TERMINAL IMPACT ANGLE AND ANGLE-OF-ATTACK CONTROL GUIDANCE FOR SURFACE-TO-AIR MISSILE USING TVC

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Abstract

This paper deals with impact angle and angle-of-attack control guidance for a two-staged surface-to-air missile. The equation of motion is simplified as pith planar motion. The vehicle can be controlled during the propellant burning phase, terminal impact angle constraint should be satisfied within the limited time. To conduct a stable stage separation, furthermore, the flight path angle and pitch attitude angle should be aligned. Namely, the angle of attack of the missile should be zero. In this paper, the guidance law to satisfy the constraints described above is obtained from an optimization tool, GPOPS. Also, the optimal path and control input results are depicted.

1 Introduction

Generally, impact angle guidance laws have been developed for anti-tank missiles or anti-ship missiles to maximize their warhead effect and penetrate the most vulnerable point of the target [1-4]. Impact angle guidance laws are also used for surface-to-air missiles to accomplish the effective engagement geometry [6]. If the missile knows the states of the target, an impact angle guidance law could be used.

For anti-ballistic missiles, multi-staged missile configuration is used such as Terminal High Altitude Area Defense (THAAD), Standard Missile-3 (SM-3) and Aster series. Especially, THAAD does not have tail fins or canard for the control and only use Thrust Vector Control (TVC) system. The missile can be controlled during the boosting phase and the impact angle constraint should be satisfied within the limited time.

After the burn-out moment, the missile and the warhead are separated. During the separation, a small angle of attack causes the aerodynamic unstable so that the warhead collides with the separated boost rocket motor. Thus, the angle of attack of the missile should be zero at the burn-out time.

In order to satisfy the impact angle constraint, many impact angle guidance laws have been developed based on the optimal control theory [1-5]. Ryoo et al. suggested impact angle control guidance law considering lag-free system and 1st order lag system [2]. In [3], they derived the optimal guidance law with the energy cost function weighted by a power of the time-to-go. Cho [5] investigated the relationship between Proportional Navigation Guidance (PNG) laws and optimal guidance laws weighted by time-to-go. Not only the impact angle, but the impact time is also adjusted [4]. Lee et al. had derived a guidance law considering impact angle and time simultaneously,.

For the surface-to-air missiles using TVC, the missile should be delivered to a predicted handover point (PHP) during the boost phase. Miss distance based on the engagement geometry between the missile and the target should be compensated by the kill vehicle, however, the magnitude of lateral acceleration of a kill vehicle is generally smaller than that of TVC systems. Also, the speed of the missile varies due to the thrust and the mass variation of the missile. In this paper, an optimal problem for the missiles considering the constraints described above is defined. Then, the optimal guidance command is calculated by the optimization tool.

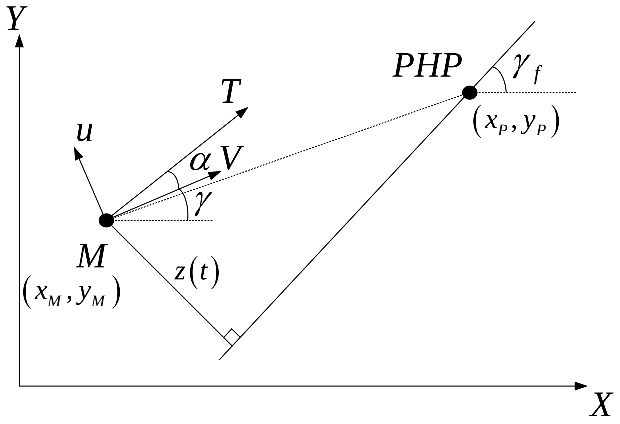

Fig. 1. Missile Engagement Geometry

This paper is composed of as follows. Section 2 deals with the problem formulation. Two formulations, general case and the missiles using TVC case are compared. Optimization results using a commercial optimization tool, GPOPS, are depicted and the conclusion of this paper is followed.

2 Problem Formulation for Optimal Impact Angle Guidance Laws

2.1 General Impact Angle Control Guidance Law

In [2-4], optimal guidance laws considered that the missile is a point mass and the engagement kinematics is defined on the impact angle frame (x_P, y_P). The missile engagement geometry is depicted in Fig. 1, where (x_M, y_M) is missile position, V is velocity, u is control input, γ is flight path angle, γ_f is terminal impact angle constraint and $z(t)$ is lateral position with respect to the impact angle frame. Thrust T and angle of attack α is described in the next section.

$$\bar{\gamma} = \gamma - \gamma_f \tag{1}$$

The missile kinematics are described as following equation.

$$\begin{aligned} \dot{z} &= V \sin \bar{\gamma}(t) \\ V\dot{\bar{\gamma}} &= a \end{aligned} \tag{2}$$

Generally, the missile velocity V is constant and a small angle approximation is used. Equation (2) can be linearized as

$$\begin{aligned} \dot{z} &= V\bar{\gamma} = v \\ \dot{v} &= a \end{aligned} \tag{3}$$

Terminal constraints for the problem are given below.

$$z(t_f) = v(t_f) = 0 \tag{4}$$

The cost function is determined to minimize the control input a.

$$\min_u J = \frac{1}{2} x^T(t_f) S_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} u^2(t)dt \tag{5}$$

where x is state vector, $u(t)$ is control input and S is weighting matrix. The optimal control for this problem is given as

$$u = -\frac{6}{t_{to}^2} y - \frac{4}{t_{go}} v \tag{6}$$

2.2 Impact Angle Guidance using TVC

In the previous section, we assume that the missile is a point mass and the velocity does not change. This paper deals with the missile using TVC, we make five basic assumptions as follows:

A1. The target (PHP) is predetermined, thus, stationary.
A2. The engagement occurs in a horizontal plane.
A3. The thrust vector is aligned with the body axial direction.
A4. The missile is controlled by using the missile angle of attack.
A5. The mass variation, aerodynamic forces and the gravity are neglected.

Under the assumptions and for a stationary target, we can obtain the engagement kinematics as follows:

$$\begin{aligned}\dot{x} &= V\cos\gamma \\ \dot{y} &= V\sin\gamma \\ \dot{V} &= \frac{T\cos\alpha}{m} \\ \dot{\gamma} &= \frac{T\sin\alpha}{mV}\end{aligned} \tag{7}$$

where T is thrust, α is angle of attack and m is mass of the missile.

As the missile approaches to the PHP, the downrange is monotonically decreases. Now we can replace the independent variable time with another monotone variable, the downrange. Then, equation (7) can be rewritten as

$$\begin{aligned}\frac{dy}{dx} &= V\tan\gamma \\ \frac{dV}{dx} &= \frac{T\cos\alpha}{mV\cos\gamma} \\ \frac{d\gamma}{dx} &= \frac{T\sin\alpha}{mV^2\cos\gamma}\end{aligned} \tag{8}$$

Following relationships are obtained by using small angle approximation.

$$\begin{aligned}&\sin\alpha \simeq \alpha \\ &\cos\alpha \simeq 1 \\ &\tan\gamma \simeq \gamma \\ &\cos\gamma \simeq 1\end{aligned} \tag{9}$$

The equation (8) is simplified by substituting equation (9) into (8).

$$\begin{aligned}\frac{dy}{dx} &= \gamma \\ \frac{dV}{dx} &= \frac{T}{mV} \\ \frac{d\gamma}{dx} &= \frac{T\alpha}{mV^2}\end{aligned} \tag{10}$$

In the assumption A5, the mass variation is neglected, so the velocity of missile can be integrated as a function of downrange.

$$V^2 = V_0^2 + \frac{2T}{m}\left(x - x_0\right) \tag{11}$$

Finally, we can obtain the equation of y-directional position and flight path angle by substituting equation (11) into (10).

$$\begin{aligned}\frac{dy}{dx} &= \gamma \\ \frac{d\gamma}{dx} &= \frac{T}{m\left(V_0^2 + T'\left(x - x_0\right)\right)}\alpha\end{aligned} \tag{12}$$

where V_0 is missile initial velocity and $T' = 2T/m$. The state boundaries are given below.

$$\begin{aligned}x(t_0) &= x_0 \quad & x(t_f) &= x_f \\ y(t_0) &= y_0 \quad & y(t_f) &= y_f \\ \gamma(t_0) &= \gamma_0 \quad & \gamma(t_f) &= \gamma_f\end{aligned} \tag{13}$$

t_f equals to the burn-out moment and subscript $(\cdot)_f$ means the terminal constraints of the states because the missile should be located at the PHP when the propellant burns out. The independent variable is changed as downrange x, equation (13) should be changed using following terms.

$$\begin{aligned}y(x_0) &= y_0 \quad & y(x_f) &= y_f \\ \gamma(x_0) &= \gamma_0 \quad & \gamma(x_f) &= \gamma_f\end{aligned} \tag{14}$$

Now, we can define the optimal control problem. As we mentioned above, the angle of attack should be zero as the propellant burn-out moment nears. The cost function is defined considering angle of attack constraint.

$$\min_{\alpha} J = \int_{x_0}^{x_f} \frac{1}{2}\frac{\alpha^2}{\left(x - x_f\right)^2}dx \tag{15}$$

The denominator decreases as the missile approaches to the target, magnitude of the control input should decrease to minimize the cost function (15).

To find the optimal control input, defined the Hamiltonian.

$$H=\frac{1}{2}\frac{\alpha^2}{\left(x-x_f\right)^2}+\lambda_y\,\gamma+\lambda_\gamma\frac{T}{m\left(V_0^2+T^{'}\left(x-x_0\right)\right)}\alpha \tag{16}$$

where λ_y and λ_γ are co-states.

The equations of co-states can be obtained from the equation (16).

$$\begin{aligned}\frac{d\lambda_y}{dx}&=-\frac{\partial H}{\partial y}=0\\ \frac{d\lambda_\gamma}{dx}&=-\frac{\partial H}{\partial \gamma}=-\lambda_y\end{aligned} \tag{17}$$

We can get the co-state equations by integrating equation (17) from x to x_f .

$$\begin{aligned}\lambda_y&=\nu_y\\ \lambda_\gamma&=\nu_\gamma+\nu_y\left(x_f-x\right)\end{aligned} \tag{18}$$

The optimality condition is obtained by partially differentiating the Hamiltonian by control input α . Then, substitutes equation (18).

$$\begin{aligned}\alpha=&-\frac{\nu_\gamma T}{m\left(V_0^2+T^{'}\left(x-x_0\right)\right)}\left(x-x_f\right)^2\\ &+\frac{\nu_y T}{m\left(V_0^2+T^{'}\left(x-x_0\right)\right)}\left(x-x_f\right)^3\end{aligned} \tag{19}$$

3 Optimization Results

To find the undetermined coefficients in equation (19) is tedious procedure. Previously, the optimal control and trajectory are obtained by a commercial optimization tool, GPOPS. This tool transcribes optimal control problem into parameter optimization problems by using pseudo-spectral method.

The optimization scenario is defined as follows: the missile is launched from the origin of the two-dimensional Euclidean plane. Initial and terminal conditions are given below.

$$\begin{aligned}x\left(t_0\right)&=0 & x\left(t_f\right)&=13km\\ y\left(t_0\right)&=0 & y\left(t_f\right)&=6km\\ V\left(t_0\right)&=10\,m/s & \gamma\left(t_f\right)&=35\deg\\ \gamma\left(t_0\right)&=85\deg\end{aligned} \tag{20}$$

In the optimization scenario, equation (7) is used but mass variation of the missile should be considered. The parameters associated with TVC are given as follows:

$$\begin{aligned}T&=20kN\\ m_0&=350kg\\ \dot{m}&=-10kg/\sec\\ t_{burn}&=20\sec\end{aligned} \tag{21}$$

The cost function is set as equation (15). The optimal trajectory, velocity profile, flight path angle, mass variation and the optimal control input are depicted from Fig. 2-6.

In Fig. 2 and Fig. 4, the missile approaches to the target with terminal flight path angle constraint. The mass varies from initial mass 350kg to 150kg with the propellant consumption rate -10kg/sec and the burn time 20 seconds.

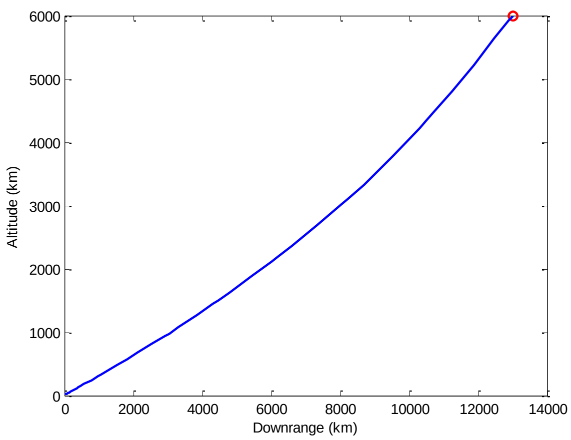

Fig. 2. Missile Trajectory

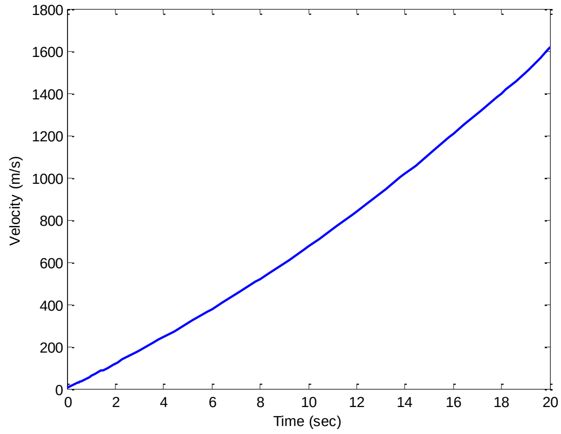

Fig. 3. Velocity Profile

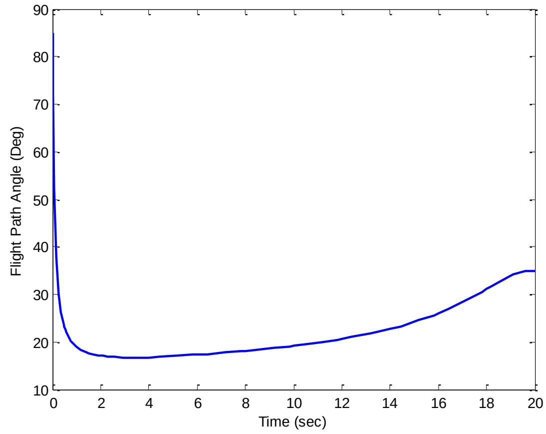

Fig. 4. Flight Path Angle Profile

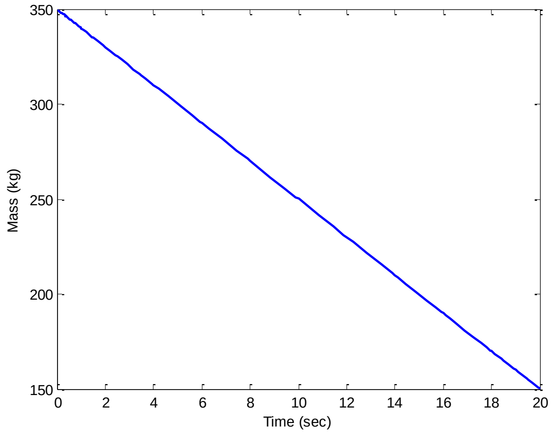

Fig. 5. Mass Variation

The angle of attack in Fig. 6 also satisfies the terminal condition. At the final time, angle of attack becomes zero. There exist jump point near 18 sec, however, it is caused by the tool.

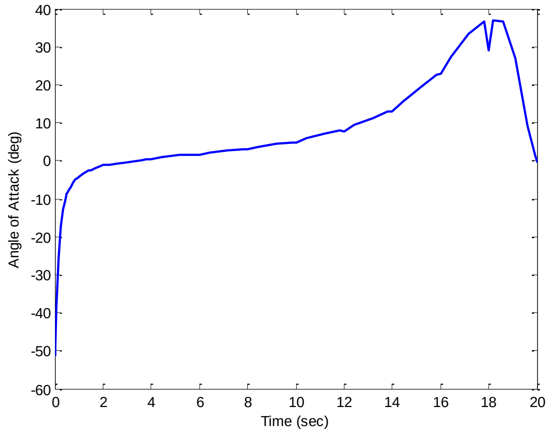

Fig. 6. Angle of attack Profile

4 Conclusion

In this paper, the optimal guidance for the missiles using TVC is dealt. Unlike the general impact angle guidance law based on the optimal control theory, the missile kinematics is defined considering the missile velocity variation and the control input is defined as angle of attack. The optimization is conducted by using the commercial optimization tool instead deriving rigorous analytic solution which is tedious procedure. The optimization results are satisfying the terminal constraints. Also, the optimal control input can be calculated.

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