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TRAJECTORY DESIGN WITH PATH PREDICTION FOR TERRAIN FOLLOWING

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Abstract

This paper deals with the trajectory designing for terrain following by using geometric methods. Considering the current flight path angle and load factor of the cruise missile, a future path can be predicted. Also with known future terrain data, this predicted path can determine whether a maneuvering command should be ordered or not. The terrain falls into two parts: concave and convex. For the convex case, predicted path method is used to find the starting and ending points of pull-up maneuver to maintain survivability. For the concave case, predicted path method is also used, but to find the starting and ending points of pull-down maneuver to maintain the minimum distance to the ground possible. By finding maneuvering starting and ending points, the whole trajectory can be designed.

1 Introduction

The development of the radar system in the last few decades created a new task for aircrafts to reach their target while avoiding radar detection. This resulted in an increase on the importance of terrain following guidance for aircrafts and missiles. The main objective of the terrain following problem is to design a path as close as possible to ground while flying safely in a vertical plane over a given terrain.

The techniques used to design such trajectories are divided into two ways: (1) design based on optimal control theory, (2) design based on geometric methods.

For the former techniques, some cost functions are set to be minimized. Menon [1] used flight time and terrain masking as performance indices, and Halpern [2] included tracking errors into performance index. The cost functions had limitations that Menon's idea needed the first and second derivatives of the terrain and Halpern took positive and negative errors as same. However Halpern used the terrain profile to get future information which led to improved performance.

The latter techniques are usually used by the idea of piecewise segments such as cubic splines and parabolic flight segments. Funk [3] used a cubic spline function to formulate the geometric relations of height, slope and curvature of terrain to determine an optimal terrain following flight under the requirement of staying as close to all the terrain points as possible. Jackson and Crouch [4] used cubic splines to interpolate specified waypoints. Asseo [5] used parabolic flight segments in trajectory designing for clearing the critical terrain points.

This paper proposes a geometric method considering load factor to design trajectory for terrain following. A minimum clearance curve is set to make sure the cruise missile does not crash into the ground. The whole terrain information is divided into two groups: concave and convex. Each group has a different objective. For the convex case, a pull-up maneuver position is going to be found to sustain survivability. Whereas for the concave case, a pull-down maneuver position is going to be found so as to prevent the aircraft from going over the minimum clearance curve of a peak point. If the cruise missile does not keep the minimum distance at the peak point, it takes more time to return to the reference terrain after the pull-down maneuver. The whole process will consider the maximum climb angle of the cruise missile in order to avoid the meaningless path creation that the cruise missile cannot follow. The terrain profiles will be used to obtain future information to know where to pull-up or pull-down maneuver should start.

2 Trajectory Design Algorithm

This part introduces how to plan a path that cruise missile maintains the minimum height while guaranteeing its survival.

It is assumed that the cruise missile is moving with a constant velocity at any time, allowing future path to be predicted at any point. The cruise missile is moving in a circular path with the maximum load factor and with a constant velocity during the maneuver. The circular path will be calculated by pull-up/pull-down radius R_U / R_D with maximum load factor $n_{\max}$. Note that the maximum load factor will be applied at lowest/highest point during pull-up/pull-down maneuver.

$$R_U = \frac{V^2}{g(n_{\max} - 1)} \quad (1)$$

$$R_D = \frac{V^2}{g(n_{\max} + 1)} \quad (2)$$

2.1 Clearance Curve

A minimum clearance curve is indispensable for setting a safety margin to the cruise missile. It is set to secure safety distance *d* from the terrain. For the terrain function $f(x)$, each terrain point (x, y) goes to newly created terrain point (x_{new}, y_{new}).

$$x_{new} = x + d\cos[\tan^{-1}(\frac{df(x)}{dx}) + \frac{\pi}{2}] \quad (3)$$

$$y_{new} = y + d\sin[\tan^{-1}(\frac{df(x)}{dx}) + \frac{\pi}{2}] \quad (4)$$

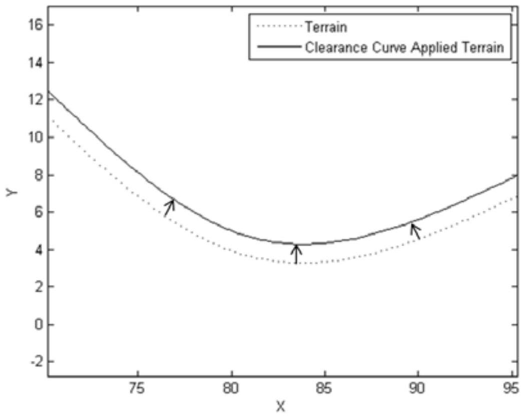

Fig. 1 Clearance Curve Applied Terrain

2.2 Pull-Down Maneuver Considered Path

When the cruise missile is in a concave terrain, a pull-down maneuver is needed to make the missile's altitude low. If there is a delay in the cruise missile begins the pull-down maneuver, it goes above the minimum clearance curve which fails to meet the main purpose of the terrain following.

The main purpose of the pull-down maneuver on a concave terrain is to not go over the peak. Therefore in this case, it is focused on finding the local maximum peaks and comparing slopes around the peak with the pull-down radius. If the slope is too steep for the cruise missile to follow, the missile will follow the pull-down maneuver path.

For the maximum peak of the clearance curve applied terrain (x_{new}, y_{new}), pull-down maneuvering path points set around this point $\{(x_D, y_D)\}$ will be obtained as:

$$\{x_D\} = x_{new} + R_D \cos\theta \quad (5)$$

$$\{y_D\} = y_{new} - R_D(1 - \sin\theta) \quad (6)$$

Where $\theta = [\frac{\pi}{2} - \theta_{\max}, \frac{\pi}{2} + \theta_{\max}]$. Since the missile has its maximum climb angle $\theta_{\max}$, the circular path out of this bound is physically meaningless. Also note that the highest point is the same as the maximum peak point of the clearance curve applied terrain.

By calculating this circular path to every local maximum point gives the option to choose between the pull-down path and the clearance curve applied terrain. Taking higher y value at the same x point gives the pull-down maneuver considered path.

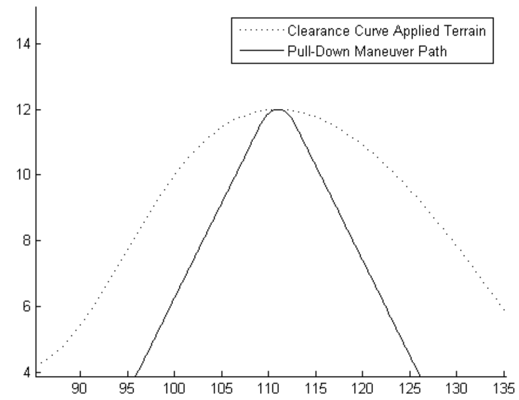

Fig. 2 Pull-Down Maneuver Path under the Clearance Curve Applied Terrain

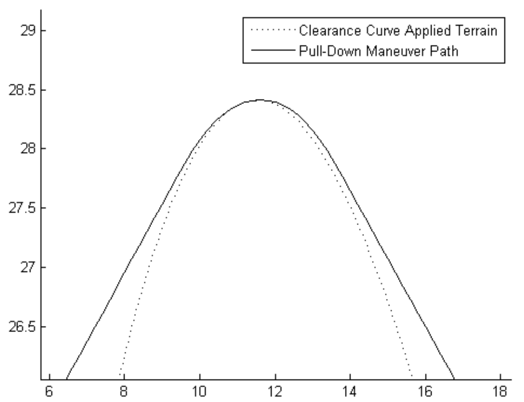

Fig. 3 Pull-Down Maneuver Path above the Clearance Curve Applied Terrain

Fig. 2 shows that clearance curve applied terrain has bigger radius than R_D so that the maximum pull-up maneuver does not need to be applied at this time. Whereas, Fig. 3 shows that without the pull-down maneuver, the cruise missile cannot follow the terrain because terrain's radius is smaller than R_D. Thus for this time, the cruise missile has to start pull-down maneuver in order to keep minimum height at the peak point.

2.3 Maximum Climb Angle Considered Path

The clearance curve applied terrain needs to be modified since the cruise missile's maximum climb angle is bounded to $\theta_{\max}$. If the clearance curve applied terrain has an interval which has inclination angle that is larger than $\theta_{\max}$, the maneuver needs to be started earlier in order to maintain survivability. Thus, from the circular path obtained from 2.2, the line with slope of $\tan\theta_{\max}$ (or $\tan(-\theta_{\max})$ if the terrain slope is negative) should be attached at both ends in order to find the larger inclination angle the terrain might have.

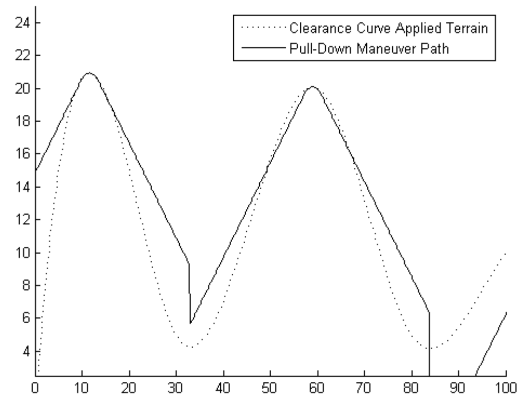

Fig. 4 Pull-Down Maneuver Path Extended

From Fig. 4, interval x = [10, 30] shows that the terrain is steeper than the missile can fly. Thus, the terrain points below the maximum climb angle line should be replaced by the newly created line as described in Fig. 5.

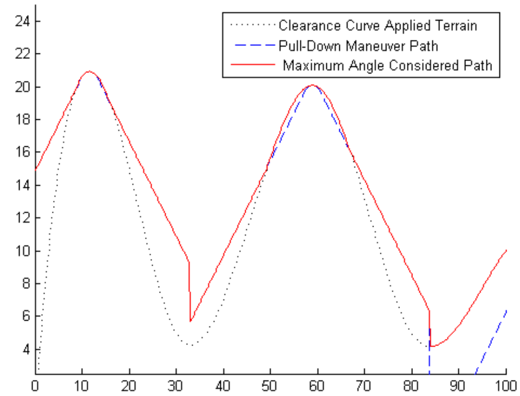

Fig. 5 Intermediate Output of Maximum Angle Considered Path

However after $x = 50$, the missile is impossible to follow the terrain as described in Fig. 6. In the interval $x = [40, 60]$, this pull-down maneuver path line cannot catch the steeper terrain points above it. Even though the missile can follow the terrain points around the peak, it is impossible to go to the start point to maneuver. For this case, the extended line should be attached to the peak terrain points not to the pull-down maneuver path. Finding tangent line to the terrain gives the final maximum climb angle considered path as described in Fig. 7.

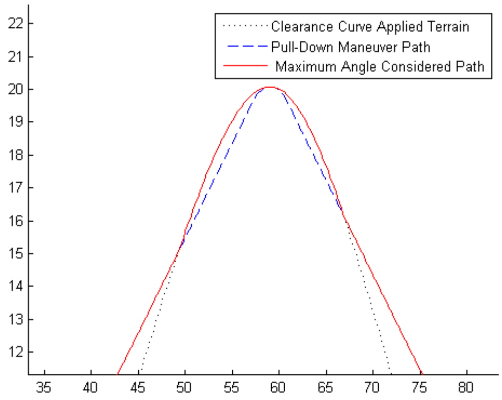

Fig. 6 Problem of Intermediate Output of Maximum Angle Considered Path

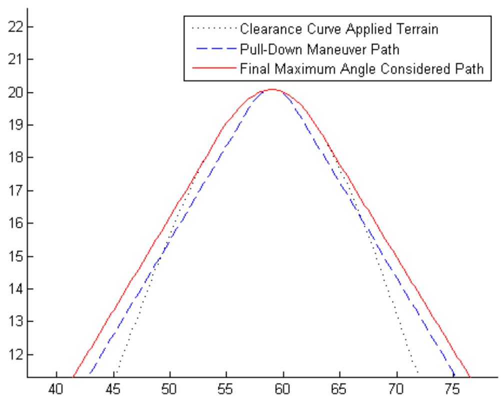

Fig. 7 Final Maximum Angle Considered Path

2.4 Pull-Up Maneuver Considered Path

So far, the path has modified with the consideration of (1) pull-down maneuver, (2) maximum climb angle. Each modification considered the cruise missile's feasibility of maneuvering while maintaining minimum height from the terrain. The newly created minimum height path (intermediate output) is depicted below.

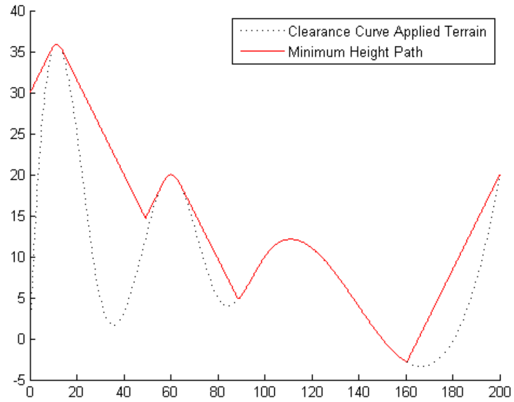

Fig. 8 Minimum Height Path (Intermediate Output)

Now, pull-up maneuver needs to be considered at the local minimum points. When the cruise missile is in the convex terrain, pull up maneuver is needed to make the missile's altitude high. The missile does not guarantee a rapid climb since it has its own positive maximum load factor. Therefore, to keep the missile safe when a sudden change of the terrain is ahead, the pull-up maneuver should occur earlier. The future terrain information is used to find the pull up maneuver start point by checking whether the predicted path meets the future terrain's minimum height path or not.

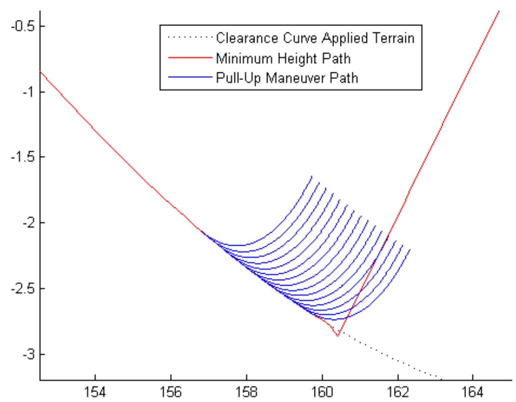

Fig. 9 Pull-Up Maneuver Path

When the predicted path does not meet the future terrain's minimum height path, the cruise missile will continue to follow the current minimum height path. However, when the predicted path does meet the future terrain minimum height path, a pull up maneuver is commanded. The predicted path is calculated at each point of minimum height path curve.

For the minimum height path function $f_{MHP}(x)$, each minimum height path point (x_{MHP}, y_{MHP}) gives predicted path points set $\{(x_U, y_U)\}$ by below equations.

$$\begin{aligned}\{x_U\} &= x_{MHP} + R_U \cos\theta \\ &+ R_U \cos[\tan^{-1}(\frac{df_{MHP}(x_{MHP})}{dx}) + \frac{\pi}{2}]\end{aligned} \qquad (7)$$

$$\begin{aligned}\{y_U\} &= y_{MHP} + R_U \sin\theta \\ &+ R_U \sin[\tan^{-1}(\frac{df_{MHP}(x_{MHP})}{dx}) + \frac{\pi}{2}]\end{aligned} \qquad (8)$$

Since path prediction starts from the current point and $\frac{df_{MHP}(x_{MHP})}{dx} < 0$, θ domain is given as

$$\theta = [\frac{3\pi}{2} + \tan^{-1}(\frac{df_{MHP}(x_{MHP})}{dx}),\ \frac{3\pi}{2} + \theta_{\max}] \qquad (9)$$

To remain as close to the ground as possible, the missile has to follow the minimum height path as long as possible. The main purpose of this step is to find a point of minimum height path that predicted pull-up maneuver path is tangent to future minimum height path.

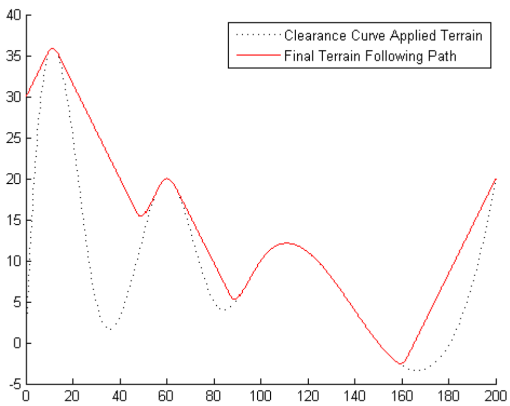

Fig. 10 Final Terrain Following Path

2.5 Algorithm Summary

Step1: For the terrain data obtained, give minimum clearance level to make clearance curve applied terrain.

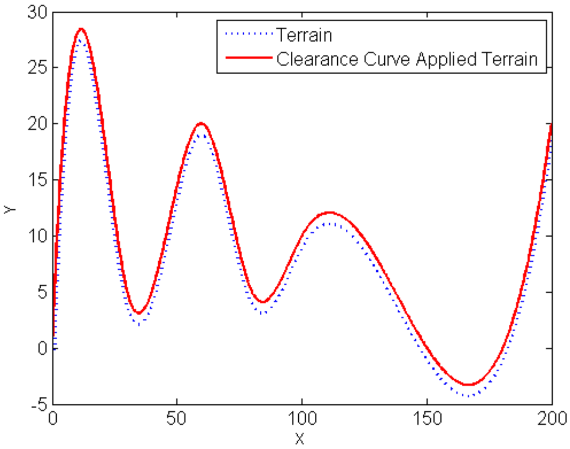

Fig. 11 Step1 Result

Step2: For every local maximum peak point, calculate predicted pull-down maneuver path that the highest y value is the peak point. x domain is bounded to where the absolute value of slope is less than $\theta_{\max}$.

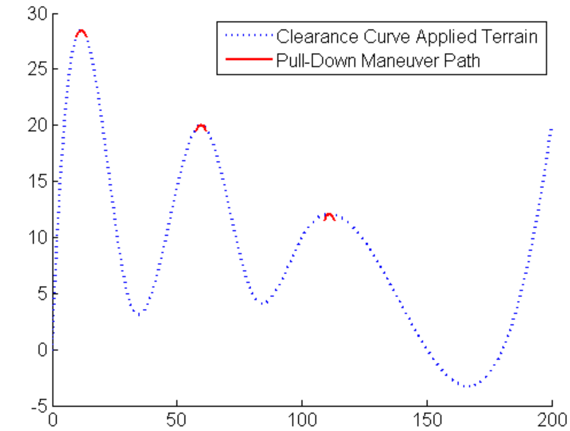

Fig. 12 Step2 Result

Step3: Even though the cruise missile do not need to do pull-down maneuver at the peak point, slope might change rapidly so that it goes over $\tan\theta_{\max}$. Thus, reference terrain has to be replaced with achievable inclination angle. A line can be extended from the pull-down maneuver end point by taking the angle of

$\tan\theta_{\max}$ and $\tan(-\theta_{\max})$ for positive and negative slope respectively.

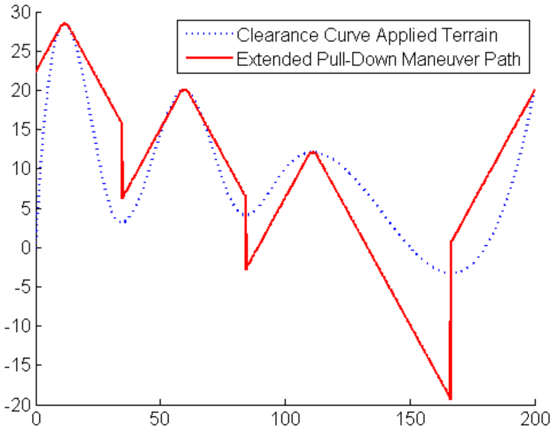

Fig. 13 Step3 Result

Step4: Compare the y value of clearance curve applied terrain and the y value obtained in step2 and 3. Take the larger value for intermediate output of maximum angle considered path.

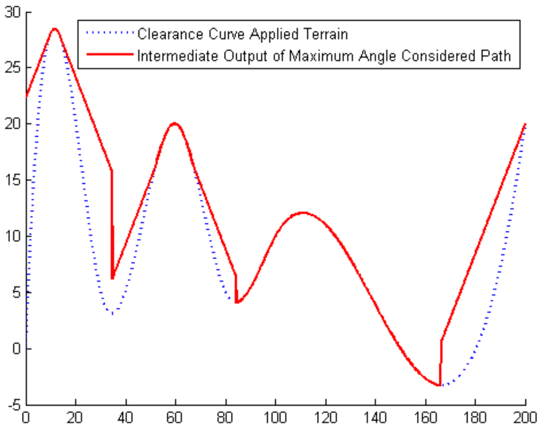

Fig. 14 Step4 Result

Step5: If the intermediate output of maximum angle considered path has slope that is larger than $\tan\theta_{\max}$ (described in Fig.6), find the point that is tangent to the intermediate output of maximum angle considered path which the inclination angle is closest to $\theta_{\max}$ but less than $\theta_{\max}$. From that point, extend the line so the line will be upper than the line from step4. This gives final maximum angle considered path (minimum height path).

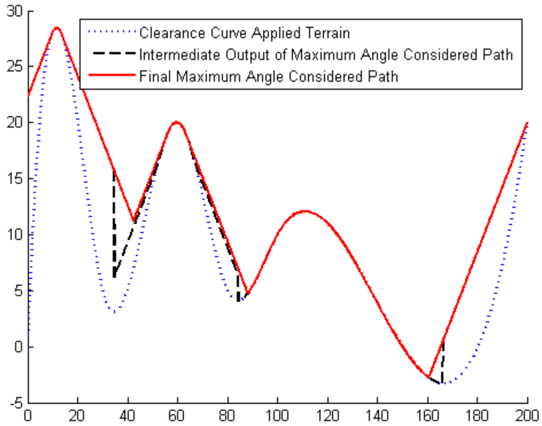

Fig. 15 Step5 Result

Step6: For every minimum height path point, calculate predicted pull-up maneuver path. It is calculated with the assumption that pull-up maneuver is going to be commanded at the current point. If the predicted path is tangent to the minimum height path, it should follow predicted pull-up maneuver path. Therefore, the final terrain following path is created.

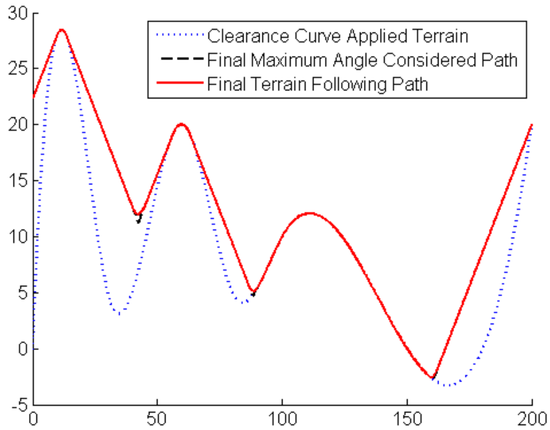

Fig. 16 Step6 Result

3 Simulation

3.1 Optimization via Pseudo-Spectral Method

Until now, the terrain following path derived from the geometric method has been discussed. In this part, the terrain following path is calculated from the optimal theory and compared with the former result.

To obtain the optimal terrain following path, the optimal control solver using Pseudo-Spectral method, 'GPOPS' is utilized. The initial conditions, terminal conditions and constraints are set as the same as geometric method for comparison.

	values
Initial conditions	$X_0 = 0$ $Z_0 = \text{free}$ $V_0 = 1$ $\gamma_0 = \text{free}$
Terminal conditions	$X_f = 200$ $Z_f = \text{free}$ $V_f = 1$ $\gamma_f = \text{free}$
Constraints	$\lvert\gamma\rvert \le \pi/6$ $\lvert a_c\rvert \le 1/4$ $\lvert z - h_t - h_c\rvert \ge 0$

Table. 1 Optimal Control Problem Conditions

The Table 1 specifies the simulation condition as an input to the GPOPS. Where γ denotes climb angle (flight path angle) and a_c is the acceleration command which can be written as

$$a_c = \frac{V^2}{R} \tag{10}$$

The velocity is normalized as 1. For simplicity, it is assumed that pull-up radius and pull-down radius are the same ($R_U = R_D = R = 4$).

The cost function is also needs to be specified. In general terrain following problem, the cost function is either a form of absolute or a quadratic of deviated altitude from the clearance level. The cost function with the integration of absolute value of a deviated altitude is selected in this problem.

$$J = \int_{t_0}^{t_f} \left| z(t) - h(x(t), z(t)) - h_{clear} \right| dt \tag{11}$$

Here, z implies altitude of vehicle at time t, $h(x,z)$ means the height of terrain at that specified vehicle location and h_{clear} is minimum clearance level given.

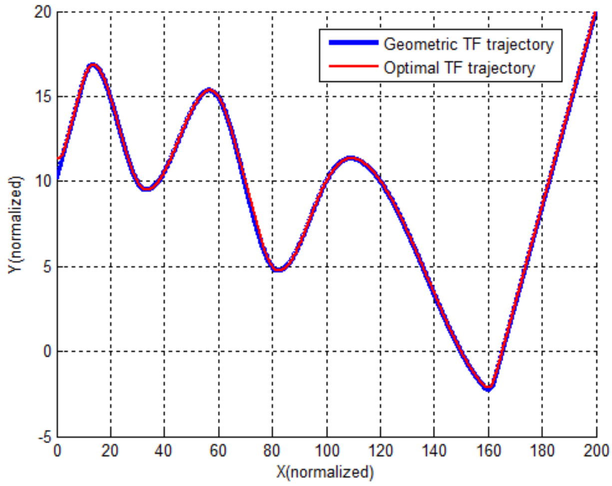

Fig. 17 Comparison of Two Trajectories

The optimal terrain following trajectory is given as Fig. 17, which is almost as the same as result derived from geometric method. These results are slightly different at initial point due to the change in flight path angle of the optimal trajectory while the geometric path keeps its flight path angle along the ascending region. In consequence, the terrain following trajectory from geometric method with maneuvering limit consideration closely resembles the optimal trajectory. Thus, it is possible to use geometric method for obtaining terrain following trajectory without solving optimal control problem in order to save time.

3.2 Performance on Radar Detection

Though the terrain following trajectory derived from the geometric method is similar to the result from the optimal solver, there are differences in the radar avoidance performance of two trajectories. Assuming that the radar is located at the end point of the terrain and if missiles are flying toward radar while flying along the terrain following trajectory, the equations identifying whether the missiles are detected by the radar are given as:

$$u = \begin{cases} 0 \text{ (not detected)} \\ 1 \text{ (detected)} \end{cases}$$
$$\text{if satisfying all equations below}$$
$$\begin{aligned} &(x_m - x_r)^2 + (z_m - z_r)^2 \le R^2 \\ &G(k) = z_r + k(z_m - z_r) - h(L(k)) > 0 \\ &L(k) = x_r + k(x_m - x_r) \ (0 \le k \le 1) \\ &\frac{z_m - z_r}{\sqrt{(x_m - x_r)^2 + (z_m - z_r)^2}} \le \sin\theta \end{aligned} \tag{12}$$

(R : detection range, $(\bullet)_m$: position of *missile*, $(\bullet)_r$: position of radar, θ : elevation angle)

The radar detects all the range and have elevation angle of $|\theta| \le 50°$. Since two calculated trajectories are given in a discrete form, the total detected time is governed by equation below.

$$t_{\text{detected}} = \int_0^{t_f} u\, dt \approx \sum_{i=1}^{n-1} \frac{u_i + u_{i+1}}{2} \Delta t \tag{13}$$

The resultant detected time graph is given as Fig. 18. The radar exposed time of two trajectories are almost identical and it can be verified through the graph Fig. 18. Because the two trajectories are identical, the difference of exposure time of the two trajectories would be small.

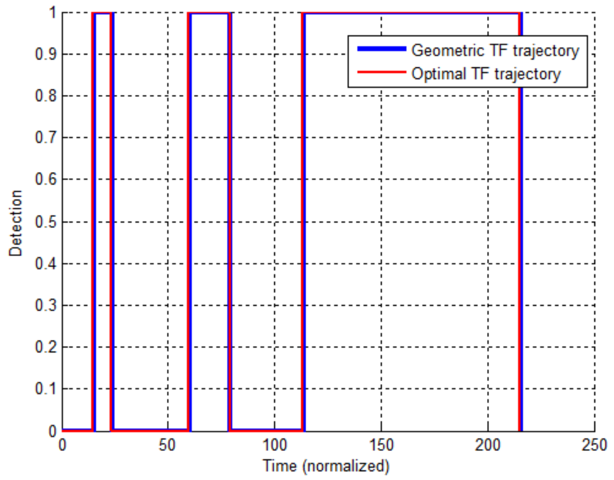

Fig. 18 Detected Time Graph of Two Trajectories

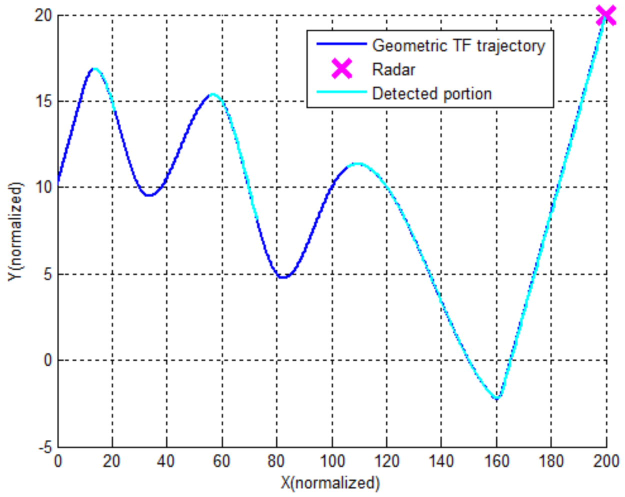

Fig. 19 Detected Portion of Geometric Path

	Geometric	Optimal
Total detected time	129.69 sec	129.02 sec

Table. 2 Detected Time of Two Paths

The total detected time of two trajectories are 129.69 seconds and 129.02 seconds each. The difference of detected time is 0.67second which is an ignorable value. The terrain following trajectory from geometric method has almost the same trajectory as trajectory from the optimal solver and it shows similar radar detection avoidance performance. Note that if the cruise missile is in a level flight condition, the detection time is 200 seconds, exactly the same as the flight time to the destination.

4 Conclusion

The terrain following path has been created by geometric way. It is divided into two parts by terrain shape: concave and convex. Since the purpose of the terrain following is to fly as close to the terrain as possible, predicted pull-down maneuver path which includes highest point at the local maximum peak point is considered in the concave part. Also for the crash prevention, predicted pull-up maneuver path is calculated to find the pull-up maneuver start/end points.

This method performs almost the same performance on radar detection as optimal path with the cost function of minimum height.

Maintaining minimum height in a range of possible flight condition is important for the detection performance because the cruise missile is not detected when it flies behind the hill. Therefore, this proposed geometric method's performance is notable in the aspect of minimizing radar detection time. Moreover, unlike optimal control method, proposed geometric method always gives terrain following path. The optimal control method might not find solution if convergence is not satisfied, whereas this proposed geometric method finds the solution without the need of convergence.

5 Acknowledgement

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