

Kisa Matsushima\*, Ryodai Namba\*, Hiroyuki Kato\*\* \*University of Toyama, \*\* Japan Aerospace Exploration Agency

Keywords: Pressure measurement, PIV, Wakes, CFD, 2.5D Poisson equation model

### Abstract

A pressure estimation system which uses the two and a half dimensional (2.5D) form of a Poisson equation and isentropic boundary conditions has been applied to a variety of flows. The primary objective of the system is to let stereo PIV measurement more advantageous for a wake survey in aerodynamic design of aircraft and other transport. In wakes, velocity and pressure distributions are no longer two dimensional, then common methods of in-plane two dimensional (2D) PIV pressure calculation do not work. The accuracy of estimated pressure by the system is investigated for both subsonic and transonic flows around a wing. The results show that the 2.5D model and isentropic boundary conditions work well for both subsonic (incompressible) flows and transonic (compressible) ones. Then, the system is applied to PIV measurement for the wake of a practical automobile. It works well, too, and indicates great promise as a non-intrusive pressure measurement merthod.

#### Nomenclature

- *x* = free stream direction coordinate axis
- y = span-wise direction coordinate axis
- *z* = altitude direction coordinate axis
- u = x component of velocity
- v = y component of velocity
- w = z component of velocity
- P = static pressure
- $C_p$  = static pressure coefficient

C = chord length of a wing section airfoil Re = Reynolds number  $\gamma$  = ratio of specific heat M =Mach number U = magnitude of a three dimensional velocity vector; i.e. =  $(u^2 + v^2 + w^2)^{\frac{1}{2}}$ AOA = the angle(s) of attack of a wing Lower suffix:  $\infty$  = indicates a value in the free stream flow

## **1** Introduction

Particle Image Velocimetry (PIV) is regarded as one of the important cores for flow-field measurement. It is an optical experimental method. Thus, it can measure undisturbed flow velocity vectors at a lot of grid points in a plane at a time, which is more advantageous than other experimental tools. On the other hand, it does not directly measure pressure which is a critical physical element other than velocity for aerodynamic analysis or design of transportation. Numerical methods have been recently studied to sophisticate PIV measurement technique [1, 2] in terms of pressure measurement based on PIV experiment. especially for aircraft aerodynamics [3-5]. In the combination technique of PIV and a computational method, the latter solves equations for pressure using PIV velocity data so that static pressure distributions on the PIV measurement plane are obtained. The pressure distributions are inevitably needed to design and develop aerodynamic shapes. In addition, using the wake integral method, span-wise distributions of drag

are obtained based on the derived pressure and the three-component velocity data from the PIV. So, we have been developing a pressure estimation method for a plane which is normal to the free stream direction in wing wakes [6-8]. We found our target flow field of the wake plane behaved three-dimensionally. The physical quantities in the wake of wings varied along the normal direction to the target plane. So, the pressure estimation results were poor without considering three-dimensional effects as shown in Section 5 of the article. Thus, we constructed a PIV-based indirect pressure measurement system which combined stereo PIV velocity measurement and computation based on Navier-Stokes equations. As for computation to obtain pressure, we realized that it was important to solve flow equations to take three-dimensional effects into account. Thus, a formulation taking the effects was conducted and it is called the 2.5D model. As input data for the 2.5D model, the PIV velocity data on three planes should be used. In this article, for test cases in the first stage, necessary data are prepared by computer simulation. Thus the velocity data on three planes at 0.1 chord length intervals are obtained for steady flows by CFD simulation about a wing, instead of those by PIV experiments. Then we apply the 2.5D model for pressure calculation using the quasi PIV data.

According to the recent reviews on PIVbased pressure calculation [9, 10], our approach of the 2.5D model for pressure estimation is rather unique. In the reviews such as Refs [9 and 10], all estimations there were done on the plane parallel to the main stream direction and the numerical method to calculate pressure was two-dimensional. Three-dimensional effects of out of plane factors were never considered for the pressure estimation in those reviews.

Therefore, the purpose of this article is firstly to introduce our research on 2.5D model pressure estimation in the wake of a wing as well as to propose a convenient method to impose boundary condition for solving the Poisson equation. This idea on the boundary condition comes from the isentropic relation on Aerodynamics. Then, the application of the 2.5D model and the isentropic boundary conditions is performed using flow-fields ranging subsonic to transonic, a complicated wakes. Finally, those are applied to actual PIV experiment in automobile wakes.

# 2 PIV Measurement in a Wake

A stereo PIV measurement in a wing wake discussed here is schematically shown in Fig. 1. The wing is rectangular with the aspect ratio of 5 and its section airfoil is NACA0012. Figure 2 illustrates the PIV measurement connected to the 2.5D model. In Fig. 2, the pressure contour map on a wake plane normal to the free stream direction is presented. In this article, we handle the halved portion by the symmetrical line of PIV planes in a wake. To clarify the difference between commonly performed planar pressure computation and that in this article, Figs. 3 and 4 are displayed. In Fig. 3, velocity component, u, v and w are measured on the x-z plane. A free stream direction is in-plane. In that case, derivatives of velocity component in the out-ofplane direction might little affect pressure. Therefore, planar pressure (2D) computation works well. Figure 4 shows the pressure distributions corresponding to the measurement plane in Fig. 3. The pressure phenomena in Fig. 4 are totally different from those in the wing wake in Fig. 2.



Fig. 1. Schematic view of stereo PIV in the y-z plane.



Fig. 2. Example pressure distributions in the *y*-*z* plane (Mach 0.35).

#### PRESSURE ANALYSIS BASED ON PIV MEASUREMENT **IN AIRPLANE WAKES**



Fig. 3. Schematic view of stereo PIV in the x-z Plane.



Fig. 4. Example pressure distributions in the x-z plane (Mach 0.82).

#### 3 Pressure Estimation Method –2.5D Model

# 3.1 Proposed Formulation (2.5D Model) [8]

For the case of wake flows, the flow-fields have significant three-dimensionality. Thus, we have to go back to three-dimensional (3D) MAC incompressible Navier-Stokes scheme and equations to formulate a basic equation for pressure estimation. At the same time, we should recall a limitation of the velocity data that PIV experiment provides. So, the second order x-derivative term of pressure in the lefthand-side of 3D Poisson equation should be eliminated. To eliminate it, we extract the following equation, Eq. (1)

$$\frac{\partial^2 p}{\partial x^2} = -\frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \frac{1}{\text{Re}} \frac{\partial}{\partial x} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$
(1)

Eq. (1) is the x-derivative of the x-momentum balance equation. After some algebra, finely, Eq. (8) is obtained.

$$\frac{\partial^{2} p}{\partial y^{2}} + \frac{\partial^{2} p}{\partial z^{2}} = -\begin{cases} \left(\frac{\partial v}{\partial y}\right)^{2} + \left(\frac{\partial w}{\partial z}\right)^{2} + \left(\frac{\partial w}{\partial z}\right)^{2} + \left(\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x}\right) + \\ u\frac{\partial E}{\partial x} + v\frac{\partial E}{\partial y} + w\frac{\partial E}{\partial z} \end{cases} + \frac{1}{\text{Re}} \left(\frac{\partial^{2} E}{\partial y^{2}} + \frac{\partial^{2} E}{\partial z^{2}}\right)$$
(2)  
where  $E = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ 

where

Equation (2) is the proposed equation to solve pressure using velocity components measured by stereo PIV experiments. In this article, it is computationally solved by a finite difference discretization and the SOR method. In Eq. (2), there are several x derivative terms which guarantee that the equation takes threedimensional physics into account. In order to evaluate them numerically, two complementary PIV planes are prepared other than the primary plane shown in Fig. 5. Equation (2) can be alternatively devised by taking the divergence of the momentum balance equations in the y and z directions. Using Eq. (8), pressure values are in a two-dimensional y-z plane solved considering three-dimensional effects in the flow-field. Therefore, it is called the two-and-ahalf- dimensional (2.5D) model.

Widely used in-plane pressure calculation equations (2D model) are derived from Eq.(2) by setting the x-partial derivative terms at zero. Then, the 2D model includes planer value of velocity components and partial derivatives. As mentioned in the introductory section, it works well in the plane parallel to the free stream direction, but nevertheless it does not work in the one normal to the free stream.



Fig. 5. PIV measurement planes connected to the 2.5D model.

# 4 Preparation of PIV Measurement Data and Correct Pressure

In this section, every research has been done by computational fluid dynamics (CFD) to evaluate the pressure estimation method of the 2D and 2.5D model. Instead of PIV experiment, threedimensional CFD simulation was conducted about a rectangular wing. It provided velocity data on several wake planes which were regarded as substitution for PIV measurement velocity. In addition, it gave pressure values which should be correct answers to evaluate the accuracy of pressures estimated by the 2D and 2.5D models. The CFD data are called pseudo PIV data. Poisson equations for pressure estimation are computationally solved using pseudo PIV velocity data. As input data, the velocity data on one wake plane are used for the 2D model, while the data of three planes are used for 2.5D one.

# 4.1 Velocity Data

Two flow fields were prepared. One was a subsonic flow where  $M_{\infty} = 0.35$ , Re=3.0million, AOA=4.94degs, and the other was a transonic one where  $M_{\infty} = 0.82$ , Re= 3.0million, AOA= 3.86degs. These were obtained by Navier-Stokes simulation for flows past a wing whose section airfoil is NACA0012 everywhere. In the article we generally use non-dimensional properties, so the length unit is C. The leading and trailing edge of a wing located at *x*=0 and *x*=1.0C, respectively. The root of a wing was at

y=0 as indicated in Fig. 5. We extracted velocity data on a target measurement plane of x=1.2C as well as two complementary ones of x=1.1C and 1.3C. The size of each plane was 4.0C in the y direction and 3.4C in the z direction.

# **4.2** Correct Pressure to evaluate the performance of the proposed model

We extracted pressure values on the target plane, x=1.2C from the simulation results. The extracted pressure data should be correct pressure distribution to assess the accuracy of estimated pressures.

# 5 Pressure Estimation Results and Analysis

# 5.1 Imposed Boundary Conditions [11,12]

To solve P of Eq. (2), boundary conditions should be imposed on each boundary edge. How to impose pressure values on a boundary is one of important aspect for PIV based pressure measurement. In this article, if the edges are out of the region affected by one of those such as the wing, its boundary layer and its trailing vortex, the pressure values calculated using the following isentropic relation of Eq. (3)

$$\frac{P}{P_{\infty}} = \left(1 + \frac{\gamma - 1}{2} M_{\infty}^{2} \left(1 - \frac{U^{2}}{U_{\infty}^{2}}\right)\right)^{\frac{\gamma}{\gamma - 1}}$$
(3)

are imposed on them. Otherwise, the Neumann boundary condition is adapted.

# 5.2 Wing Wake of Incompressible Flow [11] $M_{\infty} = 0.35$ , Re = 3.0 million, AOA=4.94degs

Figure 6 shows four plots. They are, from the top, correct  $C_p$  distributions,  $C_p$  by PIV based pressure estimation using the 2D and 2.5D models, respectively. It is obvious that the both model results are much different from each other. The 2D captures vortex but fails to detect the effect of boundary layer on the wake. The size and strength of trailing vortices by the 2D model seem to be rather excessive. On the other hand, the 2.5D model has ability to recover almost perfect  $C_p$  distributions. The bottom plot indicates the discrepancy between PIV based  $C_p$ s and correct ones in terms of relative errors.



Fig. 6. Pressure estimation for incompressible flow (M0.35).

The relative errors are evaluated using pressure value in the absolute scale not gauge scale. The errors are less than 0.1%. The largest errors are in the strong vortex region.

# 5.3 Wing Wake of Compressible Flow [12] $M_{\infty}$ =0.82, Re = 9.0 million, AOA=3.86 degs.

We applied the 2.5D model to the wake of a compressible flow. Figure 7 shows, from the top, correct  $C_p$ , a PIV based estimated  $C_p$  results by the 2D model and those by the 2.5D model, and relative error rate distributions in the target wake plane. The 2.5D model result is identical to the correct  $C_p$  distribution plot. In fact, error rate of the 2,5D model pressure is larger than that of the 2Dmodel but, it is still very small. The 2D model does not detect a wing wake following boundary layers of the wing surface. In contrast, it captures too big trailing vortices.

model formulated The 2.5D from incompressible flow equations works very well for the wake of compressible flows even having shock waves on the wing. Thus, the 2.5D model is quite promising for pressure estimation for not only incompressible flow wakes but also compressible flow wakes. We think in the wake flow-fields compressible effect is so small even if  $M_{\infty}$  is in the transonic range that the 2.5D model, which was derived from incompressible Navier-Stokes equations, stands up all right for compressible flow wakes. For the PIV based pressure measurement in wakes, the present method has advantage over existing methods introduced in Ref 13.

# 5.4 Compressible Flow of the wake of a wingfuselage configuration of NASA CRM Airplane wake $M_{\infty}$ =0.85, Re = 2.26 million, AOA=4.48 degs

The 2.5D model is being applied to PIV pressure measurement in more complicated wake flows such as flows past the NASA CRM wing-fuselage combination. Figures 8 and 9 present one of such flows where  $M_{\infty} = 0.85$ , *Re* based on MAC length=2.26million, AOA= 4.48degs. The resulted pressure estimation will be presented at the conference.



Fig. 7. Pressure estimation for compressible flow (M0.82).



Fig. 8. Perspective view of NASA CRM and a wake



Fig. 9. Velocity components distributions on the target wake plane (substitution for PIV measurement data).

### 6 Application to PIV Data from Experiment

In the previous section, the pressure estimation was done using the velocity data from CFD simulation. Those data are somewhat ideal because fluctuation noise or disturbance by experimental instruments is not there. Here, the proposed system is applied to the real PIV experiment s.



Fig. 10. Schematic view of stereo PIV in an automobile wake.



Fig. 11. Total pressure distributions directly measured by experiment and estimated using PIV velocity.

The PIV was conducted by a wind-tunnel in Honda R&D Co. Ltd as shown in Fig. 10

[14]. Parallel to the PIV, the total pressure distributions on the target wake plane were also measured [14]. The estimated total pressures are compared with the experiment in terms of contour maps in Fig. 11 and its quantitative profile along several lines in Fig. 12. Those lines of A, B and C indicated in Fig. 11. Though some tuning is needed, we think the estimation accuracy is rather good.



Fig. 12. Comparison of total pressure profiles by probe measurement and PIV based (nonintrusive) pressure measurement.

#### 7 Conclusions

The pressure estimation system based on three-plane stereo PIV experiment utilizing the proposed numerical method was examined. The numerical method consists of the 2.5D model of Poisson equation and the isentropic relation to set boundary conditions. The examination and assessment was performed using the quasi PIV data produced by CFD simulation as well as experimental data. Pressure estimations in a wake behind a wing or a vehicle using the 2.5D model were verified. Then, it is found that the proposed model and the boundary condition work very well with velocity data on three stereo-PIV planes.

#### Acknowledgments

This work was supported by Japan Society for the Promotion of Science (JSPS) KAKENHI (Grants-in-Aid for Scientific Research ) Number 24560188. The authors would like to greatly appreciate it.

#### References

- [1] Passmore, M. A., Spencer, A., Wood, D., Jowsey, L., Newnham, P. S., The Application of Particle Image Velocimetry in Automotive Aerodynamics: Wind Tunnel Developments and Simulation Tools, SAE Paper, No.2010-01-0120, pp. 1-13, April 2010.
- [2] Fujisawa, N., Satoh, A., Evaluation of Three-Dimensional Flow and Pressure field in a cold model of combustion chamber by plane PIV measurement, 4th International Symposium on PIV, Goettingen, Germany, PIV01- 1072, pp.1-7, Sep. 2001.
- [3] Aso, T., Matsushima, K., Nakahashi, K., CFD Pressure Estimation using PIV Data, KSAS-JSSAS joint international symposium, Busan, Korea, pp. 156-161, Nov. 2006.
- [4] Kato, H., Matsushima, K., Ueno, M., Koike, S., Watanabe, S., Drag and Lift Prediction Based on Wake Integration Method Using Stereo PIV, 8th International Symposium on PIV, Melbourne, Australia, PIV09-0073, pp.1-8, Aug., 2009.
- [5] AIAA, Aerodynamic Measurement Technology, Aerospace America - The Year in Review, December (2009), pp. 15-16, 2009.
- [6] Kato, H., Matsushima, K., Ueno, M., Koike, S., Watanabe, S., Lift and Drag Force Estimation by Wake Integration Using Multi Plane Stereo-PIV Data, The 43rd Fluid Dynamic Conference/ Aerospace Numerical Simulation Symposium 2011, 2A08, Tokyo, July 2011.

- [7] Matsushima, K., Yonezawa, M. and Ogawa, A. : Inverse Aerodynamic Analysis of Vehicle Wakes using PIV and CFD, Proc. 8th International Conference on Flow Dynamics, Sendai, Japan, OS6-4, Nov. 2011.
- [8] Matsushima, K., Yamaguchi, G. and Kato, H. : Assessment of 2.5 Dimensional Model to Calculate Pressure Using Stereo PIV Data in Wing Wake Flows, The 12th International Symposium on Fluid Control, Measurement and Visualization, FLUCOME 2013, (Nov.18-23) OS14-01-2 Nara, JAPAN
- [9] Charonko, J. J., King, C. V. et al., Assessment of pressure field calculations from particle image velocimetry measurement, Mea. Sci. and technol., Vol. 21, 105401(15pp), 2010.
- [10] van Oudheusden, B W., PIV-Based pressure measurement (Topical Review), Meas. Sci. and technol., Vol. 24, 032001, 2013.
- [11] Matsushima, K., Izumi, T., Kato, H., Pressure Estimation using PIV Measurement at Wing Wakes in Transonic Flows, JAXA-SP-13-011, pp.77-82, March, 2014. (in Japanese)
- [12] Yamaguchi, G., Matsushima, K, Kato, H. : Computational Experiment to Examine a Numerical Method of PIV Pressure Estimation for the Wake of a Wing Flying at Transonic Speed, Transactions of JSASS, Aerospace Technology Japan, Vol. 14, pp. 85-94, 2015.(in Japanese)
- [13] Rugni, D., Ashok, A., van Oudheusden, B.W. and Scarano, F. : Surface Pressure and Aerodynamic Loads Determination of a Transonic Airfoil Based on Particle Image Velocimetry, Meas. Sci. Technol., Vol.20, 074005 (14pp), 2009.
- [14] Fukuchi, Y., Murakumo, Y., Yonezawa, M, Matsushima, K. : Estimation of Pressure Profile from PIV Data for the Wake Flow behind Vehicle, The 18th International Symposia on Applications of Laser Techniques, Lisbon PORTUGAL, July 4–7, 2016. To be presented.Smith J, Jones B and Brown J. *The title of the book.* 1st edition, Publisher, 2001.

#### **Copyright Statement**

The authors confirm that they, and/or their company or organization, hold copyright on all of the original material included in this paper. The authors also confirm that they have obtained permission, from the copyright holder of any third party material included in this paper, to publish it as part of their paper. The authors confirm that they give permission, or have obtained permission from the copyright holder of this paper, for the publication and distribution of this paper as part of the ICAS 2016 proceedings or as individual off-prints from the proceedings.

#### **Contact Author Email Address**

mailto: kisam@eng.u-toyama.ac.jp