

# DESIGN THE GUIDANCE LAW FOR FORMATION FLIGHT OF MULTIPLE UAVS

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#### Abstract

In this paper, the algorithm for the formation flight is designed. In these days, UAVs are actively utilized in battlefield and civil area. In this point, many researches on multiple UAVs operation has been studied for various missions. In fact, formation flight gives various benefits such as high efficiency in aerodynamics and low exposure rate by opponent's radar. In this paper, the formation flight algorithm is designed based on the leader-follower method so that it is expandable to the change of number of operating UAVs. Virtual Pursuit Point concept is adapted to design guidance law and it needs only little computing power for real-time operation. Finally, the proposed formation flight algorithm is validated by MATLAB /Simulink environment simulation. Designed algorithm shows not only precise formation tracking performance but also expandability to the various number of operation UAVs.

# **1 INTRODUCTION**

For the last a few decades, Unmanned Aerial Vehicles (UAVs) have been developed for reconnaissance and surveillance purposes. In these days, UAVs get attention as a use of Unmanned Combat Aerial Vehicle(UCAV). In fact, UCAV has been used for ground attack missions using guided missiles. When applied in the battlefield, UCAV is considered as an effective platform because it can perform airstrike mission while not putting pilots' lives in danger.

Operating multiple UCAVs at the same time gives various benefits in terms of the high efficiency, speed, and reliability for completing missions. Furthermore, organizing formation flight induces a lot of advantages. During the formation flight, the vortex from the leader's wingtip lowers the drag of the followers.[1] As a result, the flight efficiency is enhanced aerodynamically. In addition, tight formation flight takes less area compared to the area taken by UCAVs that are not flying in formation. This reduces the possibility of detection by the opponent's radar by reducing the reflectible area.[2] In short, since there are a number of of formation advantages flight. various researches has been conducting to improve its efficiency in completing missions.

There are many ways to approach the autonomous formation flight depending on the way of constructing formation. The most common and easy way is to design and schedule the flight path and waypoint for each UAV. This method is just for small-scaled formation and smooth flight trajectories. Furthermore, the change of number of UAVs or trajectory causes lots of works. In this study, formation flight algorithm for operation of multiple UAVs is proposed based on the 'Leader-Follower' concept. This method expresses the formation geometric so clearly that the formation flight problem is easily turned into the control and guidance problem. For this reason, 'Leader-Follower' method is frequently mentioned in discussion of the formation flight and this paper adopted the 'Leader-Follower' method to develop the formation flight algorithm.

Proposed algorithm should be applicable not only to the loose formation, but also to the tight formation. To achieve this, Virtual Pursuit Point(VPP) concept is introduced.[3] Designed algorithm is validated by MATLAB/Simulink simulation. 5 DOF point mass model is used for the simulation environment. Based on the designed algorithm and the simulation environment, the formation flight for operating four UAVs is simulated in real-time.

# 2 Guidance Law for formation flight

#### 2.1 Virtual Pursuit Point for Formation Flight

There are a number of approach to consider for the formation flight. One method is to schedule and follow the planed flight path. This method is applicable only when the flight path has smooth curvature. In addition, it can only operate on a small number of UAVs. For this reason, the leader-follower method is proposed in this paper. As mentioned before, this method easily transforms formation flight problem as into control and guidance problem. In addition, the designed algorithm is easily applicable to various number of UAVs' operating situations.

For the formation flight algorithm, nine states following below is required.

*Attitude*(
$$\phi, \theta, \psi$$
), Velocity( $u, v, w$ ), Position(Lat, Lon, Alt)

Based on the leader's information above, geometric relation between leader and follower can be defined as shown in Fig 1.



Figure 1. Geometric relation in formation flight

In the leader-follower concept, relative location for organizing formation is defined as  $[l_c, f_c, h_c]$  as shown in Fig 1. These values mean lateral distance, longitudinal distance and height between leader and follower on the leader's body coordinate.  $\rho_e$  is defined as formation error vector for the current formation location.  $X_{VPP}^L$  is

the leader's virtual pursuit point on the vector of the leader's velocity. Based on this point,  $X_{VPP}^F$  which is follower's VPP for formation can be calculated as follows.

$$X_{Vpp}^{F} = X_{Vpp}^{L} + c_{b/r} (\phi_{L}, \theta_{L}, \psi_{L}) \begin{bmatrix} f_{c} \\ l_{c} \\ h_{c} \end{bmatrix} + K_{f} \rho_{e}$$
(1)

This formula, using formation error vector, gets  $X_{VPP}^F$  which is follower's VPP for follower to maintain the [ $l_c, f_c, h_c$ ] from leader on leader's body coordinate system.  $c_{b/r}$  is coordinate transformation matrix from reference coordinate system to body coordinate system following the sequence of 3-2-1.

#### 2.2 Guidance Law for Tracking VPP

The leader's VPP is calculated based on the leader's velocity vector. In addition, follower's VPP is obtained based on the leader's VPP. This relationship includes that the velocity of follower's VPP is same as the velocity of leader. Using formula as follows, we can calculate the velocity on the reference system.

$$V_n^{VPP} = V_{\nu pp}^L \cos\theta_L \cos\psi_l \tag{2}$$

$$V_e^{VPP} = V_{vpp}^L \cos\theta_L \sin\psi_l \tag{3}$$

$$V_d^{VPP} = -V_{vpp}^L sin\theta_L \tag{4}$$

$$V_{ref}^{VPP} = \begin{bmatrix} V_n^{VPP} \\ V_e^{VPP} \\ V_d^{VPP} \end{bmatrix}$$
(5)

Using same formula, the velocity of follower can be represented on the reference coordinate as following equations.

$$V_n^F = V_{NED}^F \cos\theta_L \cos\psi_l \tag{6}$$

$$V_e^F = V_{NED}^F \cos\theta_L \sin\psi_l \tag{7}$$

$$V_d^F = -V_{NED}^F \sin\theta_L \tag{8}$$

$$V_{ref}^{F} = \begin{bmatrix} V_{n}^{F} \\ V_{e}^{F} \\ V_{d}^{D} \end{bmatrix}$$
(9)

Using these values, we can get  $V_{ref}^{LOS}$  velocity difference between  $V_{ref}^F$  and  $V_{ref}^{VPP}$ . Using coordinate transformation matrix from reference frame to body frame,  $V_{ref}^{LOS}$  can be represented on the body coordinate as follows.

$$V_{ref}^{LOS} = \left(V_{ref}^{VPP} - V_{ref}^{F}\right) \tag{10}$$

$$V_{body}^{LOS} = C_{b/r} (\phi_F, \gamma_F, \chi_F) * (V_{ref}^{VPP} - V_{ref}^F) (11)$$

To make precise formation flight, the follower traces its VPP obtained from the leader's VPP, not the point that has  $[l_c, f_c, h_c]$  clearance away from the current leader's position. Thus, it is required to get the difference vector between follower's position and its VPP, and this value could be obtained by subtract  $X^{LOS}$  from  $X_{VPP}^{F}$ . As eq.(12) above, this value can be converted into body frame.

$$X_{body}^{LOS} = C_{b/r}(\phi_F, \gamma_F, \chi_F)(X_{VPP}^F - X^F) \quad (12)$$

On body coordinate, follower's velocity can be represented in the vector form as  $[V^F, 0, 0]$ . The angle which follower should track to make formation flight is the angle between  $X_{body}^{LOS}$  and  $X_{body}^F$ . This tracking angle, named as look angle in this paper, can be calculated as follows.

$$LookAngle = sin^{-1}(((V_{body}^F \times X_{body}^{LOS}) \times (V_{body}^F))/(V^F \operatorname{norm}(V_{ref}^{LOS}))/V^f) \quad (13)$$

When look angle is bigger than 90 degrees, it means that flying direction of leader and follower

is the opposite direction. In formation flight, it is recommended that the leader and follower have same direction. When the direction of leader and follower is opposite, although follower traces its virtual pursuit point, it take long time to make formation. Thus, when look angle is bigger than 90 degrees, it is needed to re-define  $X^{LOS}$  to organize formation faster. If the second column of  $X^{LOS}$  is bigger than zero,  $X^{LOS}$  can be redefined as follows

$$X^{LOS} = C_{r/b}(0, 0, \chi_F + \pi/2) \\ * \begin{bmatrix} norm(X^{LOS}) \\ 0 \\ 0 \end{bmatrix}$$
(14)

If the second column of  $X^{LOS}$  has negative value,  $X^{LOS}$  can be re-defined as follows.



# Figure 2. Pure pursuit guidance & Proportional navigation guidance law

In this paper, the combined guidance command of pure pursuit guidance law and proportional navigation guidance law is calculated. During follower chases the leader, pure pursuit guidance law generates acceleration command making follower's line-of-sight(LOS) to be zero. On the other hand, proportional navigation guidance law makes rate of LOS to be zero, which enables the follower to chase the leader with constant LOS value.

Acceleration command of pure pursuit guidance can be obtained as below.  $K^{PG}$  is a navigation factor for pure pursuit guidance. We have found the best results experimentally with  $K^{PG}=1$  and g is acceleration of gravity.

$$A_{cmd}^{PG} = K^{PG}(([V^L \ 0 \ 0] \times X_{body}^{LOS}) \times [V^L \ 0 \ 0])/norm(X_{body}^{LOS})/g \quad (16)$$

Pure pursuit guidance law only generates temporary tracking maneuver command to reduce the error in LOS angle. When the leader's maneuver causes changes of LOS angle, the tracking performance with pure pursuit guidance law is not enough to keep tight formation. To performance, improve the we make compensation of this change of rate of LOS by proportional navigation guidance. We have found best results experimentally with  $K^{PNG}=1$ . Difference in current positions of leader and wingman can be represented in vector and unit vector of it can be calculated.

$$DEL = \begin{bmatrix} X_L - X_F \\ Y_L - Y_F \\ Z_L - Z_F \end{bmatrix}$$
(17)

$$D\widehat{EL_{ref}} = \frac{DEL}{norm(DEL)}$$
(18)

Using  $D\widehat{EL_{ref}}$ ,  $V_{ref}^{LOS}$  and  $X^{LOS}$ , we can define  $R_{ref}$  on reference frame and  $R_{body}$  on body frame.

$$R_{ref} = (D\widehat{EL_{ref}} \times V_{ref}^{LOS}) / norm(X^{LOS})$$
(19)

$$R_{body} = C_{b/r} (\phi_F, \gamma_F, \chi_F) R_{ref} \qquad (20)$$

The command of proportional navigation guidance can be obtained by following eq. (21).

$$A_{cmd}^{PNG} = 0$$

$$K^{PNG} \begin{bmatrix} 0 \\ R_{body}(3)norm(V_{body}^{LOS}) \\ -R_{body}(2)norm(V_{body}^{LOS}) \end{bmatrix} (21)$$

Using command from pure pursuit guidance law and proportional guidance law, the acceleration command for formation flight can be obtained as follows.

$$a_{cmd} = A_{cmd}^{PG} + A_{cmd}^{PNG} - C_{b/r}(\phi_F, \gamma_F, \chi_F) \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$
(22)

The second and third column of  $a_{cmd}$  is acceleration command in the direction of y and z respectively. Using these values,  $P_{cmd}$  which is roll angular velocity commands can be obtained as below.

$$P_{cmd} = tan^{-1} (a_{cmd}^{y} / a_{cmd}^{z})$$
(23)

As mentioned above the velocity of VPP is same as the leader's velocity. Based on it, follower's velocity command can be obtained by adding command from location difference and command from change rate of distance.

$$V_{Fcmd} = V_{vpp} + K_v X_{body}^{LOS}(1) + K_{vdot} V_{body}^{LOS}$$
$$= V_L + K_v \rho_e + K_{vdot} \dot{\rho_e}$$
(24)

Now, we can get three command, velocity, roll angular rate, and Nz load factor. Wingman's state can be obtained by putting these commands into 5 DOF point mass model of MATLAB/Simulink, as shown in Figure 3.



#### Figure 3. MATLAB/Simulink simulation

# 2.3 Formation Flight Velocity Command

Using formation guidance velocity from formula (24), we can get the result of circular formation flight with velocity command of eq. (24), as shown in Fig 4.



Figure 4. Result of circular formation flight

In Fig 4, we can notice that follower which is inside of the leader's circular path moves forward than the formation. Inversely, the another follower which locates outside of the leader's circular path is pushed back from the designated formation location. This is because of difference in turning radius during circular flight. This problem can induce collision in formation flight. Thus, we solve this problem by adding feed forward control for the follower's velocity.

**Turn Radius** = 
$$\frac{V^2}{g \tan(\phi)}$$
 (25)  
 $R_W = R_L + d_{lateral}$ 

Using eq. (25), we can get the turn radius of the leader. Follower's turning radius can be obtained using this value and lateral relative location between leader and follower,  $l_c = d_{lateral}$ . Turn rate of leader can be obtained using following formula.

**Turn Rate** 
$$r = \frac{2pi}{(2R_Lpi/V_L)} = \frac{V_L}{R_L}$$
  
 $r_L = r_W = \frac{V_W}{R_W}$  (26)

To maintain the formation, leader and follower should have the same turn rate. By multiplying this turn rate to turn radius, we can get the follower's velocity command for formation flight.

$$V_{W} = r_{W}R_{W} = r_{L}(R_{L} + d_{lateral})$$
(27)

Figure 5 shows the result of simulation of circular flight using supplemented velocity command.



Figure 5 Result of modified velocity command

# **3 Formation Flight Simulation**

The performance of the proposed algorithm is verified by real time simulation environment using MATLAB/Simulink. Figure 6. Shows the structure of simulation environment. Proposed algorithm and simulation environment has easy condition to change the configuration of formation. It can also increase the number of wingman up to nine easily.



**Figure 6. Simulation Environment** 

In figure 6, red area transfers the leader's flight states, which is saved beforehand, to the follower's guidance law. Blue, green and emerald areas contain follower's guidance law and its 5 degree of freedom point mass model. Using this simulation environment, we simulated formation flight with one leader and three followers. Following contents is clearance for formation and initial position error of simulation conditions

- Formation location
   Wingman 1 = [-2m, -4m, 0m]
   Wingman 2 = [-2m, -4m, 0m]
   Wingman 3 = [-5m, -0m, 0m]
- Initialization Error of wingman Wingman 1 = [-10m, -10m, 5m] Wingman 2 = [-0m, -0m, 5m] Wingman 3 = [10m, 10m, 10m]

As given formation clearance, followers have to keep 4m distance from the leader, which value is similar with the wingspan of the small airplane. This condition can verify the performance of designed guidance law. Figure7 shows the result of tight formation flight simulation. We can find that each wingman maintains designated formation location from leader and forms diamond formation. In addition, it takes only 5 seconds to converge the assigned formation position since the simulation had been begun.

Figure 8 shows the simulation result of wingman 3. After 5 seconds, lateral/longitudinal/height errors converges to 0m,

which means that wingman 3 reaches to the assigned formation position. A little error, about 2m, occurs when the leader makes some maneuvers. In fact, common formation flight shared the intention of the maneuvers and flight path, in advance. However, proposed algorithm only uses the current position of the leader. Considering this point, it can be judged that the VPP based formation algorithm shows precise tracking performance enough to keep the formation.



Figure 7. Diamond Tight Formation simulation



Figure 8. Result of Tight Formation simulation

#### **4** Conclusion

This paper proposes the virtual pursuit point based formation guidance law using 'leaderfollower' method. Using formation error vector and rate of error vector, follower's virtual pursuit point and velocity command are generated. Especially, to compensate the error generated by the change of turn radius during circular flight, velocity command is amended by considering turn radius and turn rate.

Designed algorithm can be applied to various formation configuration and easily adaptable to various number of operating UAVs. Computing time of this algorithm is fast enough to BE applied in real time. To validate, simulation environment is designed using MATLAB/Simulink. Simulation results show that the proposed algorithm performs precise formation flight having error less than two meters.

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