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APPLICATION OF POST-BUCKLING ANALYSIS FOR COMPOSITE STIFFENED PANELS

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Abstract

The paper is a research on structure stability of composite stiffened panels of a wing. A method for analyzing post-buckling loading capacity of the composite stiffened panels is presented briefly. It is used to analyze the stability safety margin of the panels under compression or shear loads and to make a checking for structure stability of a wing under complex loading. The wing for the checking is an optimized composite wing which has not considered the stability constraint. The variety of loading conditions of the wing is known. The paper provides stability criterions for complex loading and the safety margins for the stiffened panels. This method will be integrated into the optimization system as a constraint.

1 Introduction

Based on the FEM model, the wing structure optimization has been carried out to meet the requirements of structure strength and stiffness. But the FEM model is not providing the stiffener sections for stability analysis. So the structure optimization is not including the constraint for the wing stability. An additional stability analysis should be considered for the wing. The commercial software MSC.NASTRAN has the function to evaluate structure stability, but the analysis for skin panels only uses the typical boundary conditions and not considers the real stiffener's boundary supports in a compression loading. It has not met the accuracy for buckling analysis. NATRAN doesn't have the function for post-buckling also. In fact, there is still certain loading capacity for the panels after the structure buckling. So considering structure post-buckling performance can increase the structure loading capacity and improve the efficiency of the structure. The post-buckling stability analysis is necessary during designing a structure.

This paper introduces an engineering calculation method on buckling and post-buckling of composite stiffened panels, which is partly validated by tests. Using this method verifies the buckling and post- buckling performance of a wing which is optimized with large design variables for the structure strength. An engineering structure stability analysis method has been used to analysis the safety margin of composite stiffened panels on a wing under complex loads. The result has been used to improve the wing structure which will meet the requirements of structure strength, stiffness and stability.

2 Analysis methods for buckling of composite stiffened panels

Composite stiffened panels include stiffeners and panels two parts, if to analyze composite stiffened panels buckling performance, the buckling of stiffeners and panels can be calculated separately, and then the lowest buckling load is regarded as the buckling capability of the composite stiffened panels.

2.1 Method for buckling of composite stiffeners under compression load

Stiffeners have a variety of sections, such as T, L and Ω. If to analyze the buckling of a stiffener, it needs to divide the stiffener into several small panels according to the edges of the stiffener, and then make buckling analysis for each panel. For example, a stiffener with H section can be divided into five panels, Unit1 - Unit5 respectively. Each panel of Unit1 - Unit4 has a free edge and a supported edge, Unit5 is supported by both edges, as shown in Fig.1.

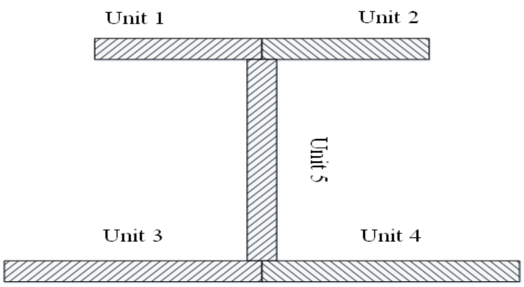

Fig.1 H-stiffener is divided into panels

If a stiffener is divided into m laminated panels, the buckling load of each panel $N_{crst1}, N_{crst2}, \cdots, N_{crstm}$ is calculated by the classic buckling load formula of the composite laminated structures [1]:

$$N_{crsti} = \frac{\pi^2 \sqrt{D_{11} D_{22}}}{b_i^2} [K - 2(1 - \frac{D_{12} + 2D_{66}}{\sqrt{D_{11} D_{22}}})] \qquad (1)$$

Where D_{ij} (i, j=1, 2, 6) are bending stiffness coefficients; b_i is the width of composite laminated panel; K is buckling coefficient, as for the panel with a free edge and a supported edges, K equals 1 or 2; as for the panel with both supported edges, the value of K can be got from reference[1].

And then take the minimum one among all panels as the stiffener buckling load,

$$N_{crst} = \min(N_{crst1}, N_{crst2}, \cdots, N_{crstm}) \qquad (2)$$

2.2 Method for buckling of composite panels under compression load

Buckling load N_{cr} of skins (or panels) can be calculated by the same formula of the composite laminated structures:

$$N_{crs} = \frac{\pi^2 \sqrt{D_{11} D_{22}}}{b_s^2} [K_c - 2(1 - \frac{D_{12} + 2D_{66}}{\sqrt{D_{11} D_{22}}})] \qquad (3)$$

Where D_{ij} (i, j=1, 2, 6) are bending stiffness coefficients. As for skin, b_s is the distance between two contiguous stiffeners; K_c is buckling coefficient, which depends on the structure parameters of the skin and the corresponding stiffeners:

$$K_c = K_c(\frac{L}{b_s}, \frac{A_{st}}{b_s t_s}, \frac{(EI)_{st}}{b_s D_s}) \qquad (4)$$

L is the length of the composite skin panel; A_{st} is the area of stiffener section; $(EI)_{st}$ is the bending stiffness of stiffeners; $b_s D_s$ is the bending stiffness of skins; K_c can be found from some figures[2] .

2.3 Method for buckling of composite panels under shear loading

According to the reference [1], the formula for shear buckling load of composite panels N_{scr} is:

$$N_{scr} = K_s \frac{\pi^2 \sqrt[4]{D_{11} D_{22}^3}}{b^2} \qquad (5)$$

The shear buckling coefficient K_s can be found from the reference [1].

3 Analysis method for post-buckling of composite stiffened panels

According to the parameter L'/ρ, three different failure models of stiffened panels which are named the short column failure, the medium-long column failure and the long column failure, The value of L' can be calculated by the formula: $L' = L/\sqrt{C}$, where L is the length of unloading side of the panels; ρ is the radius of gyration; C is the edges supporting coefficient of the panels. As they are shown in Fig.2, the medium-long column failure is the main failure model in airplane structures including the composite stiffened panels [2].

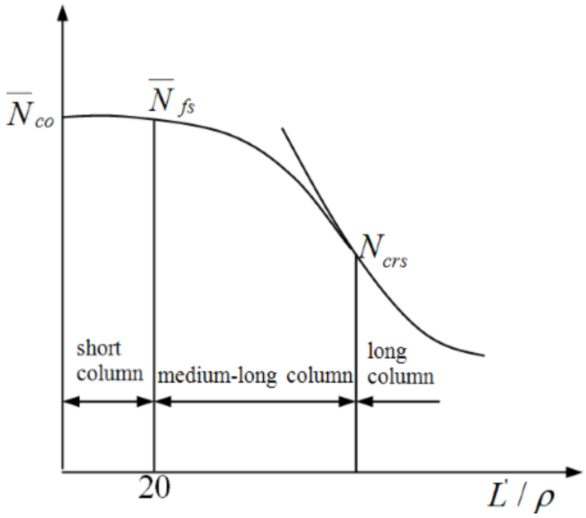

Fig.2 Failure models of stiffened panels

3.1 Method for loading capacity of composite stiffeners

The post-buckling load of stiffeners is identified as the short column failure load. Similar to the method in 2.1 section, the stiffener is divided into m panels. The post-buckling load for each panel N_{fst} is obtained by the following equation derived from theoretic analysis and experiment data [3]:

$$\begin{cases} N_{fst} = N_{crst} \times \alpha \times (\frac{N_{ed}}{N_{crst}})^n & N_{crst} < N_{fst} \\ N_{crst} = N_{fst} & N_{fst} < N_{crst} < N_{ed} \\ N_{crst} = N_{fst} = N_{ed} & N_{ed} < N_{fst} < N_{crst} \end{cases} \quad (6)$$

Where N_{ed} is edge loading capacity of the composite panels $N_{ed} = \eta N_p$; $\eta = 0.8$; N_p represents the edge max loading capacity which is obtained by Mindlin two-dimensional finite element model with 8-noded isoparametric element, the N_p is got by a FEM program; Parameters α and n equal to 0.822 and 0.575 respectively.

The failure loads of each panel $N_{fst1}, N_{fst2}, \cdots, N_{fstm}$ are calculated by Eq. (6). Take the minimum one as the average failure load for the stiffener.

$$N_{fsta} = \min(N_{fst1}, N_{fst2}, \cdots, N_{fstm}) \quad (7)$$

3.2 Method for loading capacity of composite panels

Considering the support of wing ribs and the parameters of skin panels, the post-buckling model of the wing skin are belong to the medium-long column failure model, so the failure load N_{co} is derived from the following equation:

$$\frac{\overline{N}_{co}}{\overline{N}_{fs}} = 1 - (1 - \frac{\overline{N}_{crs}}{\overline{N}_{fs}}) \frac{\overline{N}_{crs}}{\overline{N}_{el}} (\frac{N_{20}^{1/2} - N_{el}^{1/2}}{N_{20}^{1/2} - \overline{N}_{crs}^{1/2}})^2 \quad (8)$$

Where N_{20} is referred to the Euler failure load when $L'/\rho = 20$ and is given as $N_{20} = \frac{\pi^2 E_{st} t_s}{400}$; N_{fs} is the short column failure load; N_{crs} is the buckling load.

4 Stability analysis and validation strategy

Calculate all axial compression loading and shear loading capacity of the each wing skin panel using the methods for the buckling and the post-buckling which are introduced in the previous section. The working loads of each skin panel, when the wing is under the limit load and the ultimate load, are got by the finite element method. The buckling and post-buckling loads are used to compare with the working loads of the wing to verify the optimized structure whether it meets the requirements of stability.

Validation rules are that no buckling happens under the limit load and no failure occurs under the ultimate load. The paper will provide safety margin of the two aspects.

4.1 Uniaxial compression stability validation strategy.

Analysis of buckling and post-buckling under uniaxial compression include the longitudinal and transverse aspects respectively. The longitudinal direction represents the direction of wingspan while the transverse stands for the direction of chord. The buckling and post-buckling capacity should be calculated for each skin panel.

4.1.1 Buckling validation under limit load

Uniaxial compression buckling load is obtained by Eq.(2). As for the longitudinal buckling load formula, coefficient K can be referred from reference [2]; as for transverse buckling load, engineering experience shows that if the panel is under simply supported at four edges, buckling coefficient K equals 1.

In the case that the compression buckling load has been gotten, uniaxial compression stability criterion is:

$$\begin{cases} \dfrac{N_{x\lim}}{N_{crx}} < 1 \\ \dfrac{N_{y\lim}}{N_{cry}} < 1 \end{cases} \tag{9}$$

Where x represents longitudinal, y represents transverse, $N_{x\lim}$、$N_{y\lim}$ stand for horizontal and vertical working load of skin panels under the limit load.

4.1.2Post-buckling strength validation under ultimate load

The post-buckling failure load of composite stiffened panels under uniaxial compression is obtained by Eq.(8). Longitudinal failure load N_{cox} and transverse failure load N_{coy} can be calculated respectively. The criterion of post-buckling loading capacity as follows:

$$\begin{cases} \dfrac{N_{xext}}{N_{cox}} < 1 \\ \dfrac{N_{yext}}{N_{coy}} < 1 \end{cases} \tag{10}$$

Where x and y represent longitudinal and transverse respectively, N_{xext} , N_{yext} stand for longitudinal and transverse working load of skin panels under the ultimate load.

4.2 Shear load stability validation strategy

Validation rule is that the working shear load of wing skin under the ultimate load is less than the shear buckling load N_{scr} .

$$\frac{N_{s\lim}}{N_{scr}} < 1 \tag{11}$$

4.3 Buckling safety margin of complex loading under limit load.

Take the biaxial compression buckling and the compression-shear buckling into consideration in this paper.

4.3.1 Biaxial compression buckling

The buckling load of stiffened panels under biaxial compression follows this inequality:

$$R_x^{1.5} + R_y^2 \le 1 \tag{12}$$

Where $R_x = N_{x\lim} / N_{crx}$ is longitudinal compression load ratio under the limit load, and $R_y = N_{y\lim} / N_{cry}$ is transverse one; $N_{x\lim}$、$N_{y\lim}$ are longitudinal and transverse working load respectively under the limit load; N_{crx}、N_{cry} stand for longitudinal and transverse buckling load respectively. Generally, Fig. 3 can be used to calculate the composite buckling safety margin. When $N_{x\lim}$ and $N_{y\lim}$ are determined, R_x and R_y can be got. After that, the location of point M in Fig. 3 can be found out. Connecting point O and M, point N will appear in this figure. The buckling safety margin can be calculated using the following formula.

$$Ms = \frac{\overline{ON}}{\overline{OM}} - 1 \tag{13}$$

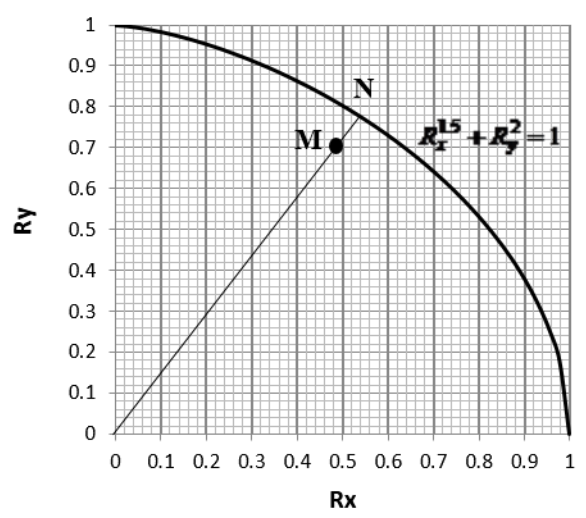

Fig. 3 Safety margins for biaxial compression buckling

4.3.2 Compression-shear buckling

Choose the status of longitudinal compression and in-plate shear for stability check.

Buckling load of composite stiffened panels under compression-shear complex loading follows this inequality:

$$R_c + R_s^2 \le 1 \tag{14}$$

Where $R_c = N_{x\lim} / N_{crx}$ is the longitudinal compression load ratio and $R_s = N_{s\lim} / N_{scr}$ is shear load ratio under the limit load; $N_{x\lim}$、$N_{s\lim}$ are longitudinal and shear working load respectively under the limit load, N_{crx}、N_{scr} are longitudinal and shear buckling load respectively.

The safety margin calculation formula is similar with formula (13).

4.4 Post-buckling safety margin of complex loading under ultimate load

The post-buckling validation of composite stiffened panels under compression-shear complex loading is operated on the basis of Fig.4.

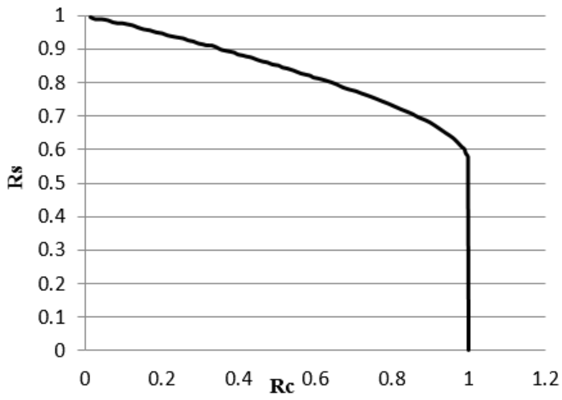

Fig.4 Compression-shear failure

The horizontal axis is $R_c = N_{xext} / N_{cox}$, vertical axis is $R_s = N_{sext} / N_{scr}$. They represent longitudinal compression ratio and shear load ratio under the ultimate load. N_{cox} is post-buckling load under longitudinal compression, N_{scr} is shear buckling load. Check the location of the point (R_c, R_s) whether it is beyond the boundary of the curve in figure 4.If the point is at the internal side of the curve, failure doesn't occur, and vice versa.

5 Case study

5.1 Force distributions under limit and ultimate load

In this paper, stability checking for upper skin of wing box which is optimized and then meets the requirements of strength and stiffness has been done. The finite element model is developed on the basis of the real wing structure, so it has engineering application value. This model has a total of 55 load cases, and the load is imposed on the finite element nodes. There are dozens of pieces of skin panels and eight stiffeners on the upper skin of wing box. The section of stiffeners and composite layers are different between each other. The section of one stiffener is shown in Fig.5. Composite material properties and load distribution are not described in this paper.

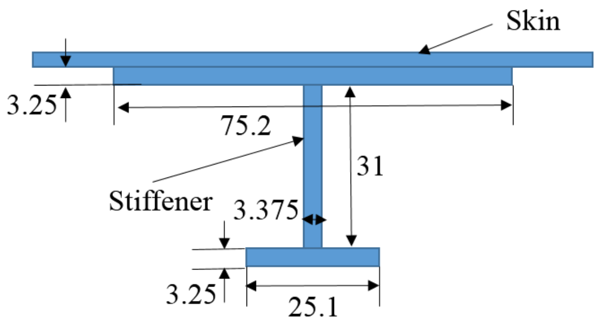

Fig. 5 Sectional dimensions of one stiffener

The internal force distributions of the wing under two serious load cases have been calculated using finite element analysis tool MSC.NASTRAN. The distributions are shown in the following figures:

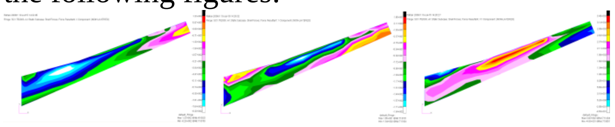

Fig. 6 Longitudinal, transverse compression and in-plate shear under limit load in case1

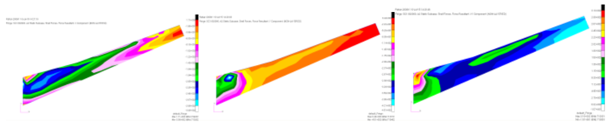

Fig. 7 Longitudinal, transverse compression and in-plate shear under limit load in case55

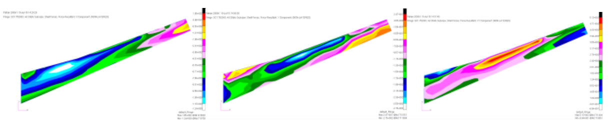

Fig. 8 Longitudinal, transverse compression and in-plate shear under ultimate load in case1

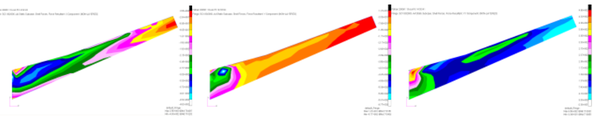

Fig. 9 Longitudinal, transverse compression and in-plate shear under ultimate load in case55

5.2 Safety margin for buckling and post-buckling

There are dozens of skin panels on upper surface of the wing, and buckling and post-buckling safety margins for each panel can be calculated. Buckling safety margins for biaxial compression buckling and compression-shear buckling of the wing skin panels under the limit load and post-buckling safety margins for compression-shear failure under the ultimate load have been calculated. The safety margins are shown in Fig. 10-Fig. 12, each point in the graphs presents a certain skin panel.

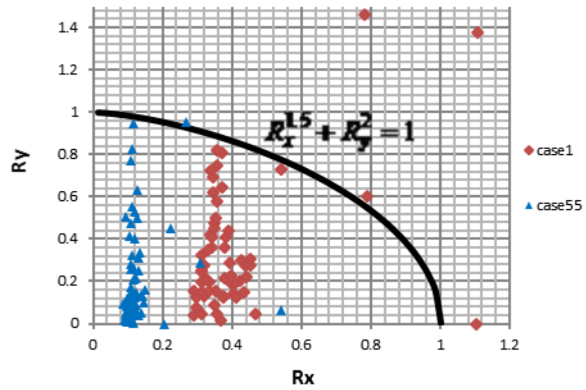

Fig. 10 Buckling safety margins for biaxial compression buckling

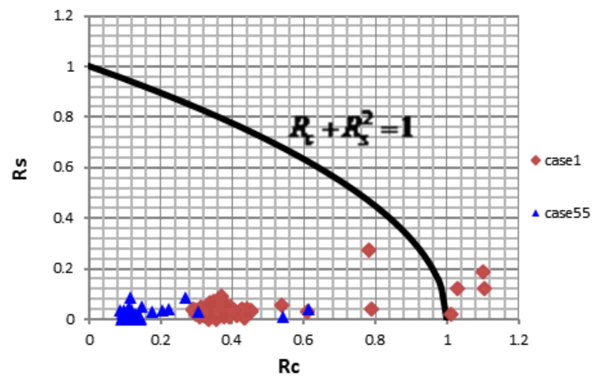

Fig. 11 Buckling safety margins for compression-shear buckling

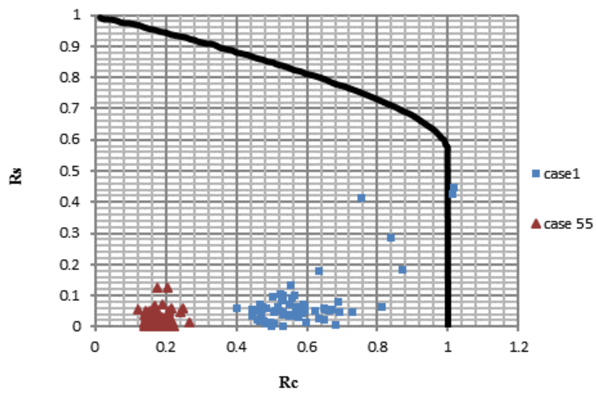

Fig. 12 Compression-shear failure

5.3 Results analysis

1. Most of stiffened panels have met the requirements of stability after the structure is optimized, but the buckling and post-buckling may be happened on some panels, so stability checking is necessary.
2. There are fewer skin panels resulting in buckling or post-buckling for the reason that the composite panels optimization are limited by the allowable strain which is quite lower than most of the buckling strain.
3. Although buckling and post-buckling don't happen on some stiffened panels under uniaxial load, they may occur on such panels under complex load. It shows that the analysis on buckling and post-buckling of stiffened panels under complex load is necessary.

Conclusion

The thin shell structure stability is the main constraint for structural static loading. The stability safety margin of composite wing should be checked to ensure whether the structure static strength is enough. This example has been verified that 98.5% of the wing skin panels after multi-objective optimization can meet the stability requirements, and the others need strengthen further. The example application shows that it is necessary to consider structure stability as a constraint during optimization. This research method of stability will be integrated into a multi-disciplinary optimization system further.

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