CLOSED-FORM COOPERATIVE GUIDANCE LAW FOR TWO MISSILES WITH COUPLED TERMINAL VELOCITY CONSTRAINTS

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Abstract

This paper presents optimal closed-form guidance laws for two-missile cooperative attack missions. Two missiles are dynamically decoupled and they cooperate to optimize a common cost function under a coupled constraint on terminal velocity vectors. Based on the linear quadratic tracking theory, we obtain the closed-form impact angle control guidance law which satisfies the coupled impact angle constraints of the two missiles. We show that the approach can be easily extended to multiple missile cases. The performance of the proposed guidance law was verified via nonlinear simulation cases with a variety of engagement scenarios.

1 Introduction

In recent times, along with the development of missile techniques, missile defense systems have made significant progress. Most of the important strategic military assets are well equipped with those defense systems. For example, battleships are equipped with advanced close-in weapon system(CIWS). CIWS is a formidable naval point-defense weapon for detecting and destroying short-range incoming missiles and enemy aircraft which have penetrated the outer defenses. Designed to engage anti-ship cruise missiles and fixed-wing aircraft at short range, it automatically engages functions which are usually performed by separate, independent systems such as search, detection, threat evaluation, acquisition, track, firing, target destruction, kill assessment and cease fire. The fast-reaction and rapid-fire defense systems dramatically drop the survivability of the missiles at the terminal phase. Therefore, it has been a challenge for a single missile to accomplish its missions [1-3].

Cooperative attack with multiple missiles is devised as a countermeasure against the formidable defense systems. For the reason that CIWS is a point-defense, a defense of a single object or a limited area, multiple missiles in a simultaneous attack is much more effective than a consecutive attack in that, other missiles can penetrate the formidable defense system while one missile encounters it. Therefore, in cooperative attack the most two important factors are the impact time and the impact angle. In controlling time for simultaneous attack, synchronization of the time-to-go is essential. [1][2] have proposed impact-time-control guidance(ITCG) law and cooperative proportional navigation(CPN) law that can synchronize the impact time of multiple missiles. To control the impact angle there have been lots of studies and applications such as Impact Angle Control Guidance(IACG) [4-6]. Also some studies have been done to control the impact time and angle together(ITACG) [3].

In a view point of communication network ITCG is a guidance law which can control the predetermined designated impact time which implies the guidance has an open loop structure without communication. CPN is a guidance law which decreases the variance of times-to-go of multiple missiles during the homing which implies multiple missiles attack a single target simultaneously in a closed loop structure with a communication network. As compared with impact time control guidance laws, for impact angle control guidance laws it is not common to construct a closed loop structure with a communication network. In other words, those guidance laws apply the optimal guidance laws derived from a single missile system to individual missile. But with the development of technology in the field of modular data links the creation of a multi-link communication network may be established between missiles and the launch platform. And the future prospect of such ad-hoc networks with many existing guidance laws makes it possible to consider the cooperative strategies of multiple missiles [7].

In this paper we consider optimal guidance laws for cooperative attack of two missiles. Two missiles cooperate to optimize a common objective which couples the terminal vertical velocities of the two missiles as a soft constraint. Firstly, a Linear Quadratic Regulation(LQR) problem is considered in which the cost function is designed to minimize the sum of vertical velocities at the terminal time with the object of enhancing the survivability. Secondly, a Linear Quadratic Tracking(LQT) problem is considered to control the impact angle of the two missiles with a desired angle. Then each guidance input consists of both missiles' state which implies that the missiles should be connected with a communication network. In that point of view this paper assumes the missiles are fully connected with a communication network in other words this is a class of centralized control problem [9-11].

This paper is organized as follows: Section 2 describes the optimal guidance law for a single missile as a preliminarily, based on [8]. In section 3 the methodology in section 2 is extended to two missiles system. Here we propose the two missile impacted angle control guidance(IACG) law and show this approach has the scalability to be extended to multiple missiles. Section 4 shows simulation results of the proposed guidance law. Finally, section 5 presents the conclusion.

2 Optimal Guidance Law for a Single Missile

Modern guidance techniques are mainly based on optimal control theory. Proportional Navigation guidance, namely, PN which is in widespread use obtained from Linear Quadratic Regulator problem where the cost is the missile's control effort with a zero miss distance imported as a terminal condition. And the perfect intercept guidance, namely, Optimal Rendezvous, is obtained with an additional terminal condition to minimize the terminal relative vertical velocity between the missile and the target. The optimal guidance laws are derived by calculus of variation, which gives the necessary condition for optimality.

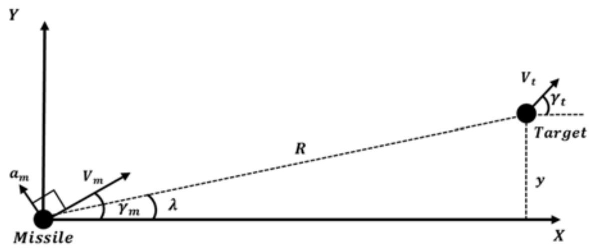

Fig. 1 Single missile-target guidance geometry

We shall make the following assumptions:

1) The missile-target conflict is two-dimensional in the horizontal plane.
2) The speed of the missile m and the target t are constant.
3) The trajectories of m and t can be linearized around their collision course.

The geometry for deriving the missile guidance general solution is depicted in Fig.1. The nominal closing velocity V_c is given by

$$V_c = -\dot{R} = V_m\, cos(\gamma_m) - V_t cos(\gamma_t) \quad (1)$$

Where V_m and V_c are the velocities and γ_m and γ_t are the nominal heading angles of the missile and the target respectively. And then the nominal terminal time is given by

$$t_f = t_0 + \frac{R(t_0)}{V_c} \quad (2)$$

Where *R(t)* is the nominal length of the nominal line-of-sight, *LOS*, at time *t* and t_0 is the initial time. Let y be the relative vertical distance between the missile and the target from the *LOS* namely, $y \equiv y_t - y_m$. Then we can find the expression for the LOS angle and its rate of change, where LOS is σ

$$\sigma(t) = \frac{y(t)}{R(t)} \tag{3}$$

$$\dot{\sigma}(t) = \frac{y}{V_c(t_f - t)^2} + \frac{\dot{y}}{V_c(t_f - t)} \tag{4}$$

Allowing the control input to be the normal acceleration, we can obtain the following state-space equation where the state vector x and control input u are defined as

$$x_1 = y \quad x_2 = \dot{y} \quad u = \ddot{y} \tag{5}$$

$$\dot{x} = Ax + Bu$$
$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

2.1 General Solution

The cost function of the optimal control problem where the object is to minimize the missile's control effort is the following:

$$J = \frac{b}{2}x_1^2(t_f) + \frac{c}{2}x_2^2(t_f) + \frac{1}{2}\int_{t_0}^{t_f} u^2(t)dt \tag{7}$$

Where $b \geq 0$ are the penalty imposed by the miss distance and $c \geq 0$ to minimize the terminal vertical velocity. Then the Hamiltonian is defined by the scalar

$$H = L + \lambda f \tag{8}$$

Where $f = AX + Bu$ and λ is a costate vector. The first-order necessary condition for optimality are the following costate equation

$$\frac{\partial \mathrm{H}}{\partial x} = -\dot{\lambda}^T$$

$$\dot{\lambda}_1 = 0 \quad \dot{\lambda}_2 = -\lambda_1 \quad u = -\lambda_2 \tag{9}$$

The terminal conditions are

$$\lambda_1(t_f) = bx_1(t_f)$$
$$\lambda_2(t_f) = cx_2(t_f) \tag{10}$$

Then the optimal control input is

$$u(t) = -[bx_1(t_f)(t_f - t) + cx_2(t_f)] \tag{11}$$

By substituting the optimal control input to the system (6) and integrating it yields

$$\begin{aligned} x_2(t) &= \frac{1}{2}b(t^2 - t_0^2)x_1(t_f) \\ &\quad - b(t - t_0)x_1(t_f)t_f \\ &\quad - c(t - t_0)x_2(t_f) + x_2(t_0) \\ x_1(t) &= \left[\frac{1}{2}b\left(\frac{t^3}{3} - \frac{t_0^3}{3}\right) - \frac{1}{2}bt_0^2(t - t_0)\right]x_1(t_f) \\ &\quad -\frac{1}{2}b(t - t_0)^2 x_1(t_f)t_f - \frac{1}{2}c(t - t_0)^2 x_2(t_f) \\ &\quad + x_2(t_0)(t - t_0) + x_1(t_0) \end{aligned} \tag{12}$$

Evaluating them at $t = t_f$ we can get the transition matrix of x from t_0 to t_f

$$x(t_f) = \Phi(t_f, t_0)x(t_0) \tag{13}$$

Then we can solve the resulting linear algebraic equations for the unknown terminal value $x(t_f)$ and thus arrive at an expression for $u(t)$ by replacing the arbitrary t_0 by t:

$$\begin{aligned} u(t) &= -[bx_1(t_f)\tau + cx_2(t_f)] \\ &= -[\tau \;\; 1]\begin{bmatrix} b & 0 \\ 0 & c \end{bmatrix}\Phi(t_f, t)x(t) \\ &= -[K_1x_1(t) + K_2x_2(t)] \end{aligned} \tag{14}$$

Where time-to-go is defined as $\tau = t_f - t$ and the optimal feedback gains are

$$\begin{aligned} K_1 &= \frac{\mathrm{b}\tau\left(\frac{1}{2}\tau c + 1\right)}{\tau^4\frac{cb}{12} + \tau c + \tau^3\frac{b}{3} + 1} \\ K_2 &= \frac{\frac{1}{3}\tau^3 cb + \tau^2 b + c}{\tau^4\frac{cb}{12} + \tau c + \tau^3\frac{b}{3} + 1} \end{aligned} \tag{15}$$

Notice that the problem can be solved by employing the associated Riccati equation:

$$-\dot{P} = PA + A^TP - PBB^TP \tag{16}$$

With the terminal conditions

$$P(t_f) = \begin{bmatrix} b & 0 \\ 0 & c \end{bmatrix}$$

Then the optimal strategy takes the form

$$u(t) = -R^{-1}B^T Px = -[K_1 \ \ K_2]x(t) \quad (17)$$

2.2 Proportional Navigation Guidance(PNG)

For Proportional Navigation Guidance(PNG) the terminal condition is the zero miss distance which can be obtained by taking $b \to \infty$ and $c = 0$ which is the velocity weighting factor. Then from Eqn. 15 the optimal feedback gains are

$$K_1 = \frac{3}{\tau^2} \quad K_2 = \frac{3}{\tau} \qquad (18)$$

Then the optimal control input is as follow, where *N=3* is the optimal navigation constant

$$\begin{aligned} u(t) &= -\frac{N(y+\dot{y}\tau)}{\tau^2} \\ &= -NV_c\dot{\sigma} \end{aligned} \qquad (19)$$

2.3 Optimal Rendezvous(OR)

For perfect intercept guidance, namely, Optimal Rendezvous(OR), the terminal conditions are zero miss distance and minimum vertical velocity, which can be obtained by taking b, $c \to \infty$ in Eqn.15. Then the optimal feedback gains are

$$K_1 = \frac{6}{\tau^2} \quad K_2 = \frac{4}{\tau} \qquad (20)$$

That gives the optimal control input as

$$\begin{aligned} u(t) &= -\frac{6y+4\dot{y}\tau}{\tau^2} \\ u(t) &= -V_c\left[4\dot{\sigma} + \frac{2\sigma}{\tau}\right] \end{aligned} \qquad (21)$$

3 Optimal Guidance Law for Two Missiles

In this section, the optimal guidance solution for a single missile described in section 2 is extended to two missiles.

The geometry for deriving two missiles guidance general solution is depicted in Fig. 2. As the two missiles are dynamically decoupled the assumptions and geometric parameters defined at section 2 are still valid to each missile identically.

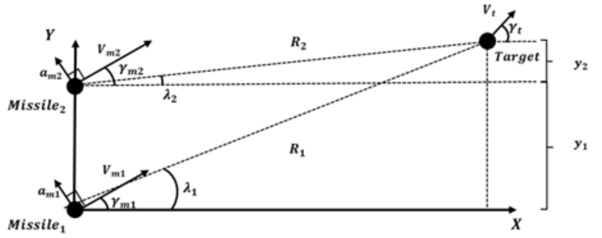

Fig. 2 Two missiles-single target guidance geometry

One additional assumption is that the closing velocity and the initial lengths of two missiles are well defined, for instance identical, so that we can handle the two missiles in an equivalent time horizon given in Eqn. 2. Then by allowing each control input to be the normal acceleration for each missile, we can obtain the extended state-space equation

$$\begin{aligned} x_1 = y_1 \ \ x_2 = \dot{y}_1 \ \ x_3 = y_2 \ \ x_4 = \dot{y}_2 \\ u_1 = \ddot{y}_1 \quad u_2 = \ddot{y}_2 \end{aligned} \qquad (22)$$

$$\dot{x} = Ax + Bu$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad (23)$$

3.1 Linear Quadratic Regulation Problem

In LQR problem the object is to minimize the sum of vertical velocities of two missiles in the terminal time to enhance the survivability at the terminal phase. For that the cost function is designed as

$$\begin{aligned} J_{LQR} = &\frac{b}{2}\left(x_1^2(t_f) + x_3^2(t_f)\right) \\ &+ \frac{c}{2}\left(x_2(t_f) + x_4(t_f)\right)^2 \\ &+ \frac{1}{2}\int_{t_0}^{t_f} u_1^2(t) + u_2^2(t)dt \end{aligned} \qquad (24)$$

Notice that at the terminal time, the velocities of the two missiles are coupled and they have to cooperate to minimize the cost function. By following the same methodology of Eqn. (8)-(13) we can obtain the transition matrix of x from t_0 to t_f

$$\Phi(t_f, t_0) = (I - A')^{-1}B' \qquad (25)$$

$$A' = \begin{bmatrix} -\frac{1}{3}b\tau^3 & -\frac{1}{2}c\tau^2 & 0 & -\frac{1}{2}c\tau^2 \\ -\frac{1}{2}b\tau^2 & -c\tau & 0 & -c\tau \\ 0 & -\frac{1}{2}c\tau^2 & -\frac{1}{3}b\tau^3 & -\frac{1}{2}c\tau^2 \\ 0 & -c\tau & -\frac{1}{2}b\tau^2 & -c\tau \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then we can arrive at an expression of $u(t)$ by replacing the arbitrary t_0 by t

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} \tau & 1 & 0 & 0 \\ 0 & 0 & \tau & 1 \end{bmatrix} \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & c & 0 & c \\ 0 & 0 & b & 0 \\ 0 & c & 0 & c \end{bmatrix} \Phi(t_f.t)x(t) \qquad (26)$$
$$= -Kx(t)$$

Where the optimal feedback matrix $K \in R^{2\times4}$ can be obtained by employing the associated Riccati equation, Eqn. 16, with the terminal condition

$$P(t_f) = \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & c & 0 & c \\ 0 & 0 & b & 0 \\ 0 & c & 0 & c \end{bmatrix}$$

3.1.1 Two missiles OR

In PNG law the weighting factors of the soft constraint in cost function are $b \to \infty,\ c = 0$ which does not handle the terminal velocity constraint. For that reason, two missiles are not cooperative and follow the PN guidance independently.

In OR law the weighting factors are $b,\ c \to \infty$ for the purpose of obtaining zero miss distance and zero sum of vertical velocities respectively. Such approach gives the feedback gain

$$\begin{aligned} K_{11} &= \frac{9}{2\tau^2} & K_{12} &= \frac{7}{2\tau} & K_{13} &= \frac{3}{2\tau^2} & K_{14} &= \frac{1}{2\tau} \\ K_{21} &= \frac{3}{2\tau^2} & K_{22} &= \frac{1}{2\tau} & K_{23} &= \frac{9}{2\tau^2} & K_{24} &= \frac{7}{2\tau} \end{aligned} \qquad (27)$$

And then the optimal control strategies are

$$\begin{aligned} u_1 &= -\left[V_{c1}\left(\frac{7}{2}\dot{\sigma}_1 + \frac{\sigma_1}{\tau}\right) + V_{c2}\left(\frac{1}{2}\dot{\sigma}_2 + \frac{\sigma_2}{\tau}\right)\right] \\ u_2 &= -\left[V_{c2}\left(\frac{7}{2}\dot{\sigma}_2 + \frac{\sigma_2}{\tau}\right) + V_{c1}\left(\frac{1}{2}\dot{\sigma}_1 + \frac{\sigma_1}{\tau}\right)\right] \end{aligned} \qquad (28)$$

Notice that the velocity constraint $x_2(t_f) + x_4(t_f) = 0$ provides a symmetric impact for the two missiles about the x-axis and the impact angle of each missile depends on the property of that. And the worst case is the singularity that both missile's impact angle can be 0.

3.2 Linear Quadratic Tracking Problem

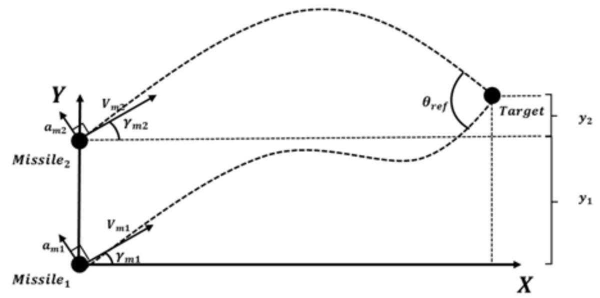

Fig. 3 Two missiles-single target guidance geometry for impact angle control

In LQT problem the objective is to control the impact angle of two missiles for the purpose of not only enhancing the survivability but also maximizing the destructibility. For that the geometry is given as Fig.3. As the trajectory of each missile can be linearized around their collision course, the relation of the heading angle(γ_m), velocity(V_m) and the state $\dot{y}$ are as follow:

$$\gamma_m = sin^{-1}\left(\frac{\dot{y}}{V_m}\right) \cong \frac{\dot{y}}{V_m} \qquad (29)$$

Assume the velocity of each missile is identical to V_m then we can obtain the relation of the desired impact angle(θ_{ref}), V_m and state $\dot{y}$ as

$$\theta_{ref} = \gamma_{m1} - \gamma_{m2} \cong \frac{\dot{y}_1 - \dot{y}_2}{V_m} \qquad (30)$$

Then by defining a new parameter $r = V_m\theta_{ref}$ we can design the cost function to control the impact angle while minimizing the control efforts as

$$\begin{aligned} J_{LQT} = &\frac{b}{2}\left(x_1^2(t_f) + x_3^2(t_f)\right) \\ &+ \frac{c}{2}\left(x_2(t_f) - x_4(t_f) - r\right)^2 \\ &+ \frac{1}{2}\int_{t_0}^{t_f} u_1^2(t) + u_2^2(t)dt \end{aligned} \qquad (31)$$

In LQT problem it is known that the optimal strategy consists of the feedback and feedforward terms. Therefore the transition of state x can be expressed

$$x(t_f) = \Phi(t_f, t)x(t) + \Psi r \tag{32}$$

Where the transition matrix $\Phi(t_f, t)$ and the forced response Ψ are given as

$$\Phi(t_f, t_0) = (I - A')^{-1}B'$$
$$\Psi = (I - A')^{-1}S$$

$$A' = \begin{bmatrix} -\frac{1}{3}b\tau^3 & -\frac{1}{2}c\tau^2 & 0 & \frac{1}{2}c\tau^2 \\ -\frac{1}{2}b\tau^2 & -c\tau & 0 & c\tau \\ 0 & \frac{1}{2}c\tau^2 & -\frac{1}{3}b\tau^3 & -\frac{1}{2}c\tau^2 \\ 0 & c\tau & -\frac{1}{2}b\tau^2 & -c\tau \end{bmatrix} \tag{33}$$

$$B' = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} \frac{1}{2}c\tau^2 \\ c\tau \\ -\frac{1}{2}c\tau^2 \\ -c\tau \end{bmatrix}$$

Then we can arrive at an expression of $u(t)$ which is given as a linear combination of x and r

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -[bx_1(t_f)\tau + c(x_2(t_f) - x_4(t_f) - r)] \\ -[bx_3(t_f)\tau + c(x_2(t_f) - x_4(t_f) - r)] \end{bmatrix}$$

Rewriting it in terms of Φ and Ψ it gives

$$\begin{aligned} &\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &= -\begin{bmatrix} \tau & 1 & 0 & 0 \\ 0 & 0 & \tau & 1 \end{bmatrix} \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & c & 0 & -c \\ 0 & 0 & b & 0 \\ 0 & -c & 0 & c \end{bmatrix} \Phi(t_f, t)x(t) \\ &+ \begin{bmatrix} 0 & c & 0 & -c \\ 0 & -c & 0 & c \end{bmatrix} \Psi r \\ &= -[Kx(t) + Vr] \end{aligned} \tag{34}$$

Where the optimal feedback gain is $K \in R^{2\times4}$ and the feedforward gain is $V \in R^{2\times1}$.

3.2.1 Impact angle control guidance(IACG)

For two missile IACG law we take the weighting factors $b,\ c \to \infty$ to obtain both the zero miss distance and the desired impact angle. Then the feedback and forward gains are

$$\begin{aligned} &K_{11} = \frac{9}{2\tau^2} \quad K_{12} = \frac{7}{2\tau} \quad K_{13} = -\frac{3}{2\tau^2} \quad K_{14} = -\frac{1}{2\tau} \\ &K_{21} = -\frac{3}{2\tau^2} \quad K_{22} = -\frac{1}{2\tau} \quad K_{23} = \frac{9}{2\tau^2} \quad K_{24} = \frac{7}{2\tau} \end{aligned} \tag{35}$$

$$V_1 = \frac{1}{\tau} \quad V_2 = -\frac{1}{\tau}$$

And then the optimal control strategies are

$$\begin{aligned} u_1 &= -\left[V_{c1}\left(\frac{7}{2}\dot{\sigma_1} + \frac{\sigma_1}{\tau}\right) - V_{c2}\left(\frac{1}{2}\dot{\sigma_2} + \frac{\sigma_2}{\tau}\right) + \frac{1}{\tau}V_m\theta_{ref}\right] \\ u_2 &= -\left[V_{c2}\left(\frac{7}{2}\dot{\sigma_2} + \frac{\sigma_2}{\tau}\right) - V_{c1}\left(\frac{1}{2}\dot{\sigma_1} + \frac{\sigma_1}{\tau}\right) - \frac{1}{\tau}V_m\theta_{ref}\right] \end{aligned} \tag{36}$$

Notice that if the closing velocities, the line-of-sights and the rates of the two missiles are identical then the optimal feedback gains follows that of the PN guidance law.

Furthermore, one advantage of this approach is the scalability that it can be easily extended to multiple missiles. Consider there are n missiles then the state space equation is

$$\begin{aligned} \dot{x} &= A_n x + B_n u \\ A_n &= diag(A, \dots, A) \quad B_n = (B_i) \end{aligned} \tag{37}$$

here $A_n \in R^{2n\times n}$ where A is defined in Eqn. 6 and $B_i \in R^{2n}$ is a column vector with all elements are zero except the $2i^{th}$ element is 1 for $i = 1,2,\dots,n$. And the state vector x and control input u are well defined with the same approach with Eqn. 22. To control the impact angles of n missiles it is necessary to constrain i^{th} missile's vertical velocity about that of $(i-1)^{th}$ and $(i+1)^{th}$ for $i = 2,\dots,n-1$. Notice that the two missile case is the special case where the missiles are at $i = 1$ and $i = n$ respectively. For that concept the cost function can be designed as [9]

$$J = \frac{b}{2}\left(\sum_{i=1}^{n} x_{2i-1}^2\right) + \frac{c}{2}\left(\sum_{i=1}^{n-1}(x_{2i} - x_{2i+2} - r_i)^2\right) + \frac{1}{2}\int_{t_0}^{t_f}\sum_{i=1}^{n} u_i^2\,dt \tag{38}$$

The result can be obtained by following the same methodology but depends on the number of missiles, n.

4 Simulation Results

In this section the performance of the IACG law is investigated. Nonlinear simulations are performed with a simple point-mass model is applied. That is

$$\dot{x}_m = V_m \sin(\gamma_m)$$
$$\dot{y}_m = V_m \cos(\gamma_m)$$
$$\dot{\gamma}_m = u_m / V_m$$

It has been used and proven useful in a wide range of literature from classical optimal guidance problems [9].

The simulations are implemented for cooperative salvo attack scenarios. For two missile IACG the cases of launched from a single platform and different platform are implemented with some interpretation on the effect with the initial heading angles.

4.1 Two missile IACG

Table 1 Scenario parameters

	$\gamma_m(t_0)$ [°]	$V_m(t_0)$ [m/s]	Launch position [x, y]
Case 1	(0, 0)	300	(0, 0), (0,0)
Case 2	(30, 15)	300	(0, 0), (0,0)
Case 3	(0, 0)	300	(0, 2000), (0,-2000)

Three cases of initial conditions are considered which are listed in Table 1 and the desired impact angles $[60°, 90°, 120°, 150°]$, $[30°, 60°, 90°]$, $[90°, 120°, 150°, 180°]$ for each case respectively.

Fig.4, 5 show the trajectories of two missiles of IACG law for case 1 in a full scale and concentrated on the terminal phase respectively. Fig. 6 shows the guidance commands of the proposed guidance law, and Table 1 shows the final impact angles of the missiles. The guidance commands are linear before the impact and increases its magnitude at the terminal phase so that it can yield the desired impact angle. The results show that two missiles impact the target properly with the desired impact angle.

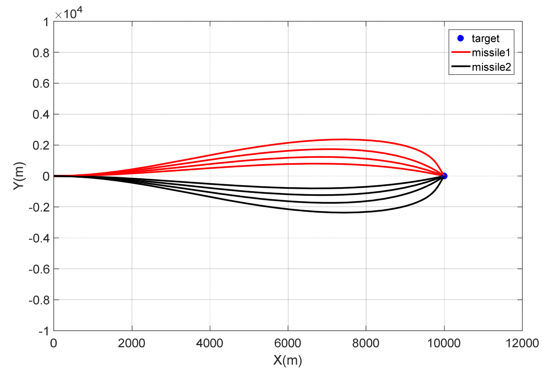

Fig. 4 Trajectories for case 1 in a full scale

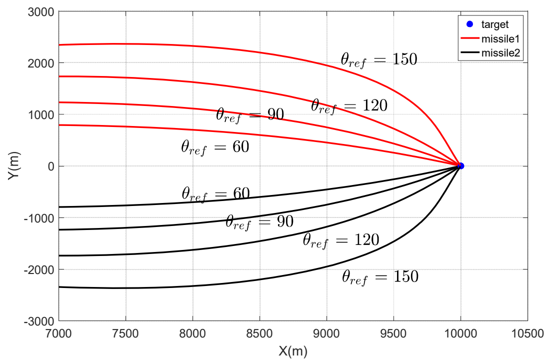

Fig. 5 Trajectories for case 1 at terminal phase

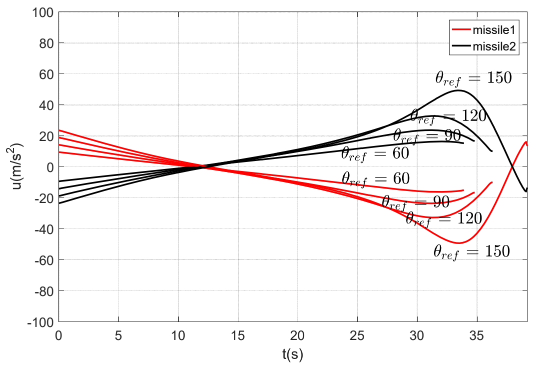

Fig. 6 Guidance command histories for case 1

Table 2 Impact angles of missiles in case 1

θ_{ref}[°]	60	90	120	150
$\gamma_1(t_f)$ [°]	-30	-45	-60	-75
$\gamma_2(t_f)$ [°]	30	45	60	75

Fig. 7, 8 show the trajectories of case 2. And Fig. 9 and Table 2 show the guidance commands and the imact angles for case 2. It is obvious that the error of impact angle increases for larger desired impact angle along with larger guidance command which can result in a severe miss distance if there is a command limit.

Comparing case 1and 2 the trajectories of case 1 are symmetric to *x*-axis that implies the two missiles are solvable in a identical time horizon which is mentioned at Section 3 while in case 2 the time horizons are different for each missile. For that more practical appraoch should be considered to handle such problems.

Fig. 10, 11 show the trajectories of case3. Fig. 12 shows the guidance command and the impact angles are listed in Table 3.

The results show that launching from different platforms is more effective in the view point of that more severe desired impact angles are obtainable properly with less guidance commands which is intuitive. In this case for 180° attack two missiles make a big turn to secure a feasible path to impact.

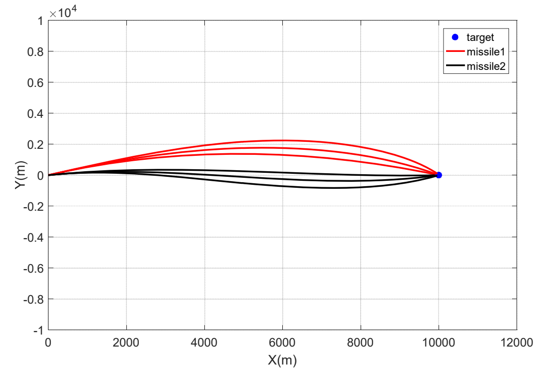

Fig. 7 Trajectories for case 2 in a full scale

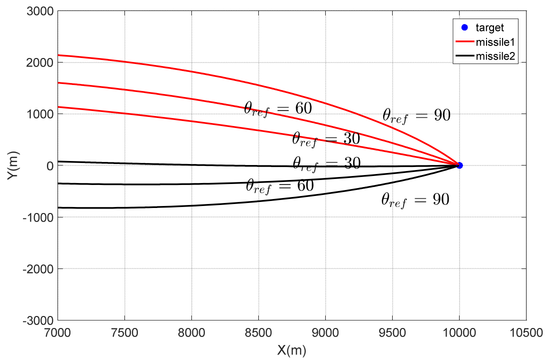

Fig. 8 Trajectories for case 2 at terminal phase

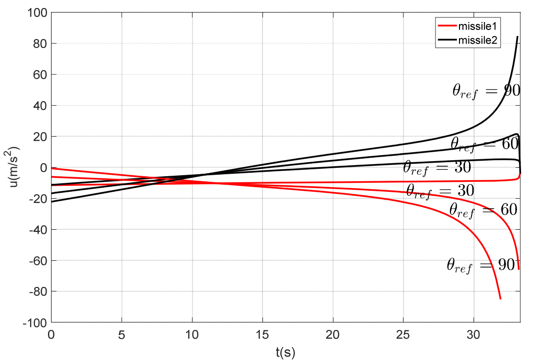

Fig. 9 Guidance command histories for case 2

Table 3 Impact angles of missiles in case 2

θ_{ref} [°]	30	60	90
$\gamma_1(t_f)$[°]	-27.5	-44.75	-65.3
$\gamma_2(t_f)$[°]	2.75	19.86	40

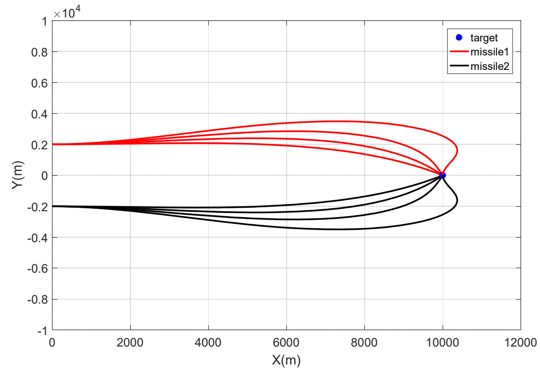

Fig. 10 Trajectories for case 3 in a full scale

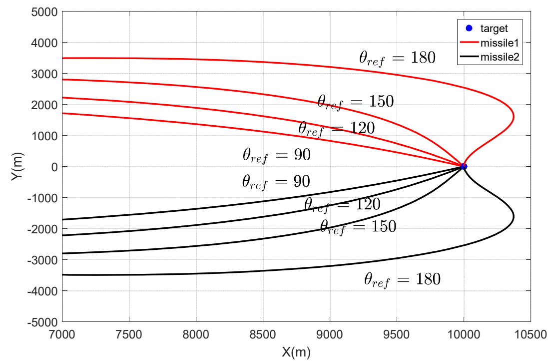

Fig. 11 Trajectories for case 2 at terminal phase

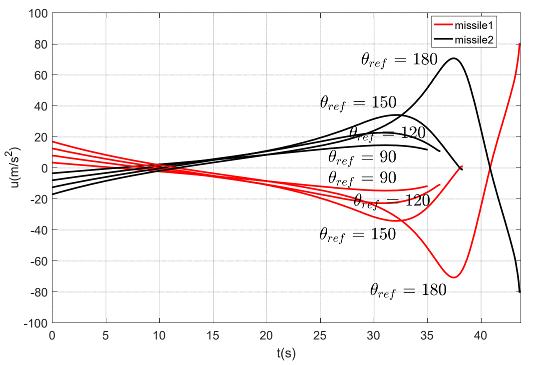

Fig. 12 Guidance command histories for case 3

Table 4 Impact angles of missiles in case 3

θ_{ref}[°]	90	120	150	180
$\gamma_1(t_f)$[°]	-45	-60	-75	-90
$\gamma_2(t_f)$[°]	45	60	75	90

5 Conclusion

In this paper, a homing guidance problem for two-missile cooperative mission is considered. We have proposed an essential approach to achieve the coupled impact angle constraints of multiple missiles. The single-missile optimal guidance law was extended to two-missile IACG by imposing a cost function with coupled terminal velocity constraints. We further show that the approach can be easily extended to multiple missile cases. The performance was investigated via nonlinear simulation cases with various impact angle constraints and initial conditions. Under mild assumptions the proposed law provides satisfactory performance for salvo missions under coupled impact angle constraints. In future studies, several practical issues, such as the time horizon difference and the communication limitation, should be addressed in order to apply the proposed guidance law in more realistic situations.

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