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# EXPERIMENTAL COMPARISON OF LIMIT CYCLE METHOD WITH PHASE RESONANCE AND PHASE SEPARATION METHODS FOR DETERMINING MODAL CHARACTERISTICS OF AIRCRAFT STRUCTURES

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## Abstract

*Classical phase resonance method (PRM) with multipoint excitation was successfully utilized in ground vibration tests (GVT) from 60-th up to 90-th years of the 20th Century. The appearance of highly productive computers resulted in replacement of this method in majority of tests by phase separation method (PSM), which requires less testing structure occupation time but needs more data evaluation and analysis time. Due to relatively complicated data processing algorithms that are required by PSM, the final results reliability is lower than in PRM. In this paper short overview of TsAGI experience in aerospace structures GVT is presented, and possible improvements of phase resonance method are discussed. The main issue is how to decrease test duration time without loss of accuracy and reliability. Under discussion is one of the possible ways, i.e. the application of feedback control of excitation force using modern data acquisition and signal generation equipment. A few examples are presented to demonstrate effectiveness and restrictions.*

## 1 Introduction

Ground vibration tests play important role during any flying vehicle development. Methods and equipment are permanently improved to increase accuracy and decrease structure occupation time. As in other countries starting from 1970 up to 1990 years in Russia was used phase resonance method (PRM) [1–4].

Equipment for this method was developed on analog electronics, so test engineer was able to see immediately the results of manipulation on the Lissajous figures. Force appropriation was not long; modes were almost normal and were measured directly by phase detector devices after electronic multiplication by generator signals. Data evaluation was very simple and short. Test time required for typical transport aircraft was about 3-4 weeks.

Starting from 80-th digital boards and computer control appeared for measurement systems. This progressive solution was applied to PRM but the result was not so good because data acquisition required some time and test engineer waited for this time before new digits or pictures appeared. The process of force appropriation became longer than on analog technique. Step by step PRM was replaced by phase separation method (PSM) [5, 6]. It needs not in accurate forces tuning, data are acquired and stored in terms of frequency response functions (FRF) in specified frequency range. Then computer calculates FRF decomposition which gives eigenfrequencies, modes and damping ratios. This way is simple compared with PRM in digital realization. Structure occupation time became about one week shorter. Unfortunately detailed data evaluation and analysis require more time than tests duration. This part of task as a rule is accomplished after test, when it is impossible to repeat doubtful measurements. Of course, main part of GVT results can be obtained with acceptable accuracy, but direct measurements in

PRM are replaced by indirect results after data evaluation procedure. These indirect results can be different when different programs (data evaluation algorithms) are used, when different data volume is evaluated, when different specialists analyze data. As a result many criteria to check the consistency of the results appeared, but final data have inevitable uncertainties.

After several words about TsAGI experience in GVT the possible way to compensate existing PRM and PSM disadvantage are considered.

## 2 TSAGI experience in GVT

GVT were accomplished by TsAGI since 30-th years of 20 century [7]. All new structures before first flight were tested by joined team of TsAGI and design bureau specialists. Since 70-th TsAGI used PRM as main method. Measurement systems and excitation equipment were manufactured by PRODERA company and Russian industry (AVDI-1N equipment). The most interesting work using PRM was an ENERGIA-BURAN system GVT (figure 1).



Fig. 1 Energia-Buran system [12]

The total mass of the system was about 1200 tons, height – 120m. Ten support units were used to suspend elastically. The lowest rigid body frequency was about 0.2 Hz. 5000 N exciters were used for excitation and about 500 accelerometers were registered during GVT.

Starting from 2000 year PRM was used in parallel with PSM, but PSM more and more replaced PRM. Test equipment was composed of PRODERA excitation systems and LMS International measurement and control system.

Typical small aircraft view during GVT when only PSM was used is presented on figure 2. Figure 3 presents the view of large transport aircraft GVT. These tests are also accomplished using PSM. It is possible to see on the picture low frequency pneumatic support systems (PSS) developed in TsAGI. They are easy in usage and make the structure suspension task short and reliable.



Fig. 2. Small aircraft GVT

In general GVT technology tendency in Russia is very close to the world tendencies. Almost the same improvements and almost the same problems (see [8], for example) appear.



Fig. 3. Large transport aircraft GVT

The main problem now is structure occupation time. Test engineers are extremely pressed by project managers, which try to

decrease project expenses. But short test time actually causes the deterioration of test results accuracy and reliability.

It is clear, that even using modern high speed digital equipment PRM cannot be improved in terms of test time better than the time achieved with old analog systems. Good properties of this method – direct measurements (modes, frequencies), mode by mode tuning and registration, simple data evaluation and objective accuracy estimation cannot be restored completely.

The improvements of PSM proposed by many authors during last years are accomplished almost all on the software level. New data evaluation algorithms, new types of excitation signals, identification of nonlinearities and so on are introduced into programs and are quickly used in practice. At the same time software become more complicated and hard in application. Test equipment (accelerometers, force transducers, amplifiers, acquisition boards) is already sufficiently accurate.

As a result it is not clear what can be improved in GVT if we want decrease structure occupation time and increase accuracy and reliability.


## 3 Feedback Control in GVT

About 50 years ago, when PRM was introduced to industrial GVT engineers investigated many interesting approaches. We mention here paper [9], where velocity signal feedback was used to investigate elements of damping matrix during GVT. Figure 4 from this paper shows how the tests were organized. Structure behavior is registered by velocity transducer in one point and specified part of this signal is directed to the exciter amplifier and hence to the exciter which is attached to other point. This is artificial correction of one element of the system damping matrix.

Compared with PRM and PSM this approach actually changes the system under test. If damping properties of tested structure can be changed, some modes can acquire almost zero damping and can be easily measured. If damping exceeds zero, system will be unstable. This is well known fact for control systems with

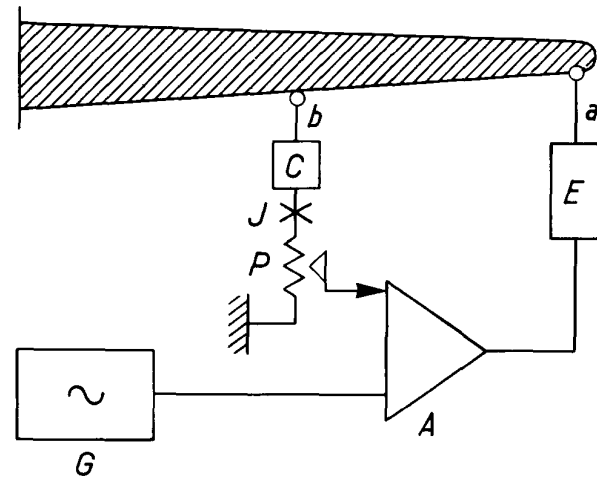feedback, but we try to use this phenomenon in structures resonance tests.



Fig. 1 - Schéma de la réalisation d'une force d'amortissement par réaction.

C. Capteur de vitesse
E. Excitateur
A. Amplificateur de puissance
P. Potentiomètre
J. Inverseur de phase
G. Générateur de commande.

Fig. 4. Test scheme from paper [9]

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Abbreviations should be spelt out in full the first time they appear and their abbreviated form included in brackets immediately after. Words used in a special context should appear between single quotation marks the first time they appear.


## 4 One DOF System with Feedback

Let we have linear system with one degree of freedom and ideal exciter exactly transforming signal to force. The governing equation is:

$$m\ddot{x}(t) + h\dot{x}(t) + kx(t) = f(t) \qquad (1)$$

Let transducer measures velocity $\dot{x}(t)$ and we apply linear feedback

$$f(t) = \alpha\dot{x}(t) \qquad (2)$$

$\alpha$ is coefficient collecting all necessary calibrations.

It is evident, that when $h - \alpha = 0$ the system is undamped with frequency $\omega_0 = \sqrt{\frac{k}{m}}$. Any nonzero initial conditions lead to constant

amplitude oscillations which can be recorded and evaluated.

If $h - \alpha < 0$, system is unstable, amplitude increases up to exciter restrictions or structure destruction. To avoid such behavior, it is necessary to add nonlinear term to the feedback which will restrict amplitude growth. There are well known equations, describing such systems.

Van der Pol equation

$$\ddot{x}(t) - \varepsilon \left( 1 - \left( x(t) \right)^2 \right) \dot{x}(t) + x(t) = 0 \quad (3)$$

Rayleigh equation

$$\ddot{x}(t) - \varepsilon \dot{x}(t) + \eta \left( \dot{x}(t) \right)^3 + x(t) = 0 \quad (4)$$

Equation with "quadratic" friction [10]

$$\ddot{x}(t) - \varepsilon \dot{x}(t) + \eta |\dot{x}(t)| \dot{x}(t) + x(t) = 0 \quad (5)$$

Van der Pol equation is less interesting because restricting part of damping depends on the displacement, i.e. for the feedback we need to measure velocity and displacement.

Equations (4) and (5) contain damping members dependent only on the velocity $v(t) = \dot{x}(t)$. Test scheme for the equation of type (4) can look as presented on figure 5. For our system the equation is as follows:

$$m\ddot{x}(t) + h\dot{x}(t) + kx(t) = F(t) = \alpha v(t) - \beta (v(t))^3 \quad (6)$$

When $h - \alpha < 0$ and $\beta > 0$ the periodic motion exists as one of solutions – limit cycle oscillations. The amplitude $v_0$ of limit cycle can be estimated in terms of first harmonic energy balance.

$$v_0 \approx \sqrt{\frac{4(\alpha - h)}{3\beta}} \quad (7)$$

In the case of quadratic friction (5) equation is

$$m\ddot{x}(t) + h\dot{x}(t) + kx(t) = F(t) = \alpha v(t) - \beta v(t)|v(t)| \quad (8)$$

and limit cycle amplitude:

$$v_0 \approx \frac{3(\alpha - h)}{8\beta} \quad (9)$$

In this experiment there are two "regulators" and cutout. Regulator $\alpha$ allows to find stability boundary and to exceed it. Regulator $\beta$ allows approximately specify desirable amplitude. Under digital control it is possible to specify a few amplitudes. Cutout allows terminate limit cycle oscillations and observe structure natural oscillations. Of course, such system needs the initial disturbance to start oscillation process. If desired amplitude must not exceed $v_0$, we need specify $\beta$ (neglecting unknown $h$ value ) not less than

$$\beta \geq \frac{4\alpha}{3v_0^2} \quad (10)$$

for Rayleigh case and

$$\beta \geq \frac{3\alpha}{8v_0} \quad (11)$$
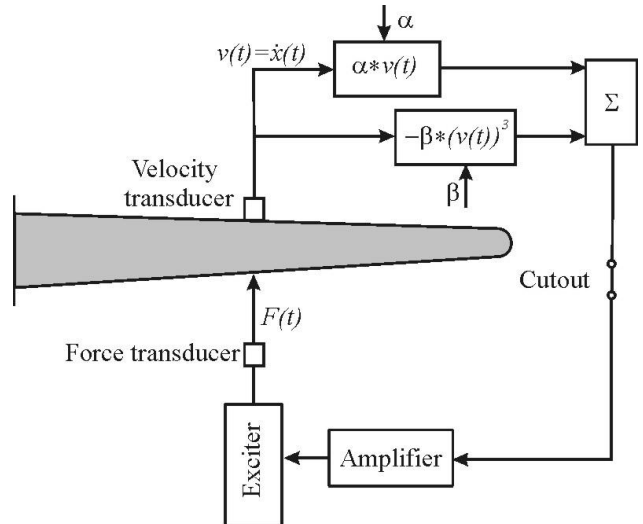
for the quadratic friction case.



Fig. 5. Test scheme with nonlinear feedback

If nothing is known about damping $h$, value $\alpha$ is found by successive increasing up to instability appearance. Value $\beta$ must be corrected simultaneously with $\alpha$ to maintain amplitude restriction.

Numerical simulation shows how it looks. On figure 6 displacement and velocity signals are presented for one degree of freedom system with quadratic nonlinear feedback member. Control system automatically switched value of $\beta$ (four levels on picture) and finally switched off feedback. Last part of process describes system free decay without any external forces.
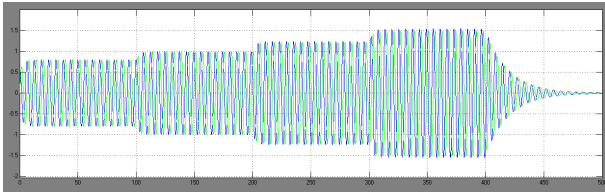
Fig. 6. Numerical simulation 1 DOF system with nonlinear feedback

One such record allows quickly determine frequency and equivalent damping for each vibration amplitude, and to control these data by free decay part of signals. Also it is easy to determine amplitudes of harmonics, to estimate the level of introduced and existing nonlinearities.

The time scale on shown record is nondimensional. Amplitude was switched after 15-16 periods, all record contains about 80 periods. The system with 1 Hz frequency can be tested within 80-100 seconds, 10 Hz – 8-10 seconds.

For better frequency estimation test time can be made as long as necessary.

Equivalent damping is determined by corresponding feedback coefficients α and β which are known exactly at any moment. If force transducer's signal $f(t)$ is also recorded, second way to estimate equivalent damping is integration of the force and velocity signals product. Free decay part allows estimate frequency and damping once more.

## 5 Many DOF System with Feedback

If one exciter feedback control is applied to any point of the system with many degrees of freedom, the approach described above can detect one mode having minimal damping between all modes which can be excited from selected point. When α coefficient exceeds level sufficient to excite one mode, we can expect more complex picture which can not be easy evaluated. We try to avoid this situation.

For MDOF numerical simulation many authors use sample theoretical model with 11 degrees of freedom [13] (figure 7). We used "nonproportional" viscous damping model, with system parameters presented in the table 1. All masses are unit
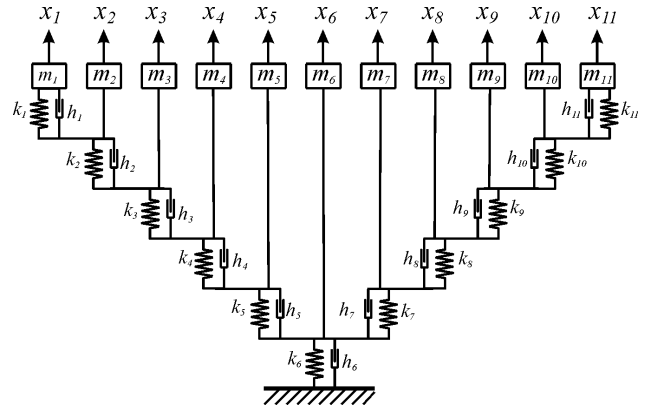


Figure 7: Numerical simulation – 11 DOF system.

System modes are composed of symmetric and antisymmetric modes. Frequencies are distributed by relatively close pairs from 2.7 to 28.5 Hz. First three normal modes are drawn on figure 8. Frequencies are: 2.733, 2.948 and 7.236 Hz. Highest frequency is 28.542.

If we apply force to mass No. 2, for example, and use for the feedback signal $\dot{x}_2(t)$, first limit cycle appears with first frequency. Figure 9 shows all signals $x_i(t)$, figure 10 presents starting part, where limit cycle stabilizes.



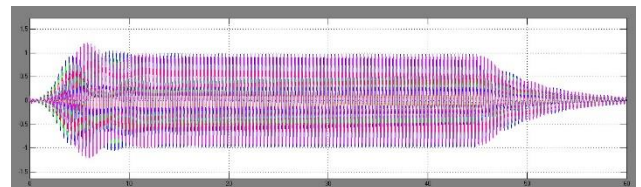Fig. 8. Numerical simulation – 11 DOF system, first 3 normal modes



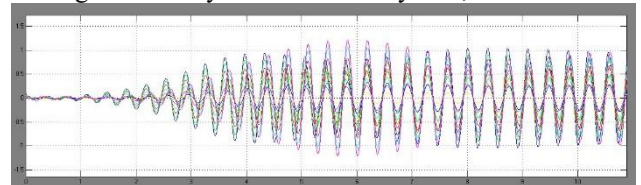Fig. 9. Limit cycle for 11 DOF system, first modes



Fig. 10. Starting part of limit cycle stabilization

Stabilized part looks close to first normal mode (all transducers are almost in phase) but more detailed analysis shows, that phases are not

strongly same (figure 11). Phase shift between signals don't exceed 5˚.
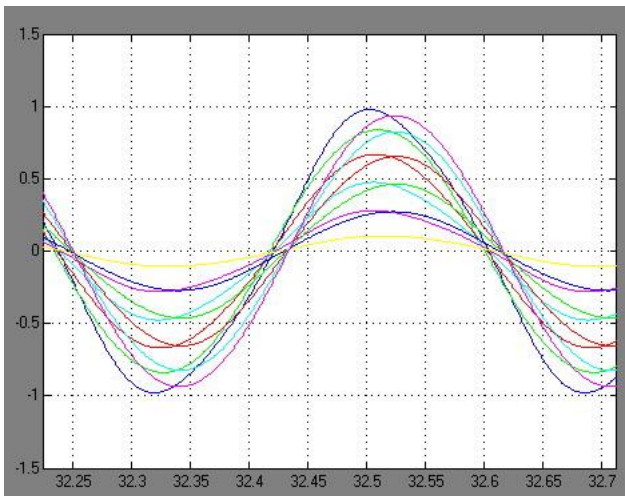


Fig. 11. Phase shift between signals

It's impossible to improve "mode measurement" using one exciter. Multipoint excitation need to be used if we want to improve estimation or if we want to measure next mode. If second exciter is applied to mass No. 10 even with same feedback signal, the phase shift will be twice lower.

Second mode appears first, if antisymmetric forces are prepared by control system.

In general case the task appears similar to PRM – to find force distribution optimal for detected normal mode.

If the mathematical model is known for tested structure, it is reasonable to use calculated modes as appropriate initial approximation.

It is necessary to note, that limit cycle appears for physical system which includes structure and all transducers (masses) and all exciters moving parts masses. Structure damping matrix is disturbed by the feedback. Strictly speaking modes appearing during limit cycle oscillations are not completely normal and are not completely complex due to the damping matrix distortion. But if the number of exciters is sufficient, it is possible decrease phase shift for necessary signals by force tuning. As a result limit cycle mode become more close to the normal mode.

## 6 Test Bench

To check experimentally described approach typical scaled model was used as an example of structure. Figure 12 shows general view of test bench. The model deformations were measured by 13 accelerometers. 50 N PRODERA exciters were used to reproduce calculated forces. Velocity transducers were used for the feedback. Control system demonstrator was prepared for manual control similar to the scheme on figure 5. Limit cycle amplitude was controlled by coefficients $\alpha$ and $\beta$. Initial perturbation to start oscillations was done by manual impulse or by small level random signal added to exciter. In general case initial conditions are important, because solutions for nonlinear system are dependent on it.



Fig. 12. Structure under test

Figure 14 demonstrates controlled limit cycle behavior. Limit cycle amplitude was decreased manually by test engineer. Length of this record is about 60 seconds.
Figure 15 shows starting part of the record where limit cycle appearance from complex combination of transient modes can be observed.

Stable limit cycle is plotted on figure 16. Cursor on this plot is positioned near amplitudes maxima. Amplitude values for each transducer are presented on left top corner. It is also easy to estimate phase shifts between signals. Maximal phase shift between mean phase value and signals phases not exceed 0.9˚.

This demonstration test shows that limit cycle approach can be used in GVT as mean to estimate important vibration characteristics. Detailed technology development require more

experience especially in the problem of orthogonalization feedback control input and output signals to already excited and measured frequencies, damping factors and modes.
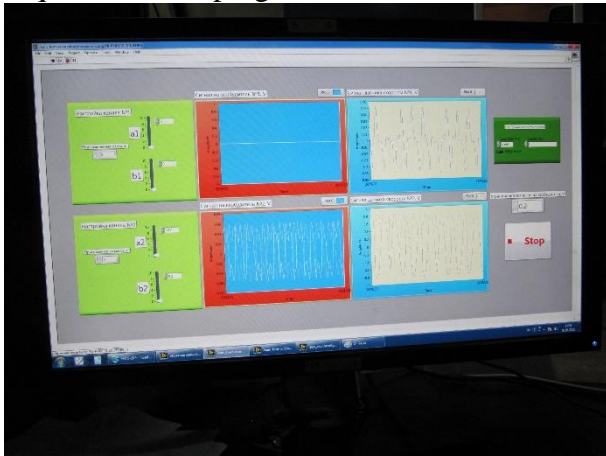

Fig. 13. View of the control panel for limit cycle oscillation tests (demonstrator)
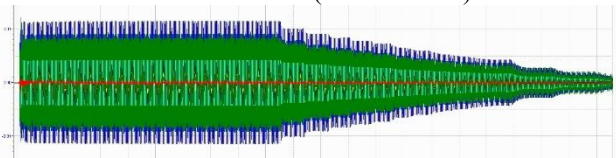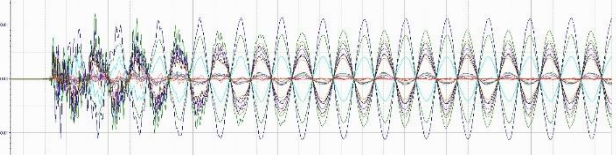

Fig. 14. Limit cycle oscillations with feedback control

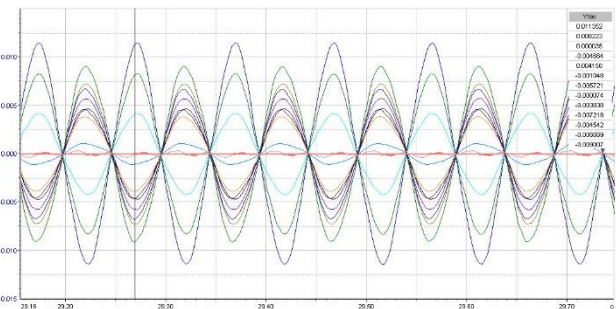
Fig. 15. Starting part of process


Fig. 16. Stabilized part of limit cycle with amplitudes of signals (mode)

60 seconds record in our example allows estimate:

- System eigenfrequency as function of vibration amplitude (figure 17),
- System damping as function of vibration amplitude
- Vibration mode (close to normal) and it dependence on the amplitude
- Scatter of results in terms of confidence intervals.

At the same time these data describe slightly disturbed nonlinear system. System damping matrix is unknown and we change it for the purpose to decrease considered mode damping up to zero. In this case measured frequency will be close to undamped frequency. Damping introduced to the system is completely determined by feedback coefficients and can be additionally controlled if force transducer's signal is also recorded. The test time (length of record) can be increased if necessary, to improve scatter characteristics.
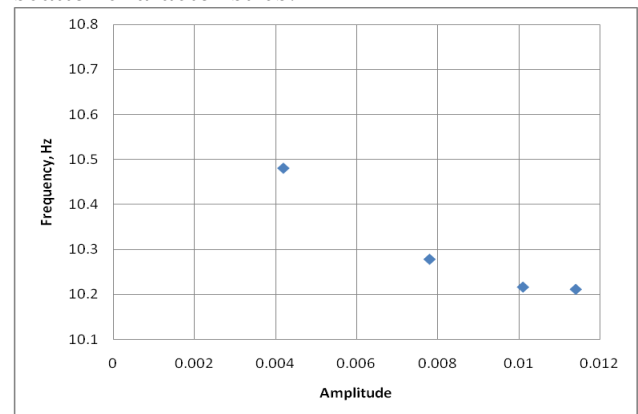

Fig. 17. Limit cycle frequency as a function of amplitude

## 7 Feedback with accelerometer's signals

Accelerometers signals can also be used for feedback. In this case linear members can be used to compensate mass of moving part of exciter or to estimate generalized masses. These two options can also improve GVT technology.

## 8 Equipment Requirements

Considered method of vibration characteristics measurement can be called "Limit cycle method" (LCM). It uses direct measurements of transducers to organize feedback. These data are transformed in the controller and then are directed to the exciters amplifier. Voltage conversion to the current and current conversion to force is accomplished by excitation system. Operations in controller are accomplished numerically with high accuracy. So final results accuracy of LCM is in general dependent on the accuracy of transducers and excitation systems accuracy.

In our LCM trainings we used PRODERA exciters [11]. To avoid additional force regulation linearity property are very important and were specially measured. Exciter EX220SC with amplifier A648/S linearity characteristic is presented on figure 18 as an example. Linearity error for this excitation equipment is less than 1%. We can expect that LCM measurements will have error of same order, if transducers with conditioners have error less than 1%.
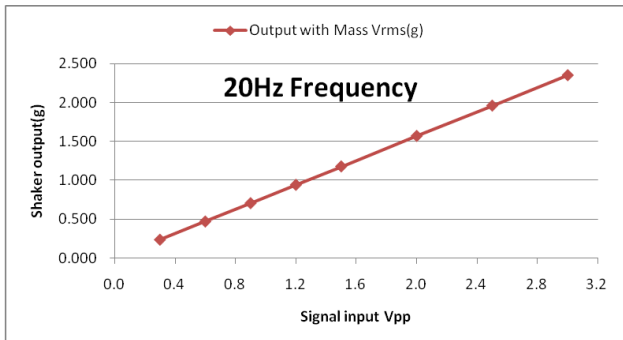


Fig. 18. Exciter EX220SC with amplifier A648/S linearity estimation

It is necessary to note that only specially designed modal shakers with low moving part mass and low nonlinearity errors in all the range of application can be used for LCM.

## 9 Conclusions

Possible improvements in industrial GVT technology were considered in view of accuracy, reliability and time of structure occupation. Phase resonance method is sufficiently accurate and reliable as direct measurement method but it requires more test time for excitation tuning. PSM is more effective, but the results are indirect, extracted after data evaluation procedure.

New test technology was investigated numerically and experimentally. It consists of nonlinear velocity feedback usage to excite limit cycle oscillation. These limit cycle oscillations can be easy controlled, registered and directly measured. Method is quick enough and simple. It can be called "Limit cycle method" (LCM), because it don't use any generator to excite structure. At the same time structure under test become nonlinear, superposition principle is not valid.

Presented illustrations shows, that LCM can be developed up to industrial technology and can supplement PRM and PSM in GVT

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