ICAS 2016
30th Congress of the International Council of the Aeronautical Sciences
DCC, Daejeon, Korea ; September 25-30, 2016

WIND VELOCITY ESTIMATION WITHOUT AN AIR SPEED SENSOR USING KALMAN FILTER UNDER THE COLORED MEASUREMENT NOISE

Yong-gonjong Park*, Chan Gook Park**
Department of Mechanical and Aerospace Eng/Automation and Systems Research Institute, Seoul National University

Keywords: *wind estimation, colored measurement noise, matrix conjugate gradient method, innovation adaptive Kalman filter*

Abstract

This paper presents a new estimation method on wind velocity without an air velocity sensor for an air vehicle. The wind velocity can be obtained by calculating the difference between the air velocity and the ground velocity observed from the navigation system. In order to estimate air velocity using only GPS/INS navigation system, extended Kalman filter is designed using 6 DOF equations of motion. The measurements of the filter are angular rate and attitude from the GPS/INS integrated system. To improve the estimation performance, we consider the colored measurement noise in Kalman filter using matrix conjugate gradient method. Numerical simulations are performed to compare the proposed algorithm with the standard Kalman filter.

1 Introduction

The gliding and control performance of an unpowered air vehicle is affected by the wind velocity. In order to maximize its gliding distance, the gliding vehicle has to fly with the velocity that minimizes the path angle. As a result, the velocity can be described to a function of wind speed. Therefore, the wind velocity is one of the most important components of the unpowered gliding vehicle to achieve an appropriate control and gliding performance. In general, in order to estimate the wind velocity, a pitot tube is widely used for measuring air speed, and the ground velocity is observed by a GPS/INS integrated navigation system. Then the wind velocity can be obtained by calculating the difference between the measured air speed and ground velocity.

Mulgund and Stengel proposed wind estimation algorithm using EKF(Extended Kalman Filter) that is based on the nonlinear longitudinal aircraft equations of motion, and it is designed to provide estimates of horizontal and vertical atmospheric wind input[1]. Langelaan and Neidhoefer described a method for estimating wind field(wind velocity, rate of change of wind velocity and wind gradient)[2]. The method utilizes sensors which are already part of a standard autopilot sensor suite. Petrich and Subbarao proposed simple methods for modeling the local wind flow that affects the vehicle’s trajectory[3]. This method deals with the estimation of the 3D wind components and shows that successful wind estimation is possible for any trajectory. Lee, Sevil, Dogan and Hullender presented and application of the Square Root unscented Kalman Filter(SR-UKF) to the estimation of aircraft system states and to the estimation of the total wind vector made up of a time-varying prevailing wind plus turbulence[4]. The estimations are computed using conventional auto-pilot sensors with exponentially correlated measurement errors.

Above papers assume that there is an air velocity sensor such as a pitot tube for obtaining wind velocity measurement. However, using a pitot tube cause increment of the power consumption, and demands of additional equipment, such as a heating system in high altitude and power supply. As a result, the installation of a pitot tube makes the cost and weight of the air vehicle increase and the gliding performance decrease[5].

In this paper, we assume that there is no air speed sensor, so the wind velocity cannot be obtained directly by calculating difference the ground velocity and air velocity. In order to estimate the wind velocity using only GPS/INS navigation system without any additional equipment, extended Kalman filter is designed using 6 DOF equations of motion. The state variables of the filter are defined to air speed, rotational angular rate and attitude of the body frame axis. The measurements of the filter are the ground velocity of the body frame, rotational angular velocity and attitude from the GPS/INS integrated system.

But there is a problem to estimate air velocity with colored measurement noise in the Kalman filter which is an optimal filter when the measurement and process error noise are white Gaussian. The estimation results of GPS/INS which are measurements of wind estimation filter have a property of colored noise then using standard Kalman filter makes the estimation performance of wind velocity degrade. So, it should be adopted to use Kalman filter with considering colored measurement noise for wind estimation.

Generally, there are two approaches to treat the colored measurement noise in the Kalman filter, which are measurement differencing and state augmentation. The measurement differencing method has developed by Bryson for the first time[6]. However, there is a 1-epoch latency in the measurement updating. Petovello, recently proposed the modified measurement differencing approach to resolve the problem of Bryson's method but Petovello's approach is more likely to diverge because it need the inverse of system matrix[7]. The state augmentation approach makes the filter diverge because of the singularity of updating error covariance matrix. Kedong Wang resolves this problem using Tikhonov Kalman filter and perturbed-P algorithm and its performance is better than measurement differencing approaches[8]. Chein-Shan Liu, Honh-Ki Hong and Satya N. Atluri are proposed novel algorithm based on the conjugate gradient method for inverting ill-conditioned matrices[9]. They insist that the method using conjugate gradient method has better performance than Tikhonov regularization for inverting ill-condition matrices.

In this paper, we use the Kalman filter based matrix conjugate gradient method for wind estimation algorithm. Because the state transition matrix of colored measurement error model and white Gaussian variance are unknown, the adaptive Kalman filter is applied additionally. Some numerical simulations are performed to compare the proposed algorithm to the result of the standard Kalman filter.

2 Adaptive Kalman Filter based Wind Estimation Algorithm

2.1 System and Measurement Model of Extended Kalman Filter

In this paper, a six degree-of-freedom model of aircraft is used for system model of extended Kalman. Its aerodynamic coefficients are nonlinear functions of position, air velocity, attitude, rotation rates and control input and the wind can be modeled random walk model whose variance is changed depending on altitude. The state variables and system model are [10]

$$x = \begin{bmatrix} v_a^T & \omega^T & \Phi^T \end{bmatrix}^T \tag{1}$$

$$\dot{x} = f\left(X, u\right) + n \tag{2}$$

$$\dot{v}_a = -\omega \times \left(v_a + v_w\right) + G + F\left(x, u\right) + n_{v_a} \tag{3}$$

$$\dot{\omega} = I^{-1}\left(-\omega \times I\omega + M\right) + n_\omega \tag{4}$$

$$\dot{\Phi} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} + n_\Phi \tag{5}$$

where v_a denotes air velocity vector along the body frame axis, ω denotes angular rate vector, Φ denotes attitude vector, ϕ, θ, ψ denote roll, pitch, yaw, v_w is wind velocity vector, F is aerodynamic force, M is the aerodynamic moment and $n_{v_a}, n_\omega, n_\Phi$ are white noise error of each states.

The measurements are angular rate along the body frame and attitude represented by roll, pitch, yaw from the INS/GPS navigation system. So, the measurement equation is

$$z = Hx + v \tag{6}$$

$$H = \begin{bmatrix} O_{3\times3} & I_{3\times3} & O_{3\times3} \\ O_{3\times3} & O_{3\times3} & I_{3\times3} \end{bmatrix} \tag{7}$$

where z denotes the measurement vector obtained from INS/GPS system, v is white Gaussian noise error of the measurement and H is the measurement matrix.

2.2 Wind Estimation Algorithm

The wind velocity can be obtained by calculation of difference air velocity and ground velocity.

$$v_w = v_a - v_g \tag{8}$$

where v_g is the ground velocity which can be measured from INS/GPS system, v_a is the air velocity which is estimated by extended Kalman filter using equation (1)~(7) by system and measurement model. Figure. 1 shows the wind estimation algorithm.

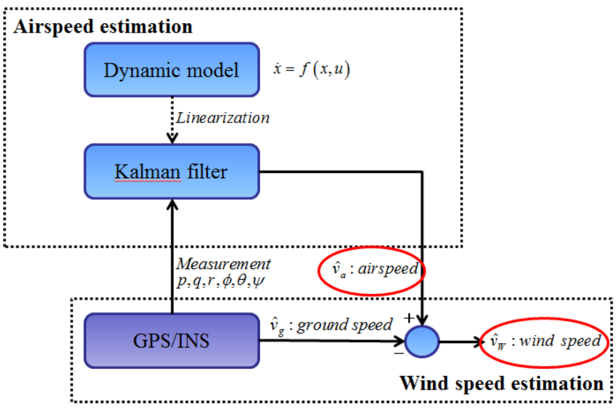

Figure. 1 Wind Estimation Algorithm Block Diagram

The wind profile is generated by using data which provided by the weather center and the wind profile is at an altitude of 10km to the ground as Figure. 2.

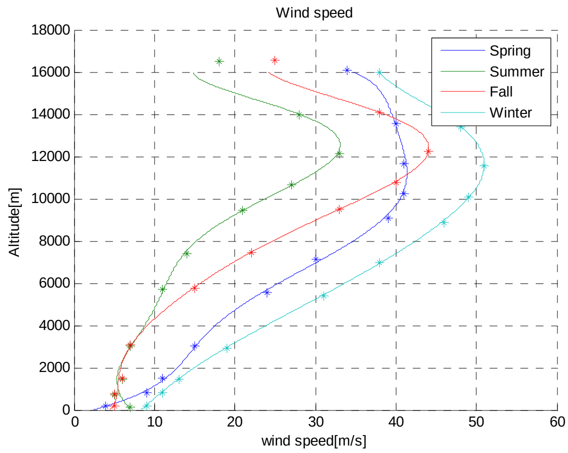

Figure. 2 Wind Speed Profile

3 Consideration of Colored Measurement Noise

3.1 Colored Measurement Noise Problem

The discrete system with the colored measurement noise error can be expressed by the following equations.

$$\begin{aligned} x_{k+1} &= F_k x_k + w_k \\ z_k &= H_k x_k + v_k \\ v_{k+1} &= \psi_k v_k + \zeta_k \end{aligned} \tag{8}$$

$$\begin{aligned} &E(w_k) = E(\zeta_k) = 0 \\ &E(w_k w_k^T) = Q_k,\ E(\zeta_k \zeta_k^T) = R_k \end{aligned} \tag{9}$$

where x_k is the state vector, F_k is the state transition matrix, w_k is the process noise vector, z_k is the measurement vector, H_k is the measurement matrix, v_k is the measurement error, ψ_k is the transition matrix of the colored noise error and ζ_k is white noise error. $E(x)$ is the expectation of the x, Q_k and R_k are the covariance matrices of w_k and ζ_k, respectively.

The system and measurement equation cannot be applied to standard Kalman filter, because the measurement error has a colored noise error. To apply the standard Kalman filter with colored measurement noise, the state vector can be augmented with the colored measurement error so that the system of eq (8) becomes

$$\begin{aligned} x_{k+1}^a &= F_k^a x_k^a + w_k^a \\ z_k &= H_k^a x_k^a \end{aligned} \qquad (9)$$

where

$$\begin{aligned} x_k^a &= \begin{bmatrix} x_k^T & v_k^T \end{bmatrix}^T, \; w_k^a = \begin{bmatrix} w_k^T & \zeta_k^T \end{bmatrix}^T \\ F_k^a &= \begin{bmatrix} F_k & O \\ O & \psi_k \end{bmatrix}, \; Q_k^a = \begin{bmatrix} Q_k & O \\ O & R_k \end{bmatrix} \\ H_k^a &= \begin{bmatrix} H_k & I \end{bmatrix} \end{aligned} \qquad (10)$$

There is no measurement error in the augmented system. If the standard Kalman filter is applied, the filter can be diverged[9]. The standard Kalman filter equation with above augmented system and measurement equations is following equations.

- Time update

$$\begin{aligned} \hat{x}_k^{a-} &= F_k^a \hat{x}_{k-1}^{a+} \\ P_k^- &= F_k^a P_k^+ F_k^{aT} + Q_k^a \end{aligned} \qquad (11)$$

- Measurement update

$$\begin{aligned} K_k &= P_k^- \left(H_k^a\right)^T \left[H_k^a P_k^- \left(H_k^a\right)^T \right]^{-1} \\ \hat{x}_k^{a+} &= \hat{x}_k^{a-} + K_k \left(z_k - H_k^a \hat{x}_k^{a-} \right) \\ P_k^+ &= \left(I - K_k H_k^a \right) P_k^- \end{aligned} \qquad (12)$$

The innovation covariance $H_k^a P_k^- \left(H_k^a\right)^T$ is singular when P_k^- is converged, so the measurement update state $\hat{x}_k^{a+}$ is easily divergent. For this reason, we choice matrix conjugate gradient method to find an inversion of $H_k^a P_k^- \left(H_k^a\right)^T$.

3.2 Matrix Conjugate Gradient Method

The conjugate gradient method is used to solve a linear system. The matrix conjugate gradient method (MCGM) is extended form of conjugate gradient method to solve matrix inversion. MCGM is used to solve the matrix Eq (13).

$$AC = I \qquad (13)$$

where C is inversion of A.

Assume an initial C_0 and calculate $R_0 = I - AC_0, P_1 = R_0$. Repeat the following iterations.

$$\begin{aligned} \alpha_k &= \frac{\left\| R_{k-1} \right\|^2}{P_k \cdot \left(AP_k \right)} \\ C_k &= C_{k-1} + \alpha_k P_k \\ R_k &= I - AC_k \\ \beta_k &= \frac{\left\| R_k \right\|^2}{\left\| R_{k-1} \right\|^2} \\ P_{k+1} &= R_k + \beta_k P_k \end{aligned} \qquad (14)$$

If C_k converges according to a given stopping criterion, $\left\| R_{k-1} \right\| < \varepsilon$, then stop. When C_k is calculated, the inversion of A is given by C_k. Because the calculation of the inverse of matrix $H_k^a P_k^- \left(H_k^a\right)^T$ in Eq (12) causes divergence of the filter, we proposed to replace A with $H_k^a P_k^- \left(H_k^a\right)^T$ to calculate inverse matrix of $H_k^a P_k^- \left(H_k^a\right)^T$.

3.3 Innovation Covariance Based Adaptive Kalman Filter

The state transition matrix of colored measurement error model and white Gaussian variance cannot be known. Therefore, adaptation logic should be applied to the filter. We choose the innovation covariance based adaptation logic and its equations as follows[11].

1) Project the state ahead

$$\hat{x}_k^{a-} = F_k^a \hat{x}_{k-1}^{a+}$$

2) Compute the innovation

$$\eta_k = z_k - H_k^a \hat{x}_k^{a-}$$

3) Estimate the innovation covariance

$$\bar{C}_k = \frac{1}{M-1}\sum_{i=k-M+1}^{k} \eta_k \eta_k^T$$

4) Compute α

$$\alpha_k = \max\left\{1, \frac{trace\left(\bar{C}_k\right)}{trace\left(H_k^a P_k^- \left(H_k^a\right)^T\right)}\right\}$$

5) Project the error covariance

$$P_k^- = \alpha_k \left(F_k^a P_k^+ F_k^{aT} + Q_k^a\right)$$

4 Simulation Results

The system is a six degree of freedom aircraft model and INS/GPS navigation system provides the angular rate and Euler angle for the measurement of the Kalman filter. The measurement error of Euler angle has the colored noise as Figure. 3.

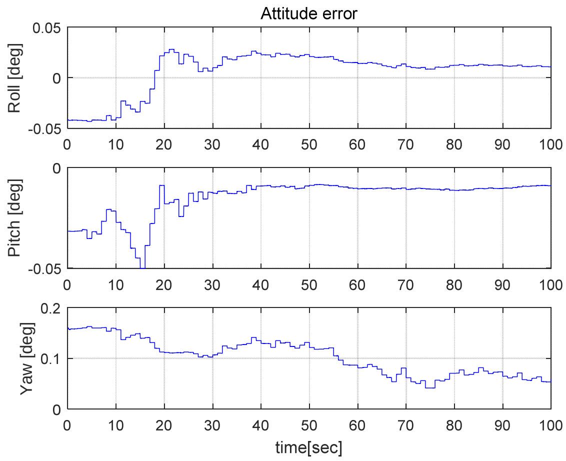

Figure. 3 Euler Angle Error

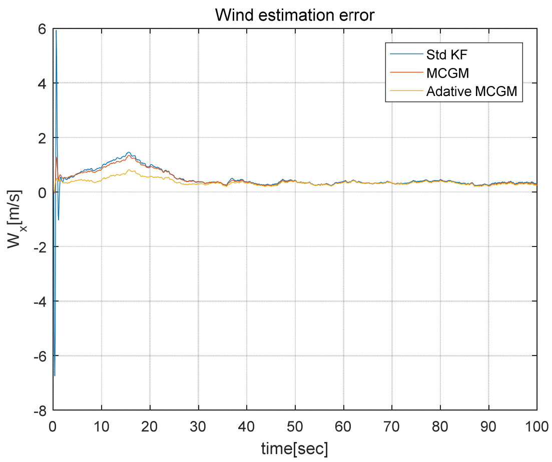

Figure. 4 Wind Estimation Error

Simulation result shows the wind estimation error. The adaptive MCGM algorithm has the best performance of wind estimation.

5 Conclusion

In this paper, we proposed the wind velocity estimation algorithm with angular rate and Euler angle of the aircraft from INS/GPS navigation system as measurements. To consider colored noise measurement error, the matrix conjugate gradient method(MCGM) was applied to calculate innovation covariance of the Kalman filter. In addition, the parameter of colored measurement error model was unknown, so we applied the innovation based adaptive logic to MCGM Kalman filter. Finally, a numerical simulation was performed to verify performance of the proposed algorithm. As a result, the adaptive MCGM Kalman filter improved the estimation performance of the wind velocity.

Acknowledgement

This research has been accomplished as a fundamental research project of LIGNex1-SNU Guidance, Navigation and Control research center.

References

[1] Mulgund S and Stengel R. Optimal Nonlinear Estimation for Aircraft Flight Control in Wind Shear. *Automatica*, Vol. 32, No. 1, pp. 3-13, 1996.

[2] Langelaan J, Alley N, Neihoefer J. Wind Field Estimation for Small Unmanned Aerial Vehicles. *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 4, pp. 1016-1030, 2010.

[3] Petrich J and Subbarao K. On-Board Wind Speed Estimation for UAVs. *AIAA Guidance, Navigation, and Control Conference*, Portland, Oregon, 2011,

[4] Lee J, Sevil H, Dogan A and Hullender D. Estimation of Maneuvering Aircraft States and Time-Varying Wind with Turbulence. *Aerospace Science and Technology*, Vol. 31, Issue. 1, pp. 87-98, 2013

[5] Kim B, Jin J, Park J, and Kim B. Optimization of Glide Performance using Wind Estimator for

Unpowered Air Vehicle without Pitot-Tube. *Journal of Institute of Control, Robotics and Systems*, Vol. 15, No. 1, pp. 1-7, 2009.

[6] Bryson A and Henrikson L. Estimation using sampled data containing sequentially correlated noise. *Journal of Spacecraft and Rockets*, Vol. 5, No. 6, pp. 662-665, 1968.

[7] Petovello M, O'Keefe K, Lachapelle G and Cannon M. Consideration of time-correlated errors in a Kalman filter applicable to GNSS. *Journal of Geodesy*, Vol. 83, No. 1, pp. 51-56, 2009.

[8] Wang K, Li Y, Rizos C. Practical Approachs to Kalman Filtering with Time-Correlated Measurement Errors. *IEEE Transactions on Aerospace And Electronic Systems*, Vol. 48, No. 2, pp. 1669-1681, 2012.

[9] Liu C, Hong H and Atluri S. Novel Algorithms Based on the Conjugate Gradient Method for Inverting Ill-Conditioned Matrices, and a New Regularization Method to Solve Ill-Posed Linear Systems. *Computer Modeling in Engineering and Sciences*, Vol. 60, No. 3, pp. 279, 2010.

[10] Garza F and Morelli E. A Collection of Nonlinear Aircraft Simulations in MATLAB, 2003.

[11] Kim K, Lee J and Park C. Adaptive two-stage extended Kalman filter for a fault-tolerant INS-GPS loosely coupled system. *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 45, No. 1, pp. 125-137, 2009

Contact Author Email Address

park0429@snu.ac.kr

Copyright Statement