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ESTIMATION OF TEMPERATURE-DEPENDANT THERMAL CONDUCTIVITY OF MATERIAL WITH GENETIC ALGORITHM

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Abstract

Estimation of temperature-dependant thermal conductivity of material is a typical inverse heat conduction problem. The temperature range can be divided into several segments and the thermal conductivity is linearly interpolated from the thermal conductivities on these segment points whose values are to be estimated by Genetic Algorithm(GA) method from some measurement information. However, under some circumstances such as the measurement noise being significant, some unphysical results of significant oscillation may come out of the estimated result. So, in this paper, physical constraints that the thermal conductivity value should be greater than zero and not decrease with the temperature rise are taken into the estimation method of GA with two strategies. One is to estimate the non-negative incremental value of the thermal conductivity with the temperature rise, the other is to implement the penalty function in the estimation process. From the numerical results of examples, it can be found that the constrained estimation methods of both strategies are feasible and effective, and the estimated values of both strategies are agreeable and generally better than the constraint-free result. The estimated result reflects the basic temperature-dependant characteristic of the material well and the method is of good prospect in engineering practices.

1 Introduction

Determination of the thermal conductivity value of the structural material plays an important role on the prediction of heating environment and heating response of airplane and aerospace vehicles. Presently, the prevailing methods to get the thermal conductivity of material include the flashing method, steady state measurement method, and sensitivity method [1,2]. In these methods, the thermal conductivity is assumed to be a constant or a predetermined function of temperature. But in many engineering circumstances, this assumption is not rational and under some circumstances, the steady state is difficult to be realized in the experiments. So, when the temperature-dependant function of the thermal conductivity is unknown, the thermal conductivity is discretized in different temperature ranges and estimated from the boundary or interior temperature measurements by bisearch method, complex-variable-differentiation method, and Genetic Algorithm(GA) in the references[3][4][5]. However, it is found in practice that when the measurement noise is significant or there are some deviation between the computational model and the real physics of the heat conduction problem, the estimated result may exhibit significant numerical oscillation, and even the unphysical result that the value of the thermal conductivity is less than zero may come out. So, it's necessary to implement some constraints in the estimation process such as the value of the thermal conductivity should be greater than 0, and for many common-used materials the value of thermal conductivity should not decrease with the temperature rise. Based on this idea, in this paper, the GA method used to estimate the thermal conductivity is modified to take the physical constraints into account, and some examples are studied.

2 Mathematic model

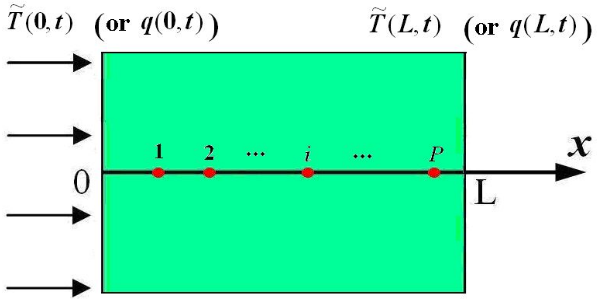


Fig.1 Sketch of thermal conductivity estimation

As far as a typical one dimensional heat conduction problem in Fig.1 is concerned, when the temperatures or heat flux are specified at the left and right boundaries, the temperature measurements at the interior points can be used to estimated the thermal conductivity of material. Without losing generality, when the temperature boundary conditions are given at the both boundaries and the temperatures at interior points of x_i(i=1,P) are measured, the control equations can be written as

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(T) \frac{\partial T}{\partial x} \right) \tag{1}$$

With boundary conditions: x=0, $T = T(0,t)$; x=L, $T = T(L,t)$; initial condition: t=0, $T = T_0$.
And the measurement equation

$$\tilde{T}(x_i,t) = T(x_i,t) + v(t)\,,\ i=1, P \tag{2}$$

where ρ is the material density, C_p is the heat capacity per unit volume and v is the measurement noise. $k(T)$ means the thermal conductivity of material is temperature-dependant.
If the temperature-dependant function of $k(T)$ is not clearly known a prior, the discretized expression can be utilized. i.e., the whole temperature range is divided into several segments and the thermal conductivity in every segment is assumed to be a linear function of temperature, illustrated in Fig.2. So, when there are M interval nodes in the whole temperature range discretization, the thermal conductivity can be expressed as,

$$k(T) = \sum_{i=1}^{M} k_i \,\varphi_i(T) \tag{3}$$

where φ_i are the base functions for linear interpolation,

$$\varphi_1 = \begin{cases} (T_2 - T)/(T_2 - T_1) & ; T \in [T_1, T_2] \\ 0 & ;\ other \end{cases}$$

$$\varphi_i = \begin{cases} (T - T_{i-1})/(T_i - T_{i-1}) & ; T \in [T_{i-1}, T_i] \\ (T_{i+1} - T)/(T_{i+1} - T_i) & ; T \in [T_i, T_{i+1}] \\ 0 & ;\ other \end{cases}$$

$$\varphi_M = \begin{cases} (T - T_{M-1})/(T_M - T_{M-1}) & ; T \in [T_{M-1}, T_M] \\ 0 & ;\ other \end{cases}$$

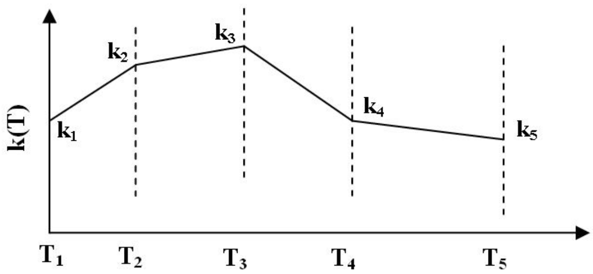


Fig.2 Sketch of temperature-dependant thermal conductivity

Provided the k_i(i=1,M) known, the Eq.(1) can be solved with Finite Control Volume(FCV) [1] method. But for the inverse problem, when k_i are unknown, the measurement equations, Eqs.(2), should be used to estimate the k_i values by changing the inverse problem to an optimization problem to minimize the following objective function,

$$J(\vec{k}) = \int_{t=0}^{t_f} \sum_{i=1}^{P} \left\{ \left[T(x_i,t,\vec{k}) - \tilde{T}(x_i,t) \right]^2 w_i \right\} dt\,; \qquad \sum_{i=1}^{P} w_i = 1 \tag{4}$$

where $\vec{k} = (k_1, \ldots, k_M)$, [0, t_f] is the time span of measurement and w_i are the weighting factors. The optimization algorithm is presented as follows.

3 Genetic Algorithm to Estimate Thermal Conductivity

Generally, for the optimization problem of Eq.(3), two methods often can be used. One is the gradient-based method, the other is the GA. Because the gradient-based method is sensitive to the initial value and prone to reach local extremum point in the optimization process when the number of parameter is large, the GA is utilized to carry out the optimization in this work. The GA method is robust and capable of finding the global extremum point of optimization theoretically. The basic underlying principle of GA is that of the Darwinian evolutionary principle of natural selection, wherein a population consisting of individuals which is composed of the parameters to be estimated is generated randomly and evolved according to the individual's objective function J, or the fitness value defined as

$$F=1/J \text{ or } F=a-J \text{ (}a\text{ is a large positive value)} \quad (5)$$

And the fittest individuals survive and are favored to produce offspring, which is analogy to the law of "the survival of the fittest". Then after several generations of evolution, the individual in the final population with the best fitness can be obtained as estimated result. The detailed evolution algorithm for the thermal conductivity estimation problem is presented as follows.

(1) Use the binary coding rule to turn the parameters k_i (i=1, M) to sequential binary code strings, called as "chromosome". Each string represents a solution point in the search space, and is composed of sub-strings that are analogous to genes.

(2) Generate the initial population consisting of N samples randomly with binary coding method. Perform the direct heat conduction problem solving for every sample in the population, and get the objective function and fitness of every sample with Eq.(4) and Eq(5).

(3) By weighted roulette wheel method, select new samples from current population and copy strings with regard to their fitness, to generate the next population.

(4) Perform the crossover and mutation operations on the new population.

(5) Solve the direct heat conduction problem for every sample in the new population. The new population usually has higher fitness values, which means that the population improves from generation to generation. Then, find the individual with the best fitness value, and decide the evolution is convergent or not. If convergent, stop the population's evolution, otherwise, return to step(3) to continue the evolution.

Two numerical examples are given as follows. The first example is the heat conduction problem of a plate with a constant heat flux Q at x=0 and insulted at x=L, there exist analytical temperature results of boundary and internal points when the thermal conductivity is a constant. So when these analytical temperatures at the left and right boundaries are given as boundary conditions and the analytical temperature at the middle point x=0.5 is both directly used and added with white noise as measurement, the thermal conductivity can be estimated by the aforementioned method. In the example, L=1, C_p=1, Q=1, and the exact value of thermal conductivity is 1. Table 1 shows the estimated result for three cases. The first case is that the analytical temperature at x=0.5 is directly used as measurement(denoted as "σ=0"), and in the other two cases, the analytical temperature at x=0.5 are added with the white noises whose standard deviation are 1% and 5% of the maximum temperature value respectively to simulate the measurements. The results show that the estimated value agrees with the exact value, even when the standard deviation of measurement noise is 5% of the maximum temperature value, the difference between the estimated and exact value is less than 3%, which verifies the feasibility of both the FCV and the GA methods.

Table 1 Exact and estimated values of const thermal conductivity

	Exact value	Estimated values		
		σ=0	σ=1%T_{max} (x=0.5)	σ=5%T_{max} (x=0.5)
k	1.0	1.0005392	1.0054315	1.0249013

In the second example, the basic parameters are given as, ρ =1000kg/m^3, C_p=1000J/kgK, L=0.01m, T_0=600K, the left and right temperature boundary conditions are shown as "x=0" and "x=10mm" in Fig.3. Two measurement points locate at the positions of x=3mm and x=6mm. When the T_i and k_i values of thermal conductivity in Eq.(3) are given in the first two rows in Table 2, the temperature histories at the two measurement points can be calculated, shown as “x=3mm” and “x=6mm” in Fig.3.

Table 2 Specified temperature-dependant thermal conductivity and estimation results

Temperature $T_i(i=1,5)$		600K	700K	800K	900K	1000K
Exact values of thermal conductivity $k_i(i=1,5)$		1.0	1.01	1.1	1.38	2.0
Estimated values of k_i (without constraints)	σ=0	0.99914	1.008	1.1047	1.3742	2.0042
	σ=4K	1.1561619	0.9800952	1.0812671	1.4738692	1.9721296
	σ=10K	1.3942136	0.9470426	1.0345725	1.6438015	1.9117926
Estimated values of k_i (Incremental method)	σ=4K	1.025508	1.025508	1.033474	1.478722	1.911060
	σ=10K	1.007471	1.007471	1.007471	1.499597	1.829573
Estimated values of k_i (Penalty method)	σ=4K	1.0342977	1.0343893	1.0343893	1.4865043	1.9250686
	σ=10K	0.9874199	0.9874199	0.9875115	1.4949276	1.7783006

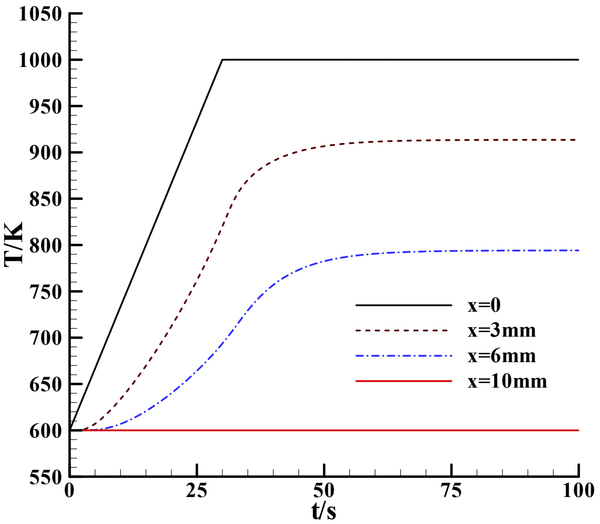


Fig.3 Temperatures of boundary condition and measurement points

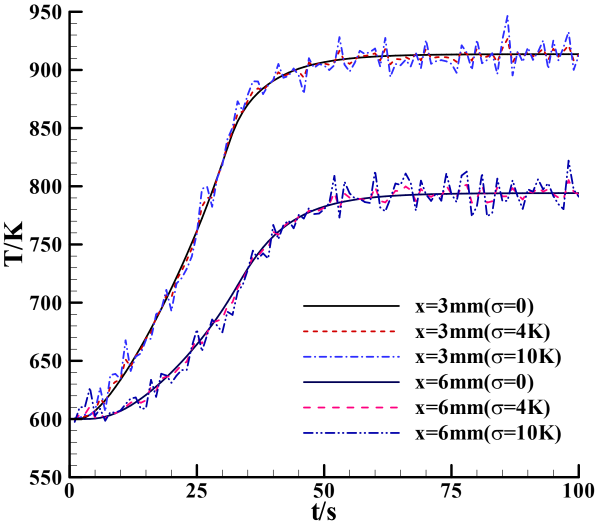


Fig.4 Temperatures with measurement noise

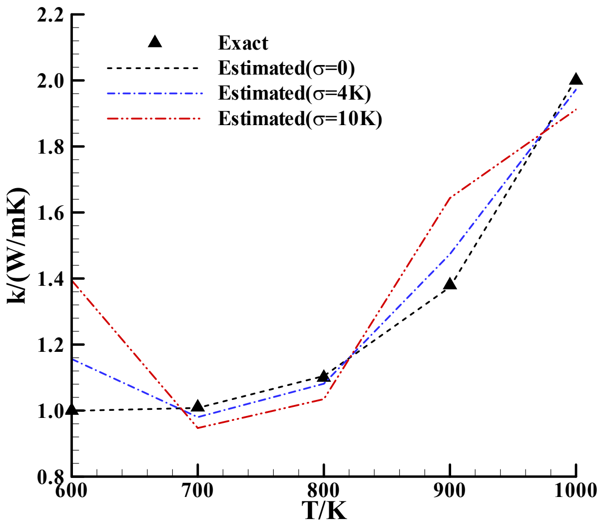


Fig.5 Estimated value of thermal conductivity

As for the inverse problem, the calculated temperature histories at the two measurement points are both directly used and added with white noise of standard deviation of 4K and 10K to be used as measurement data to estimate the thermal conductivity. In the GA algorithm, the population is chosen as 50 and the probabilities for crossover and mutation being 0.8 and 0.05 respectively. The results are shown in Fig.5 and Table 2, and it can be seen that,

(1) When the measurement noise is not added to the calculated temperature histories, the estimated result of agrees with the given value well, the relative error is less than 5‰， which also validate the effectiveness of the estimation method.

(2) When the measurement noise is considered, there exhibit some significant oscillations in the estimated results. Especially, the estimated value of k_1 is greater the value of k_2、 k_3 and k_4, which is significantly different from the real

physical situation. So, in the next section, some physical constraints of the thermal conductivity are taken into account in the estimation algorithms.

4 Estimation method with physical constraints taken into account

Physically, the value of the thermal conductivity of a material should be greater than 0, and generally, the value of the thermal conductivity may not decrease with the temperature rise. So, in the estimation algorithm, these two typical physical constraints are taken into account in the estimation algorithms. As for the first constraint, it can be implemented straightforwardly in GA by setting the lower range bounds of the parameters to be estimated to be greater than 0. For the second constraint, it can be implemented by the following two strategies.

4.1 Incremental method(IM)

The basic idea of this method is that instead of the absolute value of k_i (i=1,M), the value k_1 and the incremental values Δk_i (i=2,N) are to be estimated and the values of Δk_i are required to be not less than 0. i.e.,

$$\Delta k_i = k_i - k_{i-1}, \quad \Delta k_i \geq 0 \quad (i=2,\ M) \tag{6}$$

With Eq.(5), the constraint of thermal conductivity does not decrease with the temperature rise can be satisfied.

4.2 Penalty method(PM)

The basic idea of implementing this method in GA is to exert some penalty functions, being positive values, on the objective function to reduce the sample's fitness value when the constraint is not satisfied, and in GA the probability of the sample to be selected in the next generation can be greatly reduced. In contrary, without penalty added, the sample satisfying the constraint may have a larger fitness value, letting it won more opportunities to be selected and evolved in the following generations. Then, when the evolution is converged, the sample satisfying the constraints and with large fitness value can be obtained as estimated result. In this problem, the constraint of thermal conductivity does not decrease with the temperature rise can be rewritten as,

$$\text{For } k_m, m\in[1, M-1], k_n \geq k_m, n\in[m+1, M] \tag{7}$$

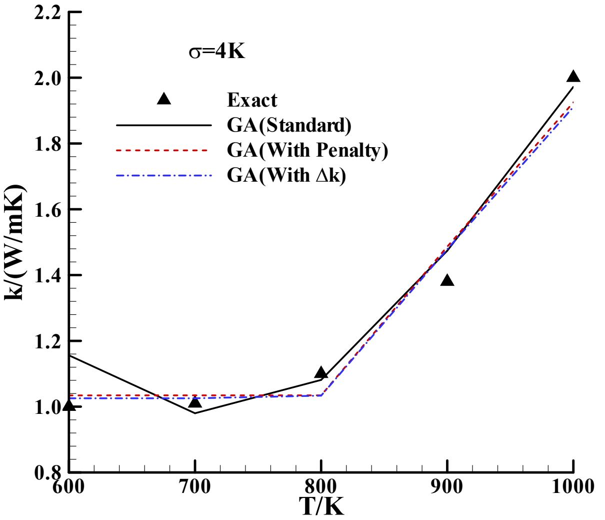
And the objective function of Eq.(4) turns to,

$$J(\vec{k}) = \int_{t=0}^{t_f} \sum_{i=1}^{P} \left\{ \left[T(x_i, t, \vec{k}) - \tilde{T}(x_i, t) \right]^2 w_i \right\} dt + \varepsilon \cdot \sum_{m=1}^{M-1} \left(\sum_{n=m+1}^{M} \max(k_m - k_n, 0) \right) \tag{8}$$

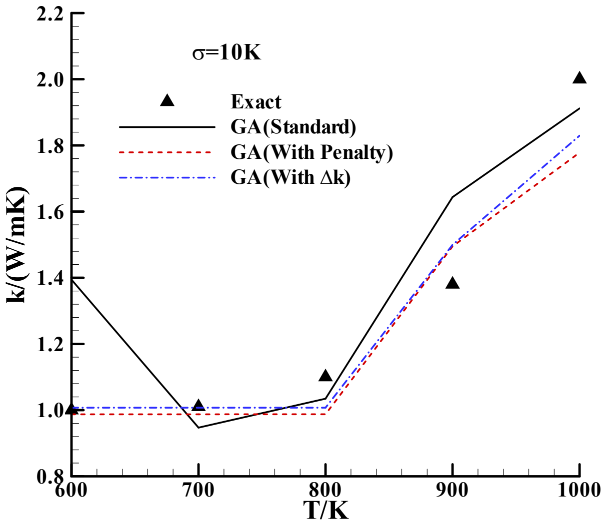
where ε is the penalty factor and its value may play an important role on the estimated result. When the value is small, the effect of penalty function may be neglected in selection process of GA and the constraints still be violated in the estimated result. While when the value is large, many potentially eligible individuals may be abandoned in the early generations of GA evolution because the constraints are slightly violated. So, in practices, this value should be adjusted according to that whether the estimated result satisfy the constraints or not.

5 Results

Still taken the second example with measurement noise considered in Section 2 to be studied, both IM and PM are implemented in GA to estimate the thermal conductivity with physical constraints. Especially, in IM, the parameters to be estimated are k_1, Δk_2, Δk_3, Δk_4, Δk_5; and in PM, the penalty factor ε is chosen to be 100. The estimated results are tabulated in Table 2 and shown in Fig.6(a)(b) denoted as "GA(With IM)" and "GA(With PM)" respectively. It can be found that, first, under these two circumstances of different measurement noise level, the estimated values of both "GA(With IM)" and "GA(With PM)" satisfy the two constraints that the value should be greater than 0 and not decrease with the temperature rise, and this results verify the feasibility of the estimation method of GA accompanied with IM and PM. Second, When the measurement noise increase, the deviation between the exact values and estimated results increase, but generally speaking the estimated results agree with the exact values well such as for the σ=10K case, the deviation between the exact values and estimated results is less than 10%. Third, the estimated values of both ”GA(With IM)” and ”GA(With PM)” are basically close, but from the view of algorithm development, the IM is simple, convenient, without free parameter while it’s experiential to determine the value of the penalty factor in the PM.



(a) σ=4K



(b) σ=10K

Fig.6 Estimated value of thermal conductivity with constraints taken into account

Conclusion

In this paper, two physical constraints that the thermal conductivity value should be greater than zero and the value should not decrease with the temperature rise are taken into the conventional estimation method of GA to develop a constrained estimation method with two strategies. One is to estimate the incremental values of the thermal conductivity with the temperature rise and restrict the incremental values to be not negative. The other is to implement the penalty function on the objective function in the GA evolution when the estimated parameters violate the constraints. From the numerical results of examples, it can be found that when the constraints is not considered, there exhibit some significant oscillations in the estimated results for the case with large measurement noise, and when the constraints is considered in estimation algorithm, the estimated results agree with the exact values well. This constrained estimation method may have a bright prospect in engineering practices.

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References

[1] Beck J V, Blackwell B, and JR Clair C R S. *Inverse heat conduction – ill-posed problems*. John Wiley&Sons, 1985.

[2] Ozisik M N, Orlande H R B. *Inverse heat transfer-fundamentals and applications*. Taylor&Francis, 2000.

[3] Zhao X L. *The research of effective conductivity in the porous media*. Master thesis, Dalian University of Technology, 2009 (In Chinese).

[4] Zhou H L, Xu X S, Li X L, Chen H L. Identification of temperature-dependent thermal conductivity for 2-D transient heat conduction problems. *Applied Mathematics and Mechanics,* Vol. 35, No. 12, pp 1341-1351, 2014 (In Chinese).

[5] Tang Z H, Qian G H, Qian W Q. Estimation of temperature-dependent function of thermal conductivity for a material. *Chinese Journal of Computational Mechanics*, Vol. 28, No.3, pp 377-382, 2011 (In Chinese).

[6] Zhou M, Sun S D. *Genetic algorithms and its applications*. National Defense Industrial Press, 1999 (In Chinese).

[7] Kramer O, Schlachter U, Spreckels V. An adaptive penalty function with meta-modeling for constrained problems, *Proceedings of 2013 IEEE Congress on*

Evolutionary Computation, Cancun, Mexico, pp.1350-1354, 2013.

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