APPLICATION OF PROPER ORTHOGONAL DECOMPOSITION FOR AVIATION SAFETY

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Abstract

In this study, the proper orthogonal decomposition reduced order method using various basis weight estimation methods is examined for efficient prediction of wake vortices. The proper orthogonal decomposition cannot guarantee the accurate prediction for the variation of a meteorological condition, in this research introduces neural network as basis weight estimation method which is capable of perceiving the nonlinear relationship between the input variables and the reduced variables. The neural network is trained with pairs of design variables and reduced variables, which are obtained from snapshot data of the LES simulation of the wake turbulence. The constructed POD/NN is applied to wake turbulence prediction for validations, and its results are compared to those of the full order analysis (LES simulation). As a result, it is found that the POD/NN has the capability to predict behavior and dissipation of wake vortices accurately.

1 Introduction

Air traffic has been forecasted to be the largest on record in this year and has grown steadily by about four percent every year. The airport logistics capacity has reached its limit with the steady increase of aviation demands. Thus a method for improving the efficiency of airport capacity has been required while ensuring the safety. For this, it is recommended to reduce the time interval of the take-off and landing of consecutive aircraft. Meanwhile, unreasonable shortening of aircraft spacing would cause an accident due to the wake turbulence. As a consequence of lift, an aircraft generates a pair of long-lived counter-rotating wake vortices which are a potential risk for the following aircraft. Dissipation and behavior of wake vortex pair are dependent on the meteorology such as clear air turbulence, horizontal wind velocity, and the strength of wake vortex pair. Therefore take-off and landing time interval must be controlled efficiently with the consideration of the meteorology condition. For these reasons, an efficient control of landing and take-off based on weather condition is required.

The reduced order model (ROM) has been implemented to improve the computational efficiency in computational fluid dynamics (CFD). Because a ROM that can replicate the output behavior of the CFD model over a limited range of input conditions can be developed [1, 2]. In particular, the ROM based on the proper orthogonal decomposition (POD) approach is valuable in areas that require high computational efforts. The main feature of the ROM based on POD is that it increases the computational efficiency and reduces the numerous degrees of freedom (DOF) of the full system by performing singular value decomposition from Navier–Stokes solutions. In other words, through POD, a small number of basis vector sets are extracted from analysis results that can express crucial features and trends of the flowfield. The solution to the ROM is quickly obtained in a low-dimensional space composed of these basis vector sets and is then transformed to the solution of the full system through a mapping function of the reduced system. Therefore, if the POD basis weights are chosen appropriately, the relevant high-fidelity system dynamics can be captured in just a few states [1,2]. Because of this significant advantage, a considerable amount of ROM-related research has been done. Lucia [2] extended the ROM to high-speed compressible fluid flows and applied it to a blunt-body problem. In his research, he developed the domain decomposition method with an internal boundary condition to treat a region containing a moving shock wave. Kim [3] proposed an efficient time domain system identification and model reduction technique for linear dynamic systems, which enables the effective construction of a ROM compared to previous schemes such as pulse/ERA [4]. Jun et al. [5] investigated the accuracy and efficiency of POD for three-dimensional Euler equations and predicted the variation in the aerodynamic performance on structural variables from the aero-structure coupling analysis. In this study, the POD based on the basis weight estimation methods using a linear combination and artificial neural network to find appropriate basis weighting method for the prediction of wake turbulence.

2 Methods

2.1 LES Simulation methods for wake turbulence

The ambient turbulent fields are obtained to simulate the wake vortices in realistic atmospheric condition by forcing the low wave numbers of the flow using a forcing with a fixed amplitude and a random phase. This forcing technique was employed for studies of vortices evolving in turbulent atmospheres [8]. Once the ambient turbulent field is obtained, a pair of vortices is superimposed to the field using Lamb-Oseen vortex model. The initial wake vortex flow is set by the vortices generated by a large aircraft which is composed of a pair of horizontal vortices of circulation Γ_0=458m^2/s, separated by b_0=47.1m, and the core size is set to r_c=0.05b_0.

For LES calculation, basic equations are the compressible Navier-Stokes equation. Inviscid numerical flux is computed based on Roe's flux difference splitting method using primitive variables interpolated by the higher-order monotonic upwind scheme for conservation laws (MUSCL) scheme. A total-variation-diminishing limiter is not used in the MUSCL interpolation. The viscous term is evaluated by the second-order central difference scheme. Smagorinsky model (C_s=0.16) was used for turbulence closure with the modification for rotational flow. The fourth-order Runge-Kutta method is used for the time integration. The accuracy of the solver was confirmed using Doswell's frontogenesis model by Misaka et al. [7]

2.2 Proper Orthogonal Decomposition

In this study, the snapshot set method is adopted to select the basis functions. The POD following the snapshot method was formulated by Sirovich [6] firstly. The problem that the POD method seeks to solve is to identify a structure in a random vector field, $U^{(i)}$. The objective is to seek a function Φ, which has a structure typical of the members of an ensemble. The solution Φ is sought from the space of functions for which the inner product <Φ,Φ> exists. Considering a set of N snapshots, a function Φ has a special form as a linear combination of the snapshots.

$$\mathbf{\Phi} = \sum_{i=1}^{N} w_i \mathbf{U}^{(i)} \quad (1)$$

$$\frac{1}{N}\sum_{i=1}^{N} \frac{\left|\left\langle \mathbf{U}^{(i)}, \mathbf{\Phi}\right\rangle\right|^2}{\left\langle \mathbf{\Phi}, \mathbf{\Phi}\right\rangle} \quad (2)$$

where, w_i needs to be determined to maximize Eq. (2). This maximization problem can be cast in an equivalent eigenvalue problem.

$$\mathbf{CW} = \lambda \mathbf{W} \quad (3)$$

$$\mathbf{C}_{ij} = \frac{1}{N}\int_{\Omega} \mathbf{U}^{(i)}(x)\mathbf{U}^{(j)}(x)dx\ , \quad \text{and} \quad \mathbf{W} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad (4)$$

C is a spatial correlation matrix which is nonnegative and Hermitian so it can be decomposed into a complete set orthogonal eigenvectors along with a set of eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N \geq 0$. To obtain the spatial correlation matrix, Eq. (4) is approximated for the discrete computational domain as follows,

$$\mathbf{C}_{ij} = \frac{1}{N}\sum_{k=1}^{N_{cell}} \mathbf{U}^{(i)}(k)\mathbf{U}^{(j)}(k)\Delta V_k \qquad (5)$$

where, ΔV_k is corresponding cell volume.

2.3 POD Basis Weight Estimation Methods

The weight estimation method of POD basis affects the accuracy of the POD approach under the condition of limited snapshots. The basis weighting method defines the relationship between input and output variables. Therefore, for the efficiency of the POD analysis, it is necessary to employ a technique that predicts more accurate POD basis weights, even with a small number of snapshots. For this reason, various studies researched the prediction of weighting coefficients [4]. Linear combination, interpolation scheme, and response model are typically used to define the relationship between input and output variables. If nonlinearity exists in the given analysis space, it should be employed in the prediction method that represents nonlinear relationships between input and output, such as a high-order interpolation scheme, high-order polynomial, and artificial neural network.

A neural network learns the relationship between input variables and unknown variables obtained from several snapshot data, and then, the trained neural network model percepts appropriate weights for specific input variables. The neural network model consists of 3 layers: the input, output, and hidden layers. The transfer function, $S(x)$, connecting information between neurons in layers is a sigmoid function as shown in Eq. (6).

$$S(x) = \frac{1}{1+e^{-x}} \qquad (6)$$

Neurons in the hidden and the output layers are calculated as shown in Eqs. (7) and (8):

$$H_j = S_{Hidden}\left(c_j + \sum_i a_{ij} X_i\right) \Rightarrow \mathbf{H} = \omega_{Hidden} \qquad (7)$$

$$Y_k = S_{Output}\left(d_k + \sum_j b_{jk} H_j\right) \Rightarrow \mathbf{Y} = \omega_{Output}\mathbf{H} \qquad (8)$$

Where **X** is the input variables vector; **Y** is the output variables vector, and **H** is the vector of the hidden nodes. The *a*, *b*, *c*, *d*, and ω are the weights of neurons. The correlation of input variables (flow condition; **X**) and output variables (unknown variables of the ROM; **Y**) is replaced with the weights of the NN (ω_{Hidden} and ω_{Output}). In this study, the weighting factors of the neural network are adjusted by training process using Levenberg-Marquardt algorithm.

2.3 Vortex identification

To identify the wake vortex pair in the flow field, the usual identification criteria based on local quantities such as velocity gradients, vorticity are intermittent due to small-scale turbulence. The main problem is to define fundamental geometrical properties of a large-scale vortex superposed on a small-scale turbulent field. To overcome this problem, in this study used two scalar functions, Γ_1, and Γ_2, derived from velocity fields developed by Laurent at al [9]. Γ_1 is used to identify the vortex center, and Γ_2 is used to determine the vortex core. These functions can characterize the locations of the center and the periphery of the large-scale vortex, by considering only the topology of the velocity field, not its magnitude. Moreover, Γ_1 and Γ_2 remove the small-scale turbulent intermittency and are sufficiently robust to process large data sets consisting of several thousand of velocity fields.

3 Results and Discussions

Using ROM based POD method according to basis weights estimation method has been performed for the prediction of wake turbulence. The snapshots are extracted in the LES simulation of wake turbulence using parameterized vortex pair. The basis vectors of the reduced-order model are obtained from the snapshots. The POD basis weight estimation model is constructed using basis vectors. Through the full-order mapping of the POD basis weights, full-order flow fields are produced at the intermediate time and compared with that of LES result. Through these comparisons, the proper weight estimation method and the number of snapshots are compared to apply the prediction wake vortex for aviation safety.

3.2 Prediction Result Comparison According to Number of Snapshots

Under the varying number of snapshots (6, 11, 21), the results of the reduced-order analyses are compared with those of the full-order LES simulation of wake vortex to investigate the effect of snapshots number. Under each condition and wake vortex position are compared with full order system.

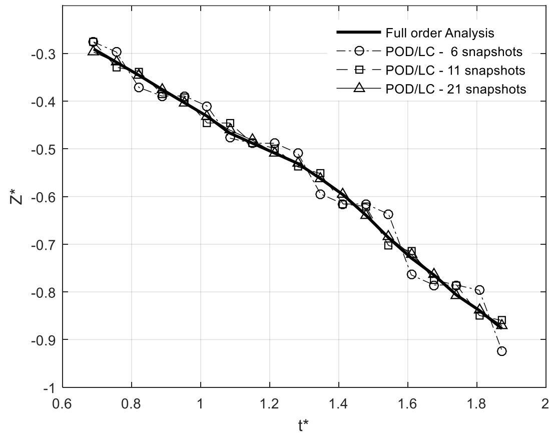

Figure 1. Comparison of vertical displacement of vortices

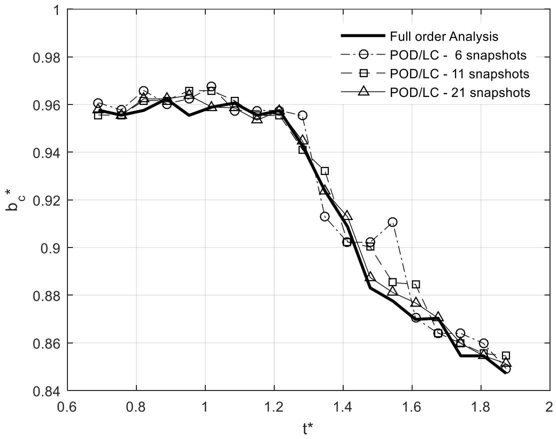

Figure 2. Comparison of lateral spacing of vortices evolution

Figure 1 and 2 shows the vortex evolution comparison of the POD/LC results according to the number of snapshots. As the number of snapshot increases, the more accurate the prediction result is. The vertical displacement is almost identical when using 21 snapshots. In lateral spacing, a significant error was seen around t*=1.5 because the lateral spacing shows nonlinear phenomena which are hard to predict with the linear combination.

3.1 Prediction Result of Using Artificial Neural Network Basis Weight Estimation Method

Under the different basis weight estimation method, the results of the reduced-order analyses are compared with those of the full-order LES simulation of wake vortex to investigate the validity of the POD/NN basis weight estimation method. Under each snapshot condition, wake vortex position and circulation are compared with full order system.

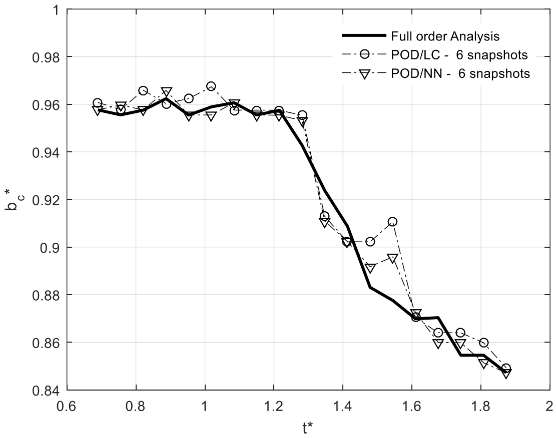

Figure 3. POD basis weight estimation method comparison

In Figure 3, the lateral spacing error reduced when using POD/NN method. The artificial neural network method was able to represent nonlinear relationships between input and output. The velocity distributions along the vortices core at t*=1.5 are compared in Figure 4. The prediction result of PON/NN with 21 snapshot shows identical velocity distribution with that of full-order analysis. It means the trained neural network model percepts appropriate weights for wake vortex prediction problem.

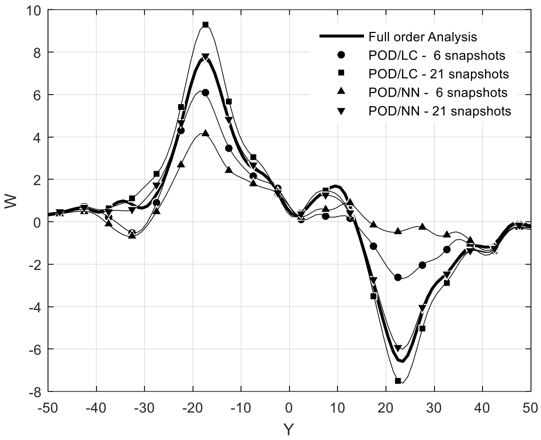

Figure 4. velocity distribution comparison along the vortices core

4 Conclusion

The artificial neural network was introduced as basis weight estimation method which is capable of perceiving the nonlinear relationship between the input variables and the reduced variables. The constructed POD/NN is applied to wake turbulence prediction for validations, and its results are compared to those of the full order analysis. The vortex pair structure of using the artificial neural network is identical to that of LES analysis result compared with the linear combination. When using the artificial neural network as a basis weight estimation method, it is possible to reconstruct the flow field precisely.

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