1999.

[3] Ragni A., Esposito B., Marrazzo M. Bellucci M. and Vecchione L. Calibration of the CIRA IWT in the high speed configuration. *43rd AIAA Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, AIAA 2005–471, 2005.

[4] Zastawny M., Mallouppas G., Zhao F. and van Wachem B. Derivation of drag and lift force and torque coefficients for non-spherical particles in flows. *International Journal of Multiphase Flow,* Vol. 39, pp 227-239, 2012.

[5] Hölzer A. and Sommerfeld M. New simple correlation formula for the drag coefficient of non-spherical particles. *Powder Technol*, Vol. 184, pp 361-365, 2008.

[6] *Oudheusden B.W., Scarano F., Roosenboom E.W.M., Casimiri E.W.F. and Souverein L.J.* Evaluation of integral forces and pressure fields from planar velocimetry data for incompressible and compressible flows. *Exp Fluids*, Vol. 43, pp 153-162, 2007.

[7] Ghaemi S., Ragni D. and Scarano F. PIV-based pressure fluctuations in the turbulent boundary layer. *Exp Fluids*, Vol. 53, pp 1823-1840, 2012.

[8] Amelyushkin I.A. Algorithms of laser sheet plane image processing in two phase flows investigation. *International scientific and technological conference «Optical methods of flow research»,* Moscow, MEI, pp 1-18, 2015. (in Russian).

[9] Amelyushkin I.A. Visualization and simulation of physical phenomena in aircraft icing process. *Proc. from the 10th Pacific Symposium on Flow Visualization and* Image (PSFVIP10), Naples, Italy, Paper ID-187, 2015.

[10] Amelyushkin I.A. Way of disperse phase numeric concentration field measurement in the aerosol flow and the apparatus of its realization. *Patent* No №2014119714 from 16.05.2014, № 2562153 (in Russian), 2014.

[11] Stryczniewicz W. An inverse problem solution for post-processing of PIV data. *Proceedings from the 10th Pacific Symposium on Flow Visualization and Image Processing (PSFVIP10)*, Naples, Italy, Paper ID-044, 2015.

[12] Nilamdeen S., Habashi W.G. Multiphase approach toward simulating ice crystal ingestion in jet engines. *J. Propulsion and Power*, Vol. 27, No. 5. pp 959–969. 2011.

[13] Dring R.P. Sizing criteria for laser anemometry particles. *Trans. ASME, J. Fluids Eng*, Vol. 104, pp 15-17, 1982.

[14] Melling A. Tracer particles and seeding for particle image velocimetry. *Meas. Sci. Technol*, Vol. 8, pp 1406-1416, 1997.

[15] Koike S., Tamura T. and Masua G. Influence of Drag Coefficients and Velocity Fluctuation on PIV Correction Method. *AIAA -75 Copyright AIAA PIV 47th AIAA Aerospace Sciences Meeting*, Orlando, Florida pp 1-17, 2009.

[16] Tedeschi G., Gouin H. and Elena M. Motion of tracer particles in supersonic flows. *Exp. Fluids*, Vol. 26, pp 288-296, 1999.

[17] Ganiev Yu.H., Lipnitskiy Yu.M., Filippov S.E., Kozlovskiy V.A., Krasilnikov A.V., Markovich D.M., Gobyzov O.A., Lozhkin Yu.A. and Ivanov I.E. PIV measurements of nonuniform supersonic flow in continuous-operated wind tunnel. *The 13th Asian Symposium on Visualization. Abstracts*, pp 67-68, 2015.

[18] Ganiev Yu. H., Lipnitskiy Yu. M., Filippov S.E., Kozlovskiy V.A., Krasilnikov A.V., Markovich D.M., Gobyzov O.A., Lozhkin Yu. A. and Ivanov I.E. PIV measurements of shock wave interaction in continuous operated supersonic wind tunnel. *E-Book Proceedings. 10th Pacific Symposium on Flow Visualization and Image Processing*, Paper ID-191, 2015.

Contact Author Email Address

Amelyushkin_Ivan@mail.ru

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