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OPTIMIZATION OF THE TRAJECTORY AND ACCUMULATOR MASS FOR THE SOLAR-POWERED AIRPLANE

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Abstract

The problems of optimal control and optimal accumulator mass for the aircraft with the electric powerplant and solar cells for the multi-day flight are investigated. The case of some restrictions on aircraft energy storage and peculiarities of its charge and discharge are also analyzed. Pontryagin's maximum principle is utilized. Optimal trajectories were obtained for the cases considered and the analysis of accumulator mass influence was conducted.

1 Introduction

By this time a set of solar-powered (SP) airplanes was designed and built (from those are “Pathfinder”, “Centurion”, “Helios”, “Solar Impuls”, SoLong”, “Zephyr” and others). But for the present state of the art the design of the SP airplane for the multi-day mission is still a serious problem because of the moderate value of the solar radiation intensity, rather low efficiency and rather high density of the solar cells, insufficient energy density of the onboard energy storage and some other factors.

The performance of the apparatus can be improved through the optimal control in the flight. Some theoretical investigations were made by authors earlier and a set of optimal control problems was solved [1]–[4]. Hypothesis of “quasi-stead” motion was used for the equations of motion simplification. Among other results, optimal altitude for the level flight and optimal trajectory for the flight at changing altitude were found analytically.

In these investigations it was assumed that the onboard energy source (“accumulator”) has infinite capacity and can accumulate any amount of energy. In fact, any storage device can accumulate only limited amount of energy depending on the storage performance

Also the efficiencies of “accumulator” charging and discharging were not taken into account. It is evident that these efficiencies affect the shape of trajectory.

Considered here is the problem of optimal control for the solar-powered airplane for the maximization of the “accumulator” energy at the end of flight for the multi-day 24-hour periodical mission taking into account capacity of “accumulator” and charging-discharging efficiency. Also, the influence of accumulator mass is investigated.

The research was made on a basis of the Pontryagin's maximum principle (to be more precise, Pontryagin's **minimum** principle) [5]. Analyzed is the case for which the dependence of the characteristics of the Sun does not change from day to day.

2 Previous Investigations

Some questions concerning the optimal multi-day flight were investigated in [1]–[4]. First of all, the optimal altitude for the flight at constant altitude was found. If the dependence of air density ρ on the altitude h is defined then the values of ρ and h can be found from

$$\rho^3 = 8\left(\eta\frac{\partial J}{\partial h}\right)^{-2}\left(\frac{G}{S}\right)^3\sqrt{\frac{A^3C_{D0}}{27}}\left(\frac{\partial\rho}{\partial h}\right)^2, \qquad (1)$$

where

η – powerplant efficiency (assumed to be constant),

$G=mg$,

m is aircraft mass,

g – acceleration of gravity,

S –wing area,

$A=1/(\pi\lambda)$,

λ – aspect ratio of the wing,

C_{D0} – drag coefficient at zero lift,

J – power obtained from the unit area of solar cell.

That equation for the isothermal atmosphere in which the density depends on the altitude by the formula defined in [1] as $\rho=\rho_0\exp(-h/h_0)$ (h_0=6374 m, ρ_0 is the air density at the reference level where h=0), gives

$$\rho = 8\left(\eta h_0\frac{\partial J}{\partial h}\right)^{-2}\left(\frac{G}{S}\right)^3\sqrt{\frac{A^3C_{D0}}{27}}.$$

The corresponding velocity V is [1]

$$V = -\sqrt{\frac{3}{AC_{D0}}\frac{\eta\rho}{2(G/S)\frac{\partial\rho}{\partial h}}\frac{\partial J}{\partial h}}, \qquad (2)$$

that for the isothermal atmosphere gives

$$V = \sqrt{\frac{3}{AC_{D0}}\frac{\eta h_0}{2(G/S)}\frac{\partial J}{\partial h}}.$$

As an example some generic aircraft was considered with the following characteristics: C_{D0}=0.01, λ=30, m=600 kg, S=200 m^2, η=0.8, solar cell efficiency – 23.2%. Taking the mean value of data from [6] and [7] one can obtain that the average value of $\partial J/\partial h$ for 24 hours is $\partial J/\partial h = 0.0025\,\mathrm{W/m^3}$. These values give the “constant optimal altitude” (COA) of about 9 km.

Then, optimal flight path was analysed for the case of non-fixed altitude with the help of Pontryagin’s maximum principle [5]. The problem was investigated numerically [1] and analytically [2]–[4]. It was found that the hypothesis of quasi-steady flight can be used for such investigation, and within this hypothesis the flight altitude and velocity are defined by the same relationships as (1) and (2) and the only difference is that the characteristics of Sun radiation are not mean values but values at the required moment of time.

The solutions were checked for the necessary optimum condition.

But during the night time there is no sun radiation, and the optimal air density is equal to infinity (corresponding altitude is equal to “minus infinity”). So, during the night time the aircraft must fly at the lowest possible altitude.

So, the shape of the optimal trajectory for the case investigated looks like in Fig. 1.

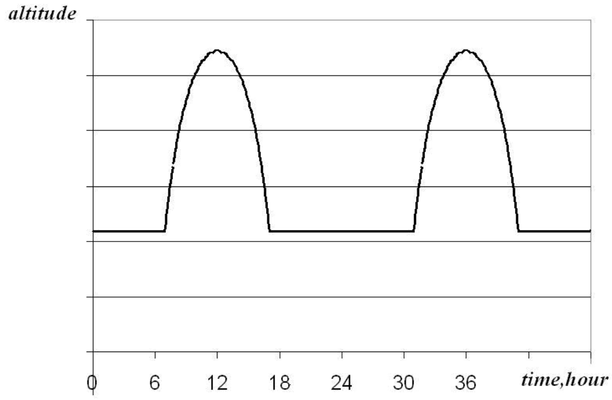

Fig. 1. Optimal trajectory shape.

Some characteristic values of the energies must be given. For the aircraft with the performance given above for the flight at the constant optimal altitude the energy consumed by the powerplant for 24 hours is about $1.3{*}10^8$ Joule. The energy obtained from solar cells for 1 day is about 10^9 Joule. The advantage in energy in the accumulators after 24 hours of flight along the trajectory shown in Fig. 1 is about 30% of the energy consumed by the motor. But in comparison with the energy obtained from solar cells it is only about 4%.

3 On-board Energy Limitation and Charge-Discharge Efficiency Influence

It is evident that the energy stored in on-board accumulators can’t be less than zero (fully discharged accumulator) and higher than some maximal amount of energy (fully charged accumulator). Mathematically these conditions can be expressed as

$\Phi_1 = E - E_{\max} \le 0$ (full discharege),

$$\Phi_2 = -E \le 0 \text{ (full charge).}$$

where E – consumed energy of accumulator

As there is no control variables in this expressions, one must use the derivatives of these equations for Pontryagin's minimum principle [5].

Assume that the part of energy α from solar cells goes to the motor and the other part of energy (1–α) goes to accumulator to charge it. Assume that the accumulator charging efficiency is γ and discharging efficiency is 1/β, η here is the efficiency of all the powerplant except accumulator and solar cells. Define that W is the power going from accumulator to the motor, so $W \ge 0$.

For this investigation the equations of motion become as follows (see [3])

$$\frac{m}{2}\dot{Z} = \left(\alpha I(h,t) + W\right)\eta -$$
$$-\left(C_{D0} + A\left(\frac{2mg}{\rho Z}\right)^2\right)\rho\frac{Z^{3/2}}{2}S - mgV_Y$$
$$\dot{E} = \beta W - \gamma(1-\alpha)I(h,t)$$
$$\dot{h} = V_Y\,,$$

where $Z=V^2$, V_Y – vertical component of aircraft velocity.

The optimized function is the energy E consumed from accumulators during 24 hours (T=24 hours) of flight

$$E = \int_0^T \left(W - I(h,t)\right)dt$$

with the condition that the solution is 24-hour periodic in the case that all the variables (Z, h, V_Y) except E are the same after 24 hours of flight. Minimum of E is required.

(For the fully periodic condition it is required that the value of E is also the same as 24 hours before. It is evident that this condition is very difficult to satisfy so one can assume that after 24 hours of flight the extra amount of energy is consumed instantaneously so for the next period the value of E is the same as 24 hours before).

Optimization problem for Pontryagin's **minimum** principle is analysed. For this problem the Hamilton function **H** for control variables α, W, V_Y is

$$\mathbf{H} = \frac{2P_Z}{m}\left(\begin{array}{l}\left(\alpha I(h,t) + W\right)\eta - \\ \left(C_{D0} + A\left(\frac{2mg}{\rho Z S}\right)^2\right)\rho\frac{Z^{3/2}}{2}S - mgV_Y\end{array}\right) +$$
$$+P_H V_Y + \left(\beta W - \gamma(1-\alpha)I(h,t)\right) +$$
$$+\mu_1\left(\beta W - \gamma(1-\alpha)I(h,t)\right) +$$
$$+\mu_2\left(\beta W - \gamma(1-\alpha)I(h,t)\right)$$
$$0<\alpha<1,\ 0<W<W_{max},$$

where P_Z and P_H are conjugate variables for Z and h, respectively.

Here $\mu_1(t) \ge 0$ for the full charge restriction, $\mu_2(t) \le 0$ for the full discharge restriction; $\mu_1(t)=0$, $\mu_2(t)=0$ inside restrictions. It is evident that μ_1 and μ_2 can't be higher than zero simultaneously.

First of all, let's find the solution within the restrictions.

Conditions for the control variables [5]:

– for W:

1) $W=W_{max}$, at $s<0$,
2) $W=W_{min}$ (W=0), at $s>0$,
3) singular control: $s=0$, $\dot{s}=0$,

where s – switch function:

$$s = \frac{2\eta P_Z}{m} + \beta\,, \qquad (3)$$

– for α:

1) $\alpha=\alpha_{max}$, at $q<0$,
2) $\alpha=\alpha_{min}$ (α=0), at $q>0$,
3) singular control: $q=0$, $\dot{q}=0$,

where q – switch function:

$$q = \frac{2\eta P_Z}{m} + \gamma\,, \qquad (4)$$

– for V_Y:

1) $V_Y = V_{Y\,max}$, at $p<0$,
2) $V_Y = V_{Y\,min}$, at $p>0$,
3) singular control $p=0$, $\dot{p}=0$,

where p – switch function:

$$p = -2gP_Z + P_H.$$

Consider the cases of singular control. It is evident that the cases of singular controls for α and W does not realize simultaneously as β>1 and γ<1 (see (3) and (4)). This means that there are the following situations.

1. Singular control for *α*. As γ<β then switch function for W is positive. It means that W must

be minimal (equal to zero in this case). So, the situation is that a part of power from solar cells goes to the motor, another part goes to accumulator and no power from accumulator goes to motor.

2. Singular control for W. In this case switch function for α is negative, and α must be maximal (α=1). This means that all the power from solar cell goes to the motor and also some power from accumulator goes to the motor.

From this one can understand that the accumulator can't charge and discharge simultaneously. On the one hand it is rather evident, but on the other hand, for the accuracy even evident things must be proved.

For each of singular control cases s=0 or q=0 the conditions of singular control $\dot{s}=0$ or $\dot{q}=0$ give the same equation

$$\dot{P}_Z=0.$$

The condition for the conjugate variable P_Z is

$$\dot{P}_Z=-\frac{\partial \mathbf{H}}{\partial Z}= =-\left(-\frac{2P_Z}{m}\left(\frac{3}{2}C_{D0}\rho\frac{Z^{1/2}}{2}S-\frac{1}{2}\frac{A(2mg)^2}{2Z^{3/2}\rho S}\right)\right) \quad (5)$$

The condition for P_H is

$$\dot{P}_H=-\frac{\partial \mathbf{H}}{\partial h}= =-\left(\frac{2P_Z}{m}\left(\alpha\frac{\partial I}{\partial h}-C_{D0}\frac{Z^{3/2}}{2}S\frac{\partial \rho}{\partial h}-\frac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\frac{\partial \rho}{\partial h}\right)+ +\gamma(1-\alpha)\left(-\frac{\partial I}{\partial h}\right)\right)$$

One can check that for any of singular controls for α or W this equation becomes

$$\dot{P}_H=-\frac{\partial \mathbf{H}}{\partial h}= =-\left(-\frac{2P_Z}{m}\left(C_{D0}\frac{Z^{3/2}}{2}S\frac{\partial \rho}{\partial h}-\frac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\frac{\partial \rho}{\partial h}\right)+\left(-\gamma\frac{\partial I}{\partial h}\right)\right)$$

This gives that the solutions for Z and h are the same as given by (1) and (2) with the substitution $\gamma\cdot(\partial I/\partial h)$ in place of $(\partial I/\partial h)$.

If we introduce the "total" powerplant efficiency η_0 with taking into account accumulator charging efficiency as

$$\eta_0=\gamma\eta,$$

we obtain the condition for the altitude in the same form as (1) with substitution η_0 in place of η.

Formula (1) for the isothermal atmosphere gives the dependency of altitude on the flight parameters as

$$\frac{h}{h_0}=2\ln\eta_0+2\ln\left(\frac{\partial J}{\partial h}\right)-3\ln\left(\frac{G}{S}\right)- -0.5\ln\left(\frac{A^3C_{D0}}{27}\right)-3\ln 2+2\ln\left(h_0\right)$$

If we assume that $(\partial J/\partial h)$ is nearly the same at any altitude for the fixed moment of time, then the optimal trajectories for the different values of η_0 are simply shifted in vertical directions but have the same shape. To estimate this shift one can say that 1% of efficiency increase is equal to the altitude increase of 127.4 m.

It must be noted that there is no symbol β in the results of this chapter. This means that the discharge efficiency does not influence on the altitude and velocity. It influences only on power consumption from accumulators. This acts only during the accumulator energy consumption (at night and in the case of insufficient solar energy).

So, for any singular control the required power depends only on the time of the day for the flight within the restrictions on the altitude. The only question is what is the source of this energy (solar cell or accumulator).

For the case of restrictions

$$\dot{E}=\beta W-\gamma(1-\alpha)I(h,t)=0$$

There are two ways to fulfill this dependence.

1. W=0, α=1. In this case all the energy from solar cells goes to the motor.

2. W>0, α<1. In this case some energy from solar cells goes to the accumulator and then it goes to motor. In this case some amount of energy losses on the way from solar cell to the motor comparing to the first case.

So, the first case must be chosen and, thus, values of two control variables W and α are defined.

The condition for V_Y control variable is:

1) $V_Y = V_{Y\max}$, at $p<0$,
2) $V_Y = V_{Y\min}$, at $p>0$,
3) singular control $p=0$, $\dot{p}=0$,

where p – switch function:

$$p = -2P_Z g + P_H. \tag{7}$$

The case of singular control is of the main interest.

As for the conjugate variables, the condition for P_Z remains the same as (6) and condition for P_H will be as

$$\dot{P}_H = -\frac{\partial \mathbf{H}}{\partial h} = -\left(\begin{array}{l} \dfrac{2P_Z}{m}\left(\alpha\eta\dfrac{\partial I}{\partial h} - \left(C_{D0}\rho\dfrac{Z^{3/2}}{2}S\dfrac{\partial\rho}{\partial h} - \dfrac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\dfrac{\partial\rho}{\partial h}\right)\right) - \\ -\gamma(1-\alpha)(1+\mu_1+\mu_2)\left(\dfrac{\partial I}{\partial h}\right) \end{array} \right)$$

As $\alpha=1$ then

$$\dot{P}_H = -\frac{\partial \mathbf{H}}{\partial h} = -\left(\frac{2P_Z}{m}\left(\eta\frac{\partial I}{\partial h} - \left(C_{D0}\rho\frac{Z^{3/2}}{2}S\frac{\partial\rho}{\partial h} - \frac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\frac{\partial\rho}{\partial h}\right)\right)\right)$$

This equation together with (5) and derivative of (6) with respect to time gives

$$\left(C_{D0}\rho\frac{Z^3}{2}S\frac{\partial\rho}{\partial h} - \frac{A(2mg)^2}{2\rho^2 S}Z\frac{\partial\rho}{\partial h}\right) - \eta\frac{\partial I}{\partial h} - -2g\left(\frac{3}{2}C_{D0}\rho\frac{Z^2}{2}S - \frac{1}{2}\frac{A(2mg)^2}{2\rho S}\right) = 0$$

The equations of motion on the limitations investigated are

$$\frac{m}{2}\dot{Z} = I(h,t)\eta - -\left(C_{D0} + A\left(\frac{2mg}{\rho(h)\cdot Z}\right)^2\right)\rho(h)\cdot\frac{Z^{3/2}}{2}S - mgV_Y$$

$$\dot{h} = V_Y .$$

Last three equations give the solution for h and Z as function of time.

As an example let's consider the case with the same conditions as in Chapter 1 and imagine that the accumulators become full at the moment when the aircraft reached the COA in the "evening". Calculations give the trajectory of airplane after this moment as shown in Fig. 2 (red line) up to the next moment of reaching the COA.

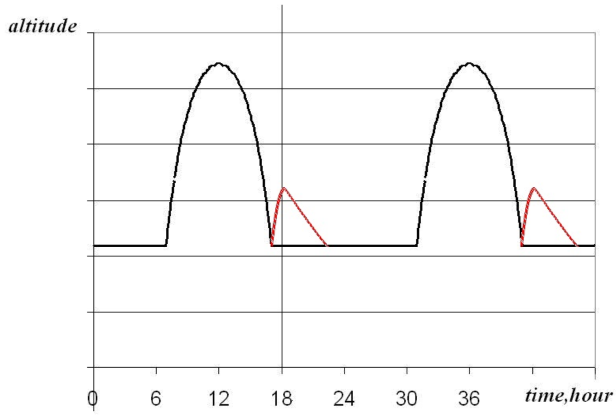

Fig.2. Trajectory for the case with restriction on the accumulator capacity

Investigation of this trajectory shows that the disadvantage of the stored energy for 24 hours (compared to the case without the restriction on the accumulator capacity) is about 2% of the energy consumed by the motors

4 Influence of accumulator mass

Previously it was assumed that the amount of accumulators is fixed. Now imagine that we can put onboard some amount of accumulators that enable the 24-hours flight. Then we decrease the amout of accumulators for some value. On the one hand, the capacity of accumulator decrease, so for some time the aircraft must fly along the trajectory corresponding to the full accumulator. This gives some disadvantage. On the other hand, the mass of aircraft also decreases, so we have some advantage in the power consumption. The following peculiarities must be taken into account

1. With the change of mass, the value of h(t) for the optimal trajectory is changed
2. With the change of mass, the moment of time for the beginning of climb from the altitude restriction also changes.

Taking into account these peculiarities the advantages in accumulator mass decreasing were investigated for the values of mass change that are corresponding to the time of full accumulator in the flight along the restriction (in the "evening time"). The investigations were conducted numerically for the aircraft with the characteristics mentioned above. The results are shown in the Fig.3.

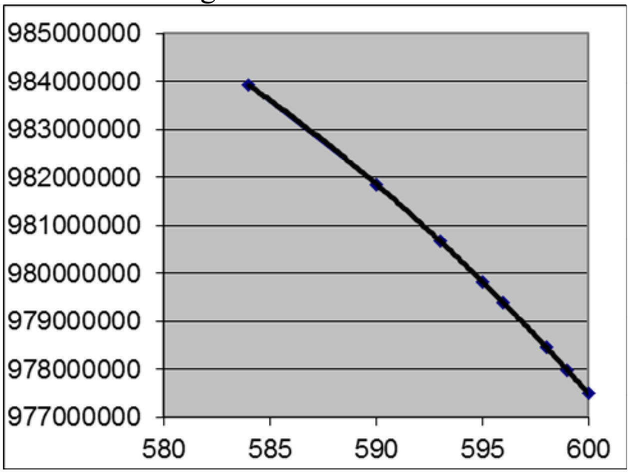

Fig. 3. Dependence of E on the mass of aircraft due to accumulator mass change

It should be mentioned that for the lowest mass of accumulators shown in Fig.3 the accumulator becomes full practically at the moment of time corresponding to the beginning of the flight at the restriction on altitude.

From this figure one can see that the advantage can be about 0.7% of the E change during 24 hours.

Also one can see that there is no exremum in this graph. So, it corresponds to the lower values of accumulator mass. This case will be analyzed in future work.

Conclusion

The analysis of optimal trajectories under the set of restrictions for the solar airplane was made.

The case of limited accumulator capacity was investigated. The characteristic value of disadvantage for the fixed aircraft mass was found

For the variable accumulator mass the question of optimal accumulator mass was analyzed.

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