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AUGMENTED POLYNOMIAL GUIDANCE FOR TERMINAL VELOCITY CONSTRAINTS

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Abstract

This paper deals with an augmented polynomial guidance law to fulfill the terminal velocity requirement of high speed maneuverable reentry vehicle. The guidance command consists of three terms; basis polynomial guidance term, velocity control term, and gravity compensation term. The polynomial guidance can lead the vehicle to the target position in specific direction, and the augmented guidance command can adjust the terminal velocity without disturbing the basis polynomial guidance. The velocity is predicted by internal simulation under non velocity control command condition. The predicted terminal velocity error is fed back to velocity control command. Some computer simulations are conducted to show the performance of the proposed algorithm.

1 Introduction

Recently, a guidance law that achieves specific terminal velocity for an unpowered flight vehicle has studied for various purposes, such as safety landing of a space ship or impact efficiency enhancement of a missile. The terminal velocity can be dissipated by maneuver of the vehicle due to aerodynamics, thus it depends on the trajectories of the vehicle. In general, the aerodynamic force is stronger at the low altitude than the high altitude, as the atmosphere is denser at sea level.

Previously, a number of researchers have studied about the impact angle control guidance law. Kim formulated a terminal guidance system for reentry vehicles with the impact angle constraints as a linear quadratic control problem, and derived a Riccati equation to calculate time-varying feedback gains [1]. A control energy optimal impact angle guidance law for missiles with a time varying velocity has studied in [2]. In the paper, the author assumed that velocity of a missile decreases exponentially. A time to go weighted control energy optimal guidance law is studied in [3], which results in a generalized vector explicit guidance law (GENEX). It consists of two parts; one is minimizing zero effort miss and the other minimizing the impact angle error, so, GENEX can control the impact angle of the vehicle,

An energy optimal impact angle guidance law for a lag system has been derived from a linear quadratic optimal control law by Ryoo [4]. He also proposes time-to-go weighted optimal impact angle guidance law [5]. A time-to-go polynomial guidance law with the impact angle constraint is proposed by Kim, in [6]. The author also proposed a downrange-to-go polynomial guidance law that can achieve the impact time and angle together in [7]. The polynomial guidance law is biased with an additional acceleration command which adjusts the trajectory of a vehicle so as to control the impact time.

An augmented guidance law using the vector explicit guidance for the impact angle and velocity constraints is introduced in [8]. It shows a stability analysis of the guidance law on the linearized dynamics. It augments a lateral additional acceleration command to the vector explicit guidance command to control the terminal velocity. The scales of the augmented lateral guidance command are determined by internal simulations and the secant method.

This paper proposes an augmented polynomial guidance law that satisfies the specific terminal velocity. The guidance command consists of an impact angle control polynomial guidance term, a bias guidance term controlling the terminal velocity of the vehicle, and the gravity compensation term. The proposed algorithm predicts the terminal velocity by an internal simulation, and feeds back the predicted terminal velocity error to the velocity control term.

This paper organized as follows; 2. The planar dynamic model of maneuverable reentry vehicle is described and problem definition follows. 3. The augmented polynomial guidance for the terminal velocity condition is explained. 4. Simulation results comes.

2 Problem Definition

2.1 Planar Guidance Model

The planar engagement dynamic model is described as follows,

$$\begin{aligned}\dot{x} &= V\cos\gamma \\ \dot{h} &= V\sin\gamma \\ \dot{V} &= -D/m - g\sin\gamma \\ \dot{\gamma} &= L/mV - g\cos\gamma/V\end{aligned} \tag{1}$$

where x is downrange variable, h is altitude, V is the velocity of the vehicle, γ is flight path angle, m is the mass of the vehicle, g is gravity acceleration, L is lift force, and D is drag force.

The aerodynamic forces are often expressed by non-dimensional coefficient.

$$\begin{aligned}D &= C_D Q S_{ref} \\ L &= C_L Q S_{ref} \\ Q &= \frac{1}{2}\rho V^2\end{aligned} \tag{2}$$

where C_D is drag coefficient, C_L is lift coefficient, Q is dynamic pressure, ρ is free stream atmosphere density, and S_{ref} is reference area.

Generally C_D-C_L is related by the polar curve. Simple drag polar model is given as,

$$C_D = C_{D\min} + K(C_L - C_{L0})^2 \tag{3}$$

where $C_{D\min}$ is parasite drag term, it is also minimum drag coefficient, and K is induced drag parameter, and C_{L0} is the lift coefficient at minimum drag.

In this paper, we employed following atmosphere density model and gravity model.

$$\rho = \rho_0 e^{-h/h_\rho}, \quad g = \frac{\mu_e}{(R_e + h)^2} \tag{4}$$

where ρ_0 is atmosphere density at sea level, h_ρ is atmosphere density scaling height, μ_e is earth gravitational parameter, and R_e is earth mean radius.

2.2 Problem Constraints

In this paper, we focused to find an acceleration command which let the vehicle have the terminal velocity and flight path angle on the final position.

The initial condition of the vehicle is given as,

$$\begin{aligned}&x(t_0) = x_0,\ z(t_0) = z_0, \\ &\gamma(t_0) = \gamma_0,\ V(t_0) = V_0\end{aligned} \tag{5}$$

where t_0 indicates initial time.

On the terminal position, the vehicle should have γ_f.

$$\begin{aligned}&x(t_f) = x_f,\ z(t_f) = z_f, \\ &\gamma(t_f) = \gamma_f,\ V_{f\min} \le V(t_f) \le V_{f\max}\end{aligned} \tag{6}$$

Note that the terminal velocity constraint is given by some boundary, not by specific value.

3 Augmented Polynomial Guidance for Terminal Velocity Constraints

3.1 Polynomial Guidance

Kim have introduced a downrange-to-go polynomial guidance, which can satisfy the impact angle condition and impact time requirement additionally, hitting the target [7].

In this chapter, we revisit the basis downrange-to-go, x_{go}, polynomial guidance.

The x_{go} polynomial guidance law assumes the guidance command as following polynomial.

$$\begin{aligned} u_{poly} &= c_n x_{go}^n + c_m x_{go}^m \\ x_{go} &= x_f - x \end{aligned} \tag{7}$$

where c_n and c_m are guidance coefficients, and x_{go} is downrange-to-go.

The linearized engagement dynamic model is given as,

$$\begin{aligned} \gamma' &= u_{poly} \\ z' &= \gamma' \end{aligned} \tag{8}$$

where $u_{poly}=a/V^2$. The apostrophe represents the derivatives by x. Thus integrating it gives the linearized trajectory of the vehicle as,

$$\begin{aligned} \gamma &= -\frac{c_n}{n+1} x_{go}^{n+1} - \frac{c_m}{m+1} x_{go}^{m+1} + c_\gamma \\ z &= \frac{c_n}{(n+1)(n+2)} x_{go}^{n+2} + \frac{c_m}{(m+1)(m+2)} x_{go}^{m+2} \\ &\qquad + c_\gamma x + c_z \end{aligned} \tag{9}$$

This trajectory should satisfy the boundary conditions. Thus, the terminal constraints determine the integral coefficients as,

$$\begin{aligned} \gamma_f &= c_\gamma \\ z_f &= c_\gamma x_f + c_z \end{aligned} \tag{10}$$

The initial conditions determine the other unknown coefficients, c_n and c_m,

$$\begin{aligned} \gamma_0 &= -\frac{c_n}{n+1} \hat{x}_{go}^{n+1} - \frac{c_m}{m+1} \hat{x}_{go}^{m+1} + c_\gamma \\ z_0 &= \frac{c_n}{(n+1)(n+2)} \hat{x}_{go}^{n+2} + \frac{c_m}{(m+1)(m+2)} \hat{x}_{go}^{m+2} \\ &\qquad + c_\gamma x_0 + c_z \\ \hat{x}_{go} &= x_f - x_0 \end{aligned} \tag{11}$$

Solve it for c_n and c_m, then,

$$\begin{aligned} c_n &= \frac{(n+1)(n+2)}{(m-n)\hat{x}_{go}^2} \begin{bmatrix} (m+1)(\gamma_f - \gamma_0)\hat{x}_{go} \\ -(m+2)(\hat{z}_{go} - \gamma_0 \hat{x}_{go}) \end{bmatrix} \\ c_m &= \frac{(m+1)(m+2)}{(n-m)\hat{x}_{go}^2} \begin{bmatrix} (n+1)(\gamma_f - \gamma_0)\hat{x}_{go} \\ -(n+2)(\hat{z}_{go} - \gamma_0 \hat{x}_{go}) \end{bmatrix} \\ \hat{z}_{go} &= z_f - z_0 \end{aligned} \tag{12}$$

Substituting these coefficients to (7) gives

$$\begin{aligned} u_{poly} &= \frac{(n+2)(m+2)}{x_{go}^2} z_{go} \\ &\quad - \frac{(n+m+3)}{x_{go}} \gamma_0 - \frac{(n+1)(m+1)}{x_{go}} \gamma_f \end{aligned} \tag{13}$$

Note that u_{poly} is the normalized command of the linearized geometry, so the dimensional guidance command should be applied with multiplication of V^2 on a guidance frame. In this paper we employed the flight path frame of the vehicle as the guidance frame. Thus, following transform is required.

$$\begin{aligned} R &= \sqrt{(x_f - x)^2 + (z_f - z)^2} \\ \lambda &= \tan^{-1}\left(\frac{z_f - z}{x_f - x}\right) \\ x_{go} &= R\cos(\lambda - \gamma) \\ z_{go} &= R\sin(\lambda - \gamma) \\ \gamma_{go} &= \gamma_f - \gamma \end{aligned} \tag{14}$$

3.2 Augmented Polynomial Guidance.

To satisfy the terminal velocity constraint and compensate the gravity acceleration, an augmented polynomial guidance is designed. The guidance command has three terms as,

$$a_{cmd} = u_{Poly} V^2 + a_{vel} + a_g \tag{15}$$

The first term is the basis x_{go} polynomial guidance term. The second term is velocity control acceleration term. The other is gravity compensation term.

Gravity compensation term is simply adopted as,

$$a_g = g_0 \cos\gamma \tag{16}$$

where g_0 is nominal gravity acceleration.

The terminal velocity control term is designed as,

$$a_{vel} = \frac{v_f - \hat{v}_f}{\hat{v}_f} n_{\max} g_0 \tag{17}$$

where v_f is nominal terminal velocity and $\hat{v}_f$ is predicted terminal velocity, and n_{max} is maximum load factor for a_{vel} term.

The $\hat{v}_f$ is a prediction under the basis polynomial guidance law and gravity compensation.

When the terminal velocity prediction is higher than the required value, this terminal velocity control term tends to reshape the trajectory downward so that the vehicle suffers from more aerodynamic forces due to the denser atmosphere. In the opposite case, the a_{vel} tends to modify the trajectory upward so that the vehicle avoids the drag more effectively.

Meanwhile, the nominal terminal velocity can be the mean value of the terminal velocity boundary.

$$v_f = \frac{V_{f\min} + V_{f\max}}{2} \tag{18}$$

3.3 Terminal Velocity Prediction

To predict the terminal velocity of the vehicle, the vehicle ought to simulate its trajectory on its own. However, it is often cost lots of effort to do so.

In order to enhance its prediction efficiency, we change the independent variable of the planar guidance dynamics from t to x as,

$$\begin{aligned} h' &= \tan\gamma \\ V' &= -D/mV\cos\gamma - g\tan\gamma/V \\ \gamma' &= L/mV^2\cos\gamma - g/V^2 \end{aligned} \tag{19}$$

As we consider the augment polynomial guidance, the required lift coefficient is calculated as

$$C_L = \frac{m(u_{poly}V^2 + g\cos\gamma)}{QS_{ref}} \tag{20}$$

Note that this procedure predicts the terminal velocity without the velocity control command.

To integrate along the downrange direction, the downrange increments are defined as,

$$\Delta x = x_{go} / N_{sim} \tag{21}$$

where N_{sim} indicates the number of the loop in the internal simulation. Then the prediction process can be expressed in discrete form as,

$$\begin{aligned} x_k &= x_{k-1} + \Delta x \\ z_k &= z_{k-1} + z'\Delta x \\ V_k &= V_{k-1} + V'\Delta x \\ \gamma_k &= \gamma_{k-1} + \gamma'\Delta x \end{aligned} \tag{22}$$

Then the final velocity becomes the predicted terminal velocity.

$$\hat{v}_f = V_{N_{sim}} \tag{23}$$

	values
initial condition	x_0 = 0 km h_0 = 30km V_0 = 2040 m/s γ_0 = -20 deg
terminal condition	x_f = 100 km h_f = 1 km V_f = v_f γ_f = 0 deg
v_f	(150, 200, 300, 400, 500) m/s

Table 1 numerical simulation scenarios.

4 Numerical Simulations

4.1 Simulation Scenarios

Numerical simulations are conducted to show the applicability of the proposed algorithm.

The vehicle starts from the given initial condition. It is a maneuverable reentry vehicle, and it needs to modify its path to the terminal position while satisfing the terminal velocity constraint.

In table 1, the initial and terminal conditions for simulation are depicted. The required final velocity differs from 150 m/s to 500 m/s for each case. The other conditions are fixed.

4.2 Simulation Results

In Fig. 1, the trajectory of each case is plotted. All vehicles eventually get to the terminal position, but the path is slightly different by cases.

Fig. 2 shows the flight path history of each case. All cases satisfied the terminal flight path angle. The flight time is different case by case, the case with low terminal velocity showed longer flight time. In cases of v_f = 150 and 200 m/s, the flight path angle smoothly attached to 0 deg. In the other cases, it sharply converged to 0 deg.

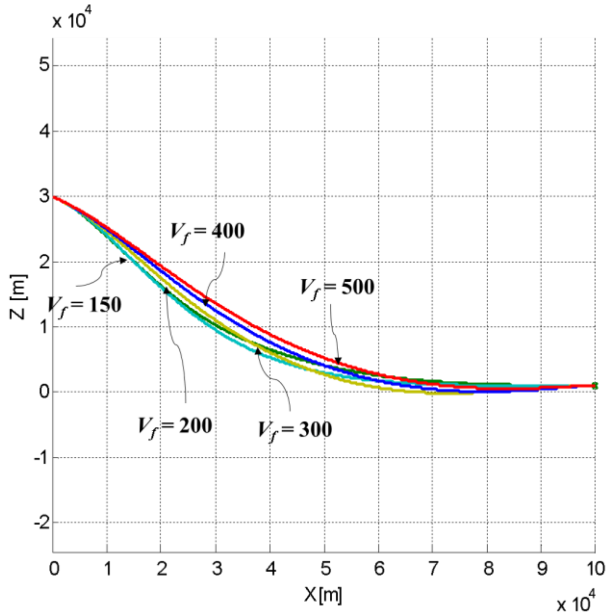

Fig. 1 trajectory of each case.

Fig. 3 shows the velocity history of each simulation. In cases of v_f = 150, 200, and 300 m/s, the terminal velocity error is less than 10 m/s. On the other hand, the other cases showed s about 10 % terminal velocity error.

Fig. 4 depicts the prediction history of the terminal velocity. The five cases show similar prediction at the beginning as their paths are similar. However, as the velocity control accelerations affect their paths, the predicted terminal velocity differs case by case. The predicted values at the final moment are identical to the final velocities.

In Fig. 5, guidance command history is shown in the load factor unit. The acceleration command is as much as -12 g at the beginning, and converges to 1 g nearby the final moment due to the gravity compensation.

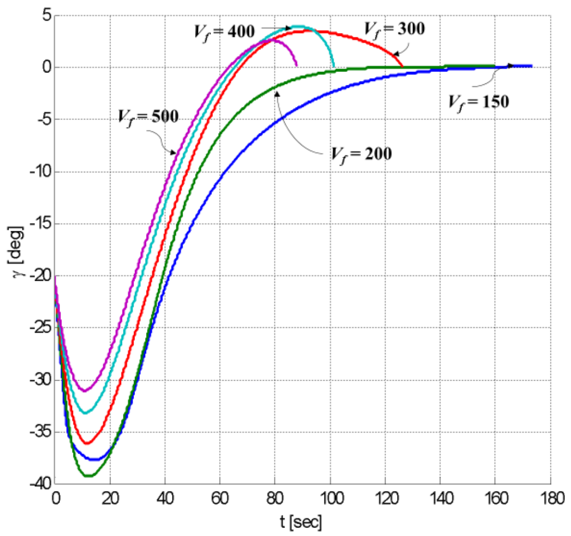

Fig. 2 flight path angle history of each case.

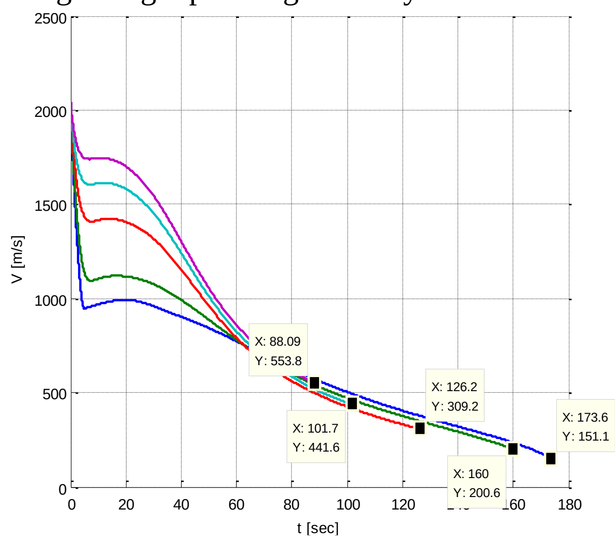

Fig. 3 velocity history of each case.

5 Conclusion

The augmented polynomial guidance is studied to control the terminal velocity of the high speed maneuverable reentry vehicle. The guidance command is composed of three parts, basis polynomial guidance, velocity control term and gravity compensation term. The terminal velocity without the velocity control command is predicted by the internal simulation. The internal simulation integrates along variable x so that it can estimate the terminal velocity quickly. The predicted terminal velocity error is fed back to the velocity control term. Numerical simulation shows that the proposed algorithm can satisfy the terminal velocity within 10% error at worst.

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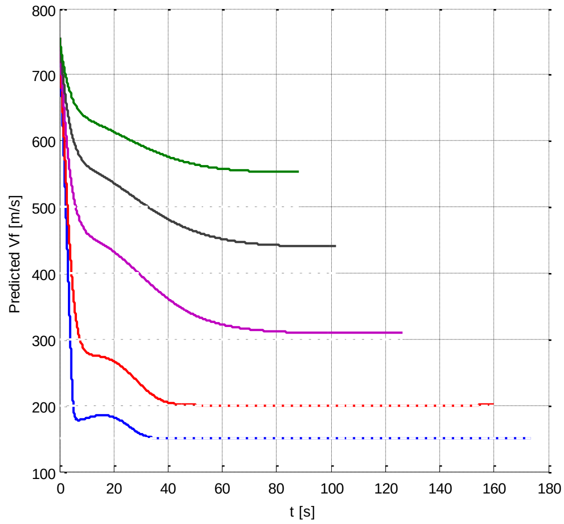

Fig. 4 terminal velocity prediction history.

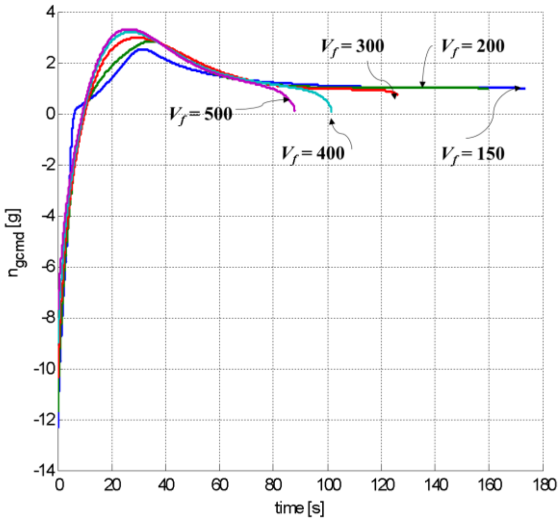

Fig. 5 non-dimensional guidance command.

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