

# AIR TRAFFIC EFFICIENCY EVALUATION BY TRAJECTORY OPTIMIZATION AND ADS-B DATA

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## Abstract

*The efficiency of jet airliner arrival flights into Munich Airport is analyzed via ADS-B data and trajectory optimization. Flight parameters of actual flights including fuel consumption are estimated from position and time in the recorded ADS-B data with meteorological and aircraft performance model. Each flight is optimized in terms of fuel and time so as to give the optimal arrival time and sequencing under an appropriate separation constraint. The comparative results demonstrate the efficiency of AMAN-assisted arrival flights, and give worthwhile information in discussing expected benefits achievable by new arrival management system.*

## 1 Introduction

Airliners have become a convenient and indispensable transportation for most people nowadays, and amount of air traffic is anticipated to be doubled within the next fifteen years for several reasons such as global population growth, economical services by LCCs (Low Cost Carriers), and economic development especially in Asian countries. Fuel consumption suppression while maintaining flight safety is an urgent issue to keep the airliner aircraft as a sustainable transportation for long years in the future. Under the NextGen program in the United States and SESAR in Europe, many research projects are conducted and many research papers have been published. In Japan, CARATS (Collaborative Actions for Renovation of Air Traffic System) has been proceeded by the government as a roadmap, and several measures are introduced to realize Japan's ideal air traffic management system [1].

In those efforts, new CNS (Communication, Navigation and Surveillance) technologies are discussed to assist realization of more efficient operations achieved by the next-generation Air Traffic Management (ATM) system.

In a congested airport all over the world, many arrival aircraft cause a complicated but extremely important problem, that is, arrival management. In many European airports, Arrival Manager called AMAN where several products have been developed by aviation industries is used to schedule time and sequence of the arrival aircraft. This system is remarkable triumph in providing desirable arrival time and sequencing to the controllers satisfying various demands for the safety, efficiency and capacity [2]. In the meanwhile, in Japan, voice-based communications between controllers and pilots still occupy a large part of air traffic control services. Previous research quantitatively revealed that serious flight efficiency deterioration and time delay occurred by "radar vectored" control at the Tokyo International Airport which is the most congested airport in Japan [3].

The main purpose of this research is to evaluate air traffic efficiency of arrival flights at an airport where the AMAN system is already introduced. This paper has originality in that ADS-B data is used to estimate fuel consumption for actual flights and to derive optimal arrival time and sequencing. These are obtained by flight trajectory optimization and arrival time assignment procedures, and compared with actual values to examine efficiency of AMAN-assisted arrival flights.

## 2 Flight Analysis for Arrival Aircraft trajectories

### 2.1 ADS-B data recorded at Munich airport

Automatic Dependent Surveillance Broadcast (ADS-B) is a precise satellite-based surveillance system [4]. ADS-B Out broadcasts aircraft position and time data measured by Global Positioning System (GPS). Other operational information such as identification code and pressure altitude are transmitted from each aircraft's transponder. Airborne aircraft and air traffic controllers on the ground can receive not only those information but weather and traffic position information via ADS-B In in the future system raised by NextGEN program. In a view of the potentiality of ADS-B as a future surveillance system, the author focuses on its wide utilization to air traffic management research.

To understand current operations by terminal radar approach control (TRACON) at an European airport and to evaluate the air traffic efficiency, the operational data was recorded via ADS-B Out system for multiple aircraft flying around Flughafen München Franz Josef Strauß commonly known as Munich Airport. The data was recorded by a simple antenna generally used for receiving digital terrestrial broadcast. The recording spot was selected at the Visitors Park located in between two runways of Munich Airport. Data recording information is listed in Table 1.

Table 1. ADS-B data recording information

	Properties
Date	18/ September/ 2015
Duration	18:40 to 20:00 (UTC)
Spot	Munich Airport Visitors Park (48.35549° N, 11.77314° E)
Device	DVB-T+DAV+FM USB tuner

Software developed by Electric Navigation Research Institute (ENRI) is used to decode the messages and to record them as analyzable data. Properties of recorded ADS-B data are indicated in Table 2.

Table 2. ADS-B data recorded at Munich Airport

	Properties
Item	ICAO 24 Bit Code, date, time, latitude, longitude, pressure altitude, ground speed, track angle
Number of data	1,038,027
Number of aircraft	584

Number of aircraft is obtained by counting number of unique ICAO 24 Bit Code. All aircraft's position in the recorded data is plotted on the googlemap [5] in Figure 1. Since the spot locates at the top of a small artificial hill where tall buildings or obstacles don't exist, the obtained data covers a sufficient range of approximately 150 [NM] for a small inexpensive antenna.

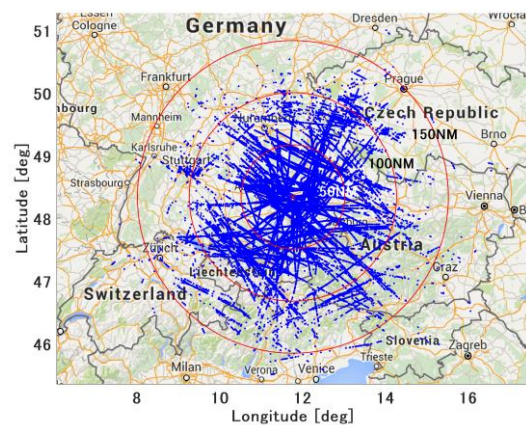


Fig. 1 Aircraft positional data recorded at Munich Airport

All these recorded data is categorized into three groups: inbound and outbound flights to/from Munich Airport and the others. 47 flights were detected as arrival at runway 26L or 26R of Munich Airport, 38 flights were departure from the same runways and the rest of the data was flights between two other airports including extremely low-quality data. All of the arrival aircraft didn't complete that day's operation because 5 flights departed from the same runway within 50 minutes. Figure 2 shows flight trajectories reconstructed from the recorded data. Departure and arrival trajectories from/to runway 26L/26R are plotted by red and blue lines respectively. Although some departure and arrival trajectories are crossing, it

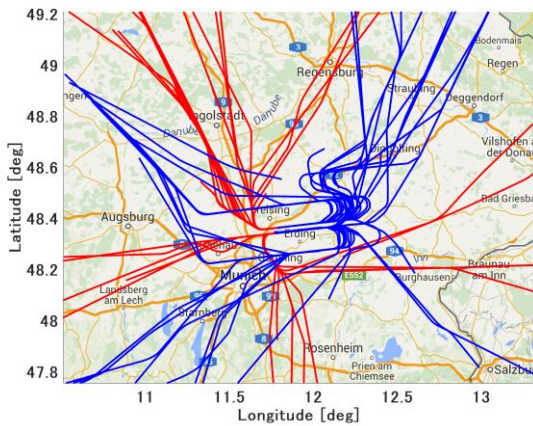


Fig. 2 Reconstructed trajectories from ADS-B data (red: departure, blue: arrival, 26L/26R Munich Airport)

seems that the areas for each use are basically separated with each other.

### 2.2 Time Interval and Separation of Arrival Aircraft

Airport capacity is another concern in the ATM research. In Munich Airport, two trombone-shaped traffic patterns are set around the runway. 12 RNAV (aRrea NAVigation) transition routes are defined on the patterns. These RNAV transition routes connect 15 STARs (Standard instrument arrival) to runway 26L and 26R. As can be seen from Figure 2, the arrival flights from northeast area which occupy a large part of all arrivals are radar vectored to fly S-shaped track to adjust arrival time interval and separation. While runway 26R was exclusively used for the arrivals from north area, some of those flights also arrived at runway 26L. It is assumed that the runway 26L was used to accept the excess arrivals from congested north area to the runway 27R in this duration. Time interval and horizontal separation at two FAFs (Final Approach Fix) of runway 26L and 26R, NELBI and GUDEG are indicated as histogram in Figures 3 and 4, respectively. Two values are detected as lower than 90 seconds at NELBI arrival. These values take over 80 seconds; besides, enough horizontal separation is maintained at more than 5 miles for both arrivals. The results suggest that all arrival flights at Munich Airport are properly controlled so as to endure a high density operation with ensuring the safety.

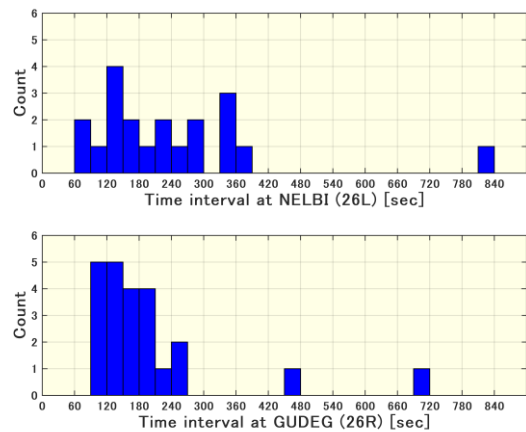


Fig. 3 Time interval of arrival aircraft at two FAFs

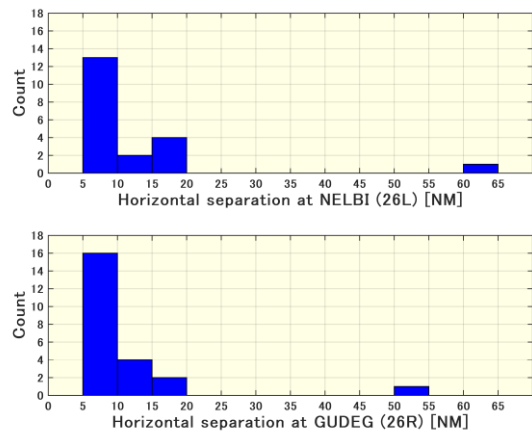


Fig. 4 Horizontal separation of arrival aircraft at two FAFs

### 2.3 Flight Parameter Estimation

Fuel consumption is an important quantity to discuss the air traffic efficiency. Ensuring a safe, economical and punctual flight is common mission for all organizations involved in civil aircraft's operation. A model based approach to estimate fuel consumption is introduced in this section. The research group in Kyushu University where the author previously belonged developed a novel and effective method to calculate fuel consumption from GPS position and time data recorded on board an airborne airliner cabin [6]. The research found that fuel consumption can be estimated with high accuracy using only aircraft position and time data. The process of flight parameter estimation from actual flight trajectories is explained in the papers [3, 7, 8]. Flight parameters such as calibrated airspeed (CAS),

true airspeed (TAS), Mach number, temperature and wind are estimated from GPS position data by applying meteorological Grid Point Value (GPV) data released from Japan Meteorological Agency (JMA) [9], and performance variables such as lift to drag ratio, thrust and fuel flow are calculated using BADA (Base of Aircraft Data) model developed and maintained by EUROCONTROL [10]. BADA version 3.11 is used in this paper. Quality of meteorological GPV data and the BADA performance model directly influence on the accuracy of estimated flight parameters. After comparing these obtained parameters with onboard flight data provided an airline, the two comparative investigations [11, 12] have revealed that those model and data are of sufficient quality to give accurate flight parameters which can be used in air traffic efficiency analysis [13].

The process of flight parameter estimation is applied to the ADS-B data recorded at Munich Airport. All the data required for parameter estimation such as time, latitude, longitude and altitude are broadcasted from all ADS-B compliant aircraft; therefore, flight parameters including fuel flow are observable for those aircraft flying within the range of antenna. This means that surveillance information is available on the ground, at the same time, each aircraft performance can be estimated accurately from the information.

### 3 Trajectory Optimization and Arrival Time Assignment by Dynamic Programming

#### 3.1 Performance Index

All 47 arrival flights are analyzed by comparing flight parameters estimated by the recorded trajectories, meteorological and performance information to the parameters of optimal trajectories calculated with using the identical meteorological data and performance model as the flight parameter estimation. The same initial and final positions and velocities are set as boundary conditions in the flight trajectory optimization. Three significant quantities, fuel consumption, flight distance and flight time are calculated and compared for both actual and

optimal trajectories to evaluate air traffic efficiency of the arrival flights. To obtain the optimal trajectories from actual trajectories, a performance index which incorporates fuel consumption and flight time is defined. The performance index  $J_k$  of the  $k$  th aircraft is defined as

$$J_k = \int_{t_{0k}}^{t_f} \left[ \mu_k(t) + \frac{m_k}{m_0} a_k \right] dt \quad (1)$$

where  $\mu_k [kg/s]$  is fuel flow and  $(m_k/m_0)a_k$  is a weighting parameter for the flight time.  $m_k$  is mass of the  $k$  th aircraft and  $m_0$  is representative mass which is set to normalize impact of time adjustment. Although the second term of Eq. (1), time cost can be evaluated equivalently with the fuel cost by introducing the transform parameter  $a_k$ ,  $(m_k/m_0)$  is required to avoid that the influence of time adjustment differs by scale of the aircraft in applying the optimization calculation to multiple aircraft. This weighting parameter is strongly associated with so-called the cost index (CI) set into Flight Management System (FMS) by a pilot. The performance index is also described as the cost of flight [dollars],

$$J_{k,dollars} = \int_{t_{0k}}^{t_f} \left[ \frac{1}{100} C_{fuel} \frac{\mu_k(t)}{0.4536} + \frac{C_{time}}{3600} \right] dt \quad (2)$$

where  $C_{fuel}$  is the fuel cost in [cent/lb] and  $C_{time}$  is the time cost in [dollars/lb]. Since the cost index is defined as time cost per fuel cost, comparing Eq. (1) and (2) introduces following relation between  $a_k$  and CI.

$$CI_k = \frac{C_{time} [dollars/lb]}{C_{fuel} [cent/lb]} = 79.37 \frac{m_k}{m_0} a_k \quad (3)$$

Figure 5 explains the relationship between CI and performance index. CI or the weighting parameter is equivalent to the slop of tangent of the feasible solutions' boundary, i.e. the optimal solutions for various weighting parameters form a Pareto front. In this manner, cost index is a free parameter for each performance index to be optimized; therefore, it is generally set according to airline operators' policies.

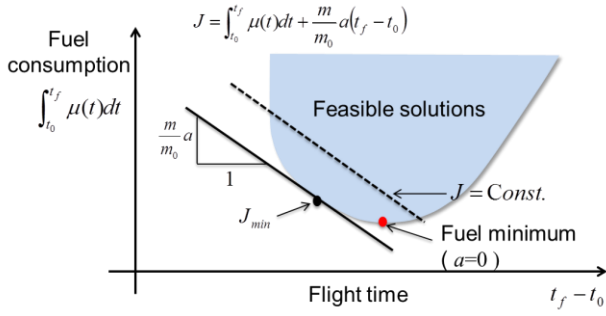


Fig. 5 Performance index for various weighing parameters

### 3.2 Trajectory Optimization by Dynamic Programming

Various flight trajectory optimization methods have been studied in the field of optimal control theory. Features of two methods are investigated in the author's previous paper [14]. A direct method, in which a finite number of parameters indicate the trajectory and are solved as a parameter optimization problem, has become increasingly used thanks to remarkable progress in the processing capability and memory capacity of recent computers. A direct method does not need any complicated formulation even if the problem includes multiple inequality constraints. Gradient based method with large sparse matrix is a well-known and relatively recent approach in a direct method. Two major advantages of the method are listed below:

- (1) A fast computation is possible even if a precise integration scheme is used.
- (2) A smooth optimal trajectory can be obtained by a numerical integration scheme.

These advantages are of assistance for practical trajectory optimization problems; nevertheless, a model should be described by a continuous function and a tabular data must be arranged so as to construct a smooth surface. Additionally, convergence is not always guaranteed. Those features are not preferable in developing an analysis tool within a short amount of time. This research uses Dynamic Programming (DP) as an easy-to-handle optimization method considering the overall applicability and usability.

In the trajectory optimization, DP is applied as a combinatorial optimization method. The optimal flight trajectory is gained by selecting state variable grid points generated in

a state space such that the performance index denoted by Eq. (1) is minimized. The simplest equidistant grid is used in the calculation. Altitude, velocity and lateral deviation from the great-circle course between the initial and final points are defined as state variables under point mass approximation. Corresponding control inputs are flight-path climb angle, thrust and flight-path heading angle. The detail is explained in the previous literatures [15, 16].

### 3.3 Arrival Time Assignment

Each flight is optimized in terms of performance index. If the terminal time is set to be free, conflicts would inevitably occur at the merging point. To maintain safe arrival time separation, the following Eq. (4) should be imposed as an inequality constraint on the two neighboring aircraft at the terminal point.

$$\begin{aligned} |t_{f_k} - t_{f_l}| > t_{\min, separation}, \text{ for any } k, l (k \neq l) \\ t_{\min, separation} = 90 \text{ [sec]} \end{aligned} \quad (4)$$

Other constraints such as air routes and spatial conflicts with the other aircraft are not considered in this analysis to investigate the influence of arrival time assignment on overall benefits.

In the rush hour at the airport, cost index based on some rules should be used, in other words, the time weighting parameter should be adjusted to consider the arrival sequence fairly. Fuel consumption values increase by changing the arrival time from the fuel minimum point; hence, to adjust arrival time of each aircraft, a summation of each performance index denoted by  $J^*$  in Eq. (5) is redefined and minimized in terms of all arrival flights.

$$\begin{aligned} J^* &= \sum_k \frac{m_0}{m_k} J_k \\ &= \sum_k \left[ \frac{m_0}{m_k} \int_{t_{0k}}^{t_{fk}} \mu_k(t) dt + a_k \int_{t_{0k}}^{t_{fk}} dt \right] \end{aligned} \quad (5)$$

If the performance index is defined by simply summing Eq. (1), the arrival time of heavier aircraft is adjusted to less than that of a lighter aircraft when the separation condition is violated; consequently, heavier aircraft has priority. To avoid such unfair evaluation,  $m_0/m_k$  is introduced before summing each

performance index; thereby, priority does not depend on the scale of aircraft.

The two problems; flight trajectory optimization and arrival time assignment construct a so-called bi-level optimization problem. The upper problem, in which the optimal arrival time is assigned on each flight by minimizing summation of all weighted performance indices, includes terminal-free flight trajectory optimization as the lower problem. As a practical optimization method, dynamic programming is also used in the arrival time optimization. In the lower problem, optimal flight trajectories are generated for various weighting parameters denoted in Eq. (1). The optimal trajectories gained by changing the weighting parameter  $a_k$  are Pareto optimal solutions for different combination of fuel consumption and flight time. In the upper problem, the optimal combination of arrival time is searched from a set of optimal flight trajectories which are already obtained for each arrival flight in the lower problem. The concept of arrival time assignment by DP is explained in Figure 6. The combinatorial optimization problem is solved for two free parameters: flight identification number and weighting parameter  $a_k$  or CI. Sequence from the first arrival to the last arrival is set as the independent variable. This approach for arrival time assignment is proposed in the papers [7, 8]. These researches extended the approach given in reference [17] by adding one more variable to optimize the sequence of arrival flight.

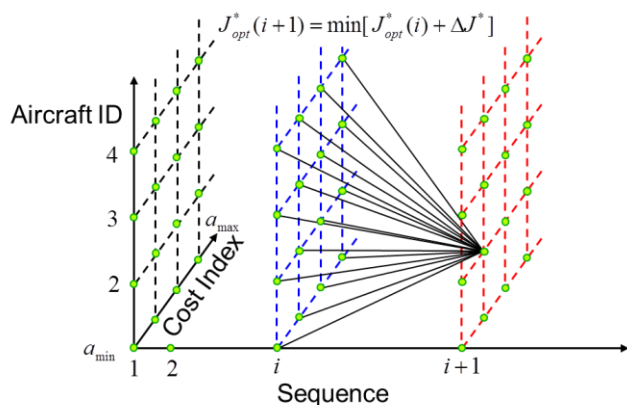


Fig. 6 Arrival time optimization by DP

## 4 Application Results of Optimization

### 4.1 Optimal Trajectory of Scheduled Flight

The arrival flights to runway 26L and 26R are respectively optimized following the procedures explained in chapter 3. As can be seen from Figure 2, if the trombone-shaped patterns would not be used at all and only two FAFs, NELBI and GUDEG, would be set as final position of optimal trajectories, some flights from west area will probably fly above the runway and arrive at FAFs from opposite directions. To avoid this, the final position of those flights is set on two fixes, DM455 and DM425 in the trajectory optimization calculation, i.e. the flight paths between the intermediate fix and the FAF are not optimized. In Figure 7, the optimal flight paths are compared to actual flight paths denoted by ADS-B data in the legend.

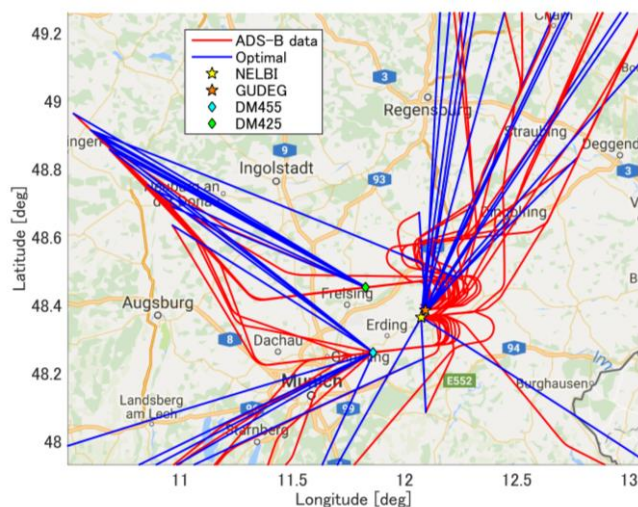


Fig. 7 Optimal and actual path of arrival flight

All of the optimal paths do not use lateral deviation in spite that flight time is adjusted from the fuel minimum point by the arrival time assignment; consequently, they are simple straight lines which connect initial and final positions.

### 4.2 Optimal Arrival Time and Sequencing

In the arrival time assignment process, combinations of weighting parameter  $a_k$  and original sequencing ID are optimized so as to

minimize the total performance index denoted by Eq. (5) under the safe separation constraint.

From the viewpoint of actual operations, the flights which have longer time than fuel

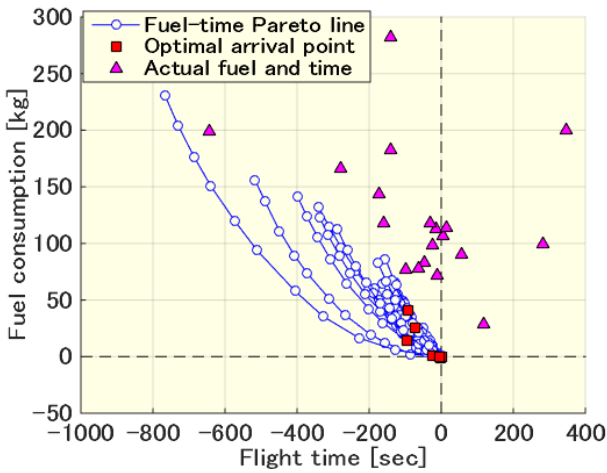


Fig. 8 Fuel and time difference of actual flight relative to fuel minimum point (NELBI, 26L)

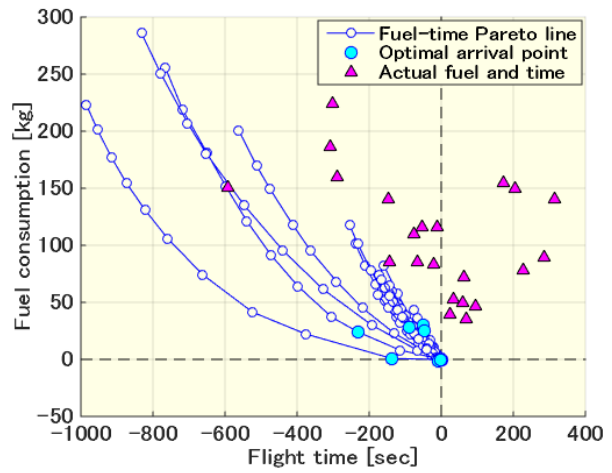


Fig. 9 Fuel and time difference of actual flight relative to fuel minimum point (GUDEG, 26R)

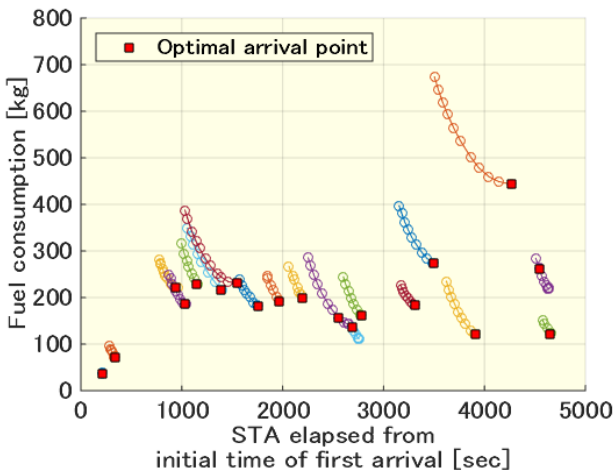


Fig. 10 Optimal arrival point assigned on each Pareto front (at NELBI, 26L)

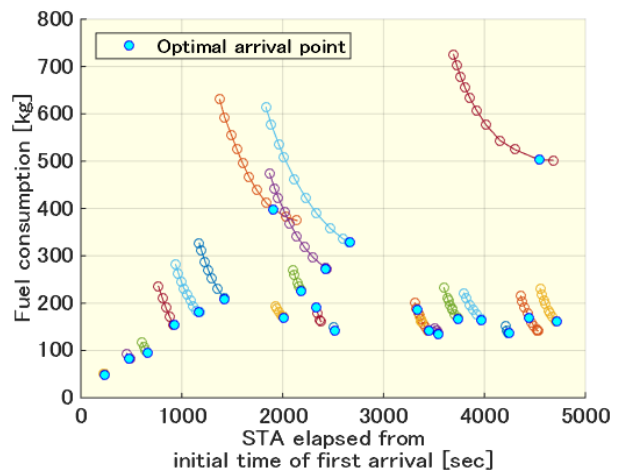


Fig. 11 Optimal arrival point assigned on each Pareto front (at GUDEG, 26R)

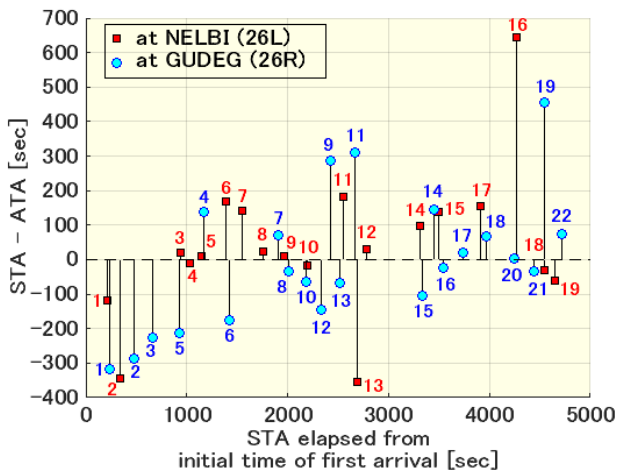


Fig. 12 Difference of STA from ATA and original sequencing ID

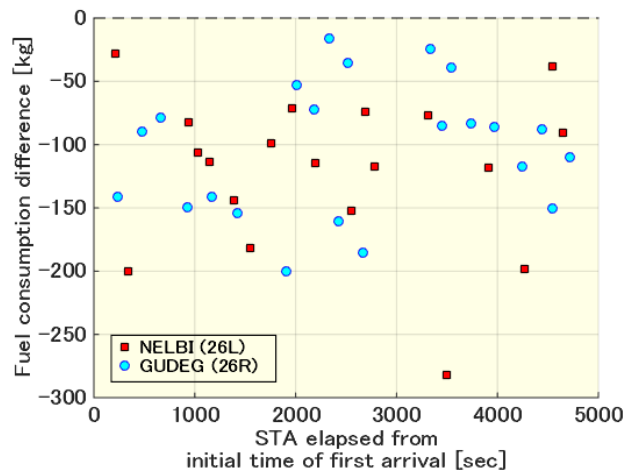


Fig. 13 Fuel consumption difference of optimal trajectories relative to actual flights

minimum flight are excluded from the candidates of optimal arrival flight; thereby, the plus parameters from 0 to 1 are adopted as  $a_k$ , which give shorter flight time.

Figures 8 and 9 show the relationship between fuel-time Pareto line of each flight and actual fuel and time. The red and cyan markers plotted on the lines are the final optimal solutions which give optimal arrival time and sequencing. Differences of flight time and fuel consumption from fuel minimum point are set as horizontal and vertical axes respectively. On the one hand, many cases of actual arrival flights to runway 26L take shorter time than the fuel minimum point; on the other hand, ratio of shorter and longer flight than fuel minimal flight is fifty-fifty in the arrival to runway 26R. The number of GUDEG arrivals which take longer time than fuel minimum is larger than that of NELBI arrivals. This suggests that many cases of the arrival flights from northern area have more flight distance extension. This interpretation is considered to be reasonable because the arrivals from northeast area are forced to gain long flight time by the radar-vectoring. Figures 10 and 11 indicate that the optimal arrival point is near to the fuel minimum point on each Pareto line to save the fuel consumption as much as possible. Naturally, the constraint of minimum time separation expressed by Eq. (4) is satisfied for all neighboring aircraft. The minimum value is 91 seconds between 12th and 13th arrival flights at NELBI, and 93 seconds between 11th and 12th arrival flights at GUDEG respectively.

Figure 12 represents differences of scheduled time of arrival (STA) from actual time of arrival (ATA) with the original sequencing ID. STA means the optimal arrival time, and original sequencing ID is the same as aircraft ID shown in Figure 6. The result indicates that both time-delayed and time-advanced flights are used as optimal arrivals. In NELBI arrival, a major part of the optimal flights takes longer time than actual flight. These time-delayed optimal arrivals mean that many cases of actual flights have less extension of flight distance and pass through the FAF comparatively earlier by higher airspeed descent. Moreover, only twice replacement of arrival

sequencing suggests that the original sequencing is almost optimal and an efficient arrival management is performed in the operation for runway 26L. GUDEG arrival, on the other hand, chooses time-advanced flight in many cases and replacement of sequencing happens more frequently. It is considered that such efficiency deterioration is caused by further extension of flight distance mainly occurred in the northeast area. Figure 13 plots fuel consumption difference between the scheduled optimal trajectories and actual flights. The average and the maximum of all 41 flights' fuel savings take the value of 111 [kg] and 282 [kg] respectively. Although the range of analysis is about 150[NM], where some flights are in the cruise phase or the powered flight phase, the fuel savings are sufficiently small even if many flights are vectored to fly longer distance. It can be said that fuel efficient operations such as continuous descent approach (CDA) is performed with the capability of flight time adjustment in the arrival flights to Munich airport.

## 5 Concluding Remarks

This research evaluated air traffic efficiency of arrival aircraft inbound to Munich Airport using ADS-B data. Fuel consumption of each arrival flight is estimated by applying the meteorological data and performance model to actual position and time of the recorded data. Additionally, each flight trajectory is optimized in terms of fuel consumption and time with the identical meteorological data and performance model, setting the same initial and final positions and velocities as the actual flights. Optimal arrival time and sequencing can be obtained for all arrivals inbound to runway 26L and 26R by solving a simple combinatorial optimization problem, where multiple cost indices are set to adjust the arrival time.

The bi-level optimization approach can give the optimal solutions which minimize a summation of total performance indices with maintaining the safe separation constraint. Comparison with the actual arrivals has revealed the operational efficiency deterioration of runway 26R indicated as larger ATA values



and frequent replacement of arrival sequencing. The reason is considered to be strongly related with the radar-vectoring longer flight occurred at northeast of the runway; nevertheless, fuel-efficient flight with reduced thrust was performed in most arrival flights as shown in the result of small fuel saving values.

This paper has a value in the point that ADS-B data has been used to evaluate the arrival management efficiency of AMAN-assisted airport through flight parameters estimation, flight trajectory optimization and arrival time assignment. It can be concluded that the AMAN system is successfully working to achieve high efficiency and density for the arrival flights inbound to Munich Airport. More analyses are required to understand maximum performance of AMAN. The AMAN system seems to be sophisticated; though, it might be some scope of improvements. Finally, this paper adds a remark that if an arrival management system would be introduced to an airport, other concerns such as reconstruction of airspace and implementation of CDA for all arrivals should be carefully considered to attain maximum performance of the system.

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### **References**

[1] Study Group for the Future Air Traffic Systems, Long Term Vision for the Future Air Traffic Systems, CARATS, Collaborative Actions for Renovation of Air Traffic Systems, URL: <http://www.mlit.go.jp/common/000128185.pdf>, 2010.

[2] EUROCONTROL, Implementation Guidelines and Lessons Learned, URL: <https://www.eurocontrol.int/sites/default/files/article/content/documents/nm/fasti-aman-guidelines-2010.pdf>, 2010

[3] Harada A, Kozuka T, Miyazawa Y, Wickramasinghe N K, Brown M and Fukuda Y. Analysis of Air Traffic Efficiency using Dynamic Programming Trajectory Optimization, *International Council of the Aeronautical Sciences (ICAS 2014)*, St. Petersburg, ICAS2014\_0743, 2014.

[4] Automatic Dependent Surveillance-Broadcast, NextGEN programs, Federal Aviation Administration, RUL: <http://www.faa.gov/nextgen/programs/adsb/>

[5] Zohar Bar-Yehuda, plot\_google\_map.m, URL: <https://www.mathworks.com/matlabcentral/fileexchange/27627-zoharby-plot-google-map>

[6] Totoki H, Wickramasinghe N K, Hamada T and Miyazawa Y. Estimation of Flight Trajectories by Using GPS Data Measured in Airliner Cabin, in Japanese, *JSASS Kouku-Uchu Gijutu (Aerospace Technology)*, Vol. 11, pp 43-50, 2012.

[7] Miyazawa Y, Matsuda H, Shigetomi S, Harada A, Kozuka T, Wickramasinghe N K, Brown M and Fukuda Y. Potential Benefits of Arrival Time Assignment, *11th USA/Europe Air Traffic Management Research and Development Seminar (ATM Seminar 2015)*, Lisbon, No. 434, 2015.

[8] Matsuda H, Harada A, Kozuka T, Miyazawa Y and Wickramasinghe N K. Arrival Time Assignment by Dynamic Programming Optimization, *ENRI International Workshop on ATM/CNS (EIWAC 2015)*, Tokyo, EN-A-019, 2015.

[9] Japan Meteorological Business Support Center Online Data Service, URL: <http://www.jmbbc.or.jp/hp/online/f-online0.html>

[10] Eurocontrol Experimental Center, User Manual for the Base of Aircraft Data (BADA) Revision 3.11: EEC Technical/Scientific Report No. 13/04/16-01, May 2013.

[11] Totoki H, Kozuka T, Miyazawa Y and Funabiki K. Comparison of JMA Numerical Prediction GPV Meteorological Data and Airliner Flight Data, in Japanese, *JSASS Kouku-Uchu Gijutu (Aerospace Technology)*, Vol. 12, pp 57-63, 2013.

[12] Harada A, Miyazawa Y and Funabiki K. Accuracy evaluation of an aircraft performance model with airliner flight data, *Transactions of JSASS Aerospace Technology Japan*, Vol. 11, pp 79-85, 2013.

[13] Miyamoto Y, Wickramasinghe N.K, Harada A, Miyazawa Y and Funabiki K. Analysis of Fuel Efficient Airliner Flight by Dynamic Programming Trajectory Optimization, *Transactions of JSASS Aerospace Technology Japan*, vol. 11, pp 93-98, 2013.

[14] Harada A. Numerical Investigation on Flight Trajectory Optimization Methods, *ENRI International Workshop on ATM/CNS (EIWAC 2015)*, Tokyo, EN-A-052, 2015.

[15] Miyazawa Y, Wickramasinghe N K, Harada A and Miyamoto Y. Dynamic Programming Application to Airliner Four Dimensional Optimal Flight Trajectory, *AIAA Guidance, Navigation, and Control (GNC) Conference*, Boston, AIAA-2013-4969, 2013.

[16] Wickramasinghe N K, Miyamoto Y, Harada A, Kozuka T, Shigetomi S, Miyazawa Y, Brown M and Fukuda Y. Flight Trajectory Optimization for Operational Performance Analysis of Jet Passenger

Aircraft, *Transactions of JSASS Aerospace Technology Japan*, vol. 12, pp a17-a25, 2014.

- [17] Bayen A M, Callantine T, Tomlin C J, Zhang J. Optimal Arrival Traffic Spacing via Dynamic Programming, *AIAA Guidance, Navigation and Control Conference*, Province, AIAA-2004-5228, 2004.

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