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# OPTIMAL SEQUENCING IN ATM COMBINING GENETIC ALGORITHMS AND GRADIENT BASED METHODS TO A BILEVEL APPROACH

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## Abstract

*A bi-level algorithm is developed to determine the optimal arrival sequence w.r.t a specific upper level objective function, e.g. minimizing pollutant emissions or maximizing runway throughput. Within a sequence it is assumed that all aircraft are operated optimally (lower level optimization), minimizing individual fuel consumption. However, all aircraft trajectories adhere to constraints imposed by the upper level, such as time and distance separation. The algorithm combines a direct optimal control method for solving the lower level problem with a genetic algorithm, which is applied to the upper level combinatorial problem.*

*The lower level problems are fully discretized by applying a trapezoidal collocation scheme provided by* FALCON.m *[1] and subsequently solved utilizing the interior point NLP solver IPOPT [2]. The discretization is performed during initialization of the algorithm and reused during execution to minimize computational effort. For the upper level problem an efficient mutation operator is introduced, which exploits the information contained within the Lagrange multiplier of the discretized arrival time constraint.*

*The algorithm is validated against a test case scenario comprising five aircraft destined for runway 08L of Munich airport, which enter the terminal maneuvering area within four minutes at multiple way points. Aircraft dynamics are represented by a model derived from BADA 3. The performance of the algorithm allows the application on a pre-tactical level for a limited number of arriving aircraft.*

## 1 Introduction

The sustaining increase of air traffic around the world is driving current air traffic control systems to their limits. The demand resulting from this growing traffic has set the focus of research on methods to enlarge air space capacity. For instance, in 2005, a high-level goal of the Single European Sky ATM Research Joint Undertaking (SESAR JU) has been declared to triple ATM capacity by the year 2020 [3]. Comparable research programs conducted by the USA (NextGen) and Japan (CARATS) include similar goals.

A bottleneck of air traffic systems are the terminal maneuvering areas (TMA) of large aerodromes, especially of hub airports [4]. Increasing congestion dictates that aircraft are guided to the runway in a most efficient manner. Advanced procedures and systems, such as Point Merge and Extended Arrival Managers (E-AMAN) help air traffic control officers to decrease separation and im-

Matthias Rieck holds a scholarship of Munich Aerospace e.V.

prove aircraft sequencing, thus increasing runway throughput.

SESAR's User Preferred Routing and i4D trajectories, as well as comparable solutions within aforementioned research programs, are designed to enable innovative approaches to further increase capacity and mitigate effects on the environment. Free routing enables less constrained trajectories, imposing a higher performance with respect to specific objective functions.

The subject of this paper is to combine

- direct optimal control methods to find optimal trajectories for a specific aircraft (lower level)
- a genetic algorithm to determine the best sequence of all arriving aircraft (upper level).

Each level can be solved with respect to a specific objective function. Hereby, the solution of the upper level imposes constraints on the lower level, e.g. the first time a specific aircraft is allowed to reach a certain point in space, such as the final approach fix. Moreover, the objective function of the upper level depends on the solution of the lower level. A scheme of the overall algorithm is displayed in Fig. 1.

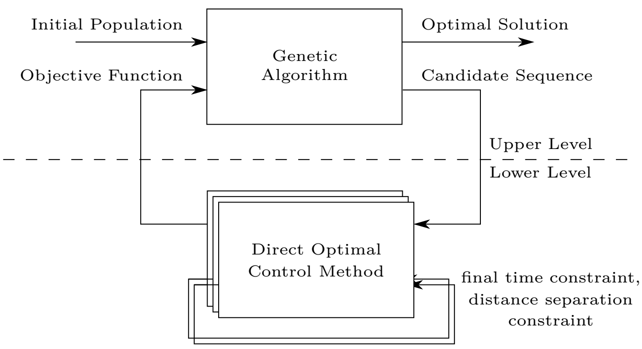


**Fig. 1** Architecture of bi-level optimization algorithm

The application of optimal control theory to optimize trajectories has become a well researched topic within the context of ATM. Different methods have been applied, including indirect approaches [5], dynamic programming [6] and direct methods [7, 8]. In particular, collocation methods have been used to determine optimal approach trajectories for scenarios containing multiple aircraft [9, 10]. To ensure feasible solutions with regard to safety, the implementation of specific constraints is required, e.g. to enforce a distance separation between two aircraft.

Similarly, the combinatorial problem in ATM has been investigated for several years, particularly utilizing evolutionary strategies. In [11], genetic algorithms are applied to solve conflict resolution problems and find an optimal take-off and landing sequence of ten aircraft. An efficient cross-over operator for a multi-runway approach scenario is introduced in [12]. Aircraft scheduling with respect to the wake turbulence category has been performed in [13]. Moreover, genetic algorithms have been applied to similar problems, which can be transferred to air traffic management. For instance, a scheduling problem for a machine process has been solved in [14], respecting deadline constraints.

This work is focused on combining both methods to generate optimal sequences respecting an optimized operation of each aircraft individually. The dynamic aircraft model used within the optimal control problems is introduced in section 2. Subsequently, the algorithms to solve the lower level optimal control and upper level combinatorial problems are introduced in section 3 and 4, respectively. Results from case studies are presented in section 5 before concluding the work in section 6.

## 2 Model

The validity of the solution to an optimal control problem w.r.t. the real system is strongly dependent on the quality of all mathematical models involved. Besides the objective function, modeling of constraints is an important task. These include the dynamic equations that represent the aircraft as well as separation constraints, which are particularly important when solving problems associated with ATM.

## 2.1 Aircraft

The aircraft are modeled as a point mass moving in three-dimensional space. The fuel consumption, the aerodynamic and propulsion forces are derived from the Base of Aircraft Data family 3 (BADA 3), published by EUROCONTROL [15]. Accordingly, the aircraft model comprises seven states as shown in Tab. 1.

The aircraft model is controlled by the kinematic angle of attack, the kinematic bank angle and the thrust lever position. The lift coefficient, which can be used as an input to the aerodynamic force model derived from BADA 3, is calculated using a linear model that depends on the aerodynamic angle of attack. The latter is determined by comparing the coefficients of the transformation matrix

$$\mathbf{M}_{BA} = \mathbf{M}_{BK} \cdot \mathbf{M}_{KO} \cdot \mathbf{M}_{OA}. \qquad (1)$$

The indices denote the body (B), aerodynamic (A), kinematic (K) and NED (O) coordinate frames, respectively, which are defined in [16].

**Tab. 1** States and control inputs for the dynamic aircraft model

| | Description | Min | Max |
|---|---|---|---|
| $\varphi$ | latitude | $-90°$ | $90°$ |
| $\lambda$ | longitude | $-180°$ | $180°$ |
| $h$ | altitude | $5000\,ft$ | $\infty$ |
| $V_K$ | kinematic speed | $V_{min,k}$ | $V_{max,k}$ |
| $\chi_K$ | azimuth | $-180°$ | $180°$ |
| $\gamma_K$ | path climb angle | $-\frac{\pi}{2}$ | 0 |
| $m$ | aircraft mass | 0 | $m_0$ |
| $\alpha_K$ | kinematic angle of attack | $-8°$ | $12°$ |
| $\mu_K$ | kinematic bank angle | $-30°$ | $30°$ |
| $\delta_T$ | thrust lever position | 0 | 1 |

## 2.2 Separation Constraints

While capacity has to be increased to cope with the growing number of aircraft movements, the safety of every flight has to be assured. An important part of maintaining an acceptable level of safety is the satisfaction of separation constraints, both temporal and spatial.

### *2.2.1 Time Separation*

The time separation between two landing aircraft is assured by enforcing a slot time for each aircraft $l$ that represents the first point in time that is eligible for aircraft $l$ to reach the final approach fix:

$$t_{Slot,l} = t_{1,l-1} + \Delta t_{Separation}\left(w_l, w_{l-1}\right). \qquad (2)$$

The time separation at the runway threshold $\Delta t_{Separation}$ was established by ICAO [17] to mitigate hazards caused by the wake vortex turbulences of the preceding aircraft $l-1$. Accordingly, this time separation depends on the wake turbulence categories $w_l$ and $w_{l-1}$ of both aircraft, respectively. Since negligible time delay is expected during the final approach due to elaborate procedures, the time separation is assumed to be valid likewise at the final approach fix.

### *2.2.2 Distance Separation*

Like the time separation, the distance separation is modeled according to ICAO Doc. 4444 [17], which defines minimum separation for the vertical and the horizontal distance, separately. While vertical separation is fixed to $d_h = 1000\,ft$, horizontal distance $d_x$ is dependent on the wake turbulence category of both aircraft involved.

To enforce both the vertical and horizontal separation minima, the maximum norm can be applied

$$\left\| \left( \sqrt{\Delta x^2 + \Delta y^2}, \frac{d_x}{d_h}\Delta h \right) \right\|_\infty - d_x \leq 0, \qquad (3)$$

where $\Delta x$ and $\Delta y$ are the horizontal distance components in north and east direction, respectively. $\Delta h$ represents the altitude separation that has to be scaled by the quotient $\frac{d_x}{d_h}$ to adjust for the lower vertical separation minimum. Using this formulation, the separation inequality constraint takes the form of a cylinder, which must not be violated by another aircraft at any time.

It is assumed that the critical encounters occur between two adjacent aircraft within the sequence. Thus, the constraint is evaluated for the preceding aircraft $l-1$, respectively, where the first aircraft of a sequence is not constrained. In order to determine the distance $(\Delta x, \Delta y, \Delta h)^T$ to the previous aircraft, the optimal trajectory is stored after a solution has been found. Using a cubic spline interpolation between the time points $t_l$, the geodetic position $(\lambda, \phi, h)_l^T$ of the preceding aircraft $l-1$ can be evaluated at all time points. Finally, the geodetic coordinates are transformed to the Cartesian distance. [18]

Because the analytic Hessian of the Lagrangian is used to solve the optimal control problem, the formulation of every constraint must be twice continuously differentiable. Hence, the spatial separation constraint is relaxed to

$$\left(\left(\sqrt{\Delta x^2 + \Delta y^2}\right)^p + \left(\frac{d_x}{d_h}\Delta h\right)^p\right)^{\frac{1}{p}} \leq d_x, \quad (4)$$

which has to be fulfilled in every time step. The parameter $p$ can be adjusted to ensure sufficiently smooth edges of the cylindrical separation constraint model to avoid numerical difficulties.

## 3 Lower Level Optimization

The lower level optimization represents non-cooperative action of aircraft operators who, are determined to minimize the aircraft's operating cost. Yet, constraints imposed by the air traffic control, i.e. the upper level optimization problem, are adhered. The task of optimally operating dynamic systems can be formulated as an optimal control problem.

### 3.1 Optimal Control Problem

The solution to the lower level optimal control problem defines the optimal state trajectory

$$\mathbf{x}_l(t) \in \mathbb{R}^n \quad (5)$$

and the corresponding control history

$$\mathbf{u}_l(t) \in U_l \subseteq \mathbb{R}^m \quad (6)$$

of each aircraft $k$ over the time

$$t \in [t_{l,0}, t_{l,1}] \subseteq \mathbb{R}. \quad (7)$$

from the point of entering the TMA $t_0$ until reaching the final approach fix at $t_1$, where the latter is a parameter of the optimization problem. Optimality is defined by a specific objective function, which can be represented by a Mayer formulation

$$J = -m(t_1) \quad (8)$$

in the context of this paper, where the Mayer term represents the negated final mass of each aircraft. Maximizing the final mass is equivalent to minimizing the fuel consumption according to the dynamic model described in section 2.1, which in turn can be consulted to account for approximate pollutant emissions.

The optimal control problem is subject to a set of constraints, which contain the differential equation governing the dynamic system

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), t). \quad (9)$$

Additionally, boundary conditions for the state trajectory are imposed in the form

$$\Psi(\mathbf{x}(t_0), \mathbf{x}(t_1)) = (\Psi_0, \Psi_1)^T \leq 0. \quad (10)$$

The final boundary conditions correspond to the *MAGAT* final approach fix of runway 08L [19], displayed in Tab. 2:

$$\Psi_1 = \mathbf{x}(t_1) - \mathbf{x}_1 = 0. \quad (11)$$

The velocity $V_{k,app}$ at the final approach fix is dependent on the aircraft type.

**Tab. 2** Final boundary conditions for the final approach fix (*MAGAT*)

| | state | final value |
|---|---|---|
| $\varphi$ | latitude | $48° \ 20' \ 30''$ |
| $\lambda$ | longitude | $11° \ 29' \ 48''$ |
| $h$ | altitude | $5000 \ ft$ |
| $V_K$ | kinematic speed | $V_{k,app}$ |
| $\chi_K$ | azimuth | $82°$ |
| $\gamma_K$ | path climb angle | $-3°$ |

Similarly, the initial boundary conditions are formulated for the entry point of the TMA. Four way points [19] have been used as shown in Tab. 3. Aircraft are assigned to one of these way points, which are set as initial boundary condition for the aircraft's position. Remaining states are chosen to resemble realistic aircraft operation and partially depend on the aircraft type.

**Tab. 3** Initial way points

| way point | latitude | longitude |
|---|---|---|
| *AKANU* | 49° 03′ 06″ | 10° 39′ 30″ |
| *ANORA* | 48° 57′ 00″ | 10° 32′ 54″ |
| *RIXED* | 48° 49′ 48″ | 10° 25′ 06″ |
| *ABGAS* | 48° 35′ 36″ | 10° 23′ 30″ |

Finally, algebraic equalities

$$c_{eq}\left(\mathbf{x}\left(t\right), \mathbf{u}\left(t\right), t\right) = 0 \tag{12}$$

and inequalities

$$c_{ineq}\left(\mathbf{x}\left(t\right), \mathbf{u}\left(t\right), t\right) \leq 0 \tag{13}$$

are introduced as additional constraints to reflect typical limitations in ATM, such as restricting the altitude by introducing an inequality condition for the path climb angle

$$\gamma_K \leq 0, \tag{14}$$

thus disallowing aircraft to climb during the approach (see Tab. 1). Similarly, limitations on the flight performance are introduced, e.g. by limiting the load factor in the z-direction of the body frame

$$0.8 \leq \left(n_z\right)_B \leq 1.2. \tag{15}$$

To ensure feasibility of the bi-level algorithm, the time and distance separation constraints are crucial. The final time is subject to the inequality constraint

$$t_{Slot} - t_1 \quad \leq \quad 0, \tag{16}$$

where $t_{Slot}$ is determined as described in section 2.2.1. Similarly, the distance separation is ensured by introducing the separation constraint modeled in section 2.2.2.

## 3.2 Direct Collocation Methods

Among numerous numerical approaches to solve optimal control problems, which can be found in extensive literature, e.g. [20, 21, 22], a good performance of direct collocation methods has been observed for comprehensive problems [8, 9, 10]. The method implies that the augmented objective function is discretized at a number of collocation points, where state defects are introduced to account for the system dynamics. Along with further constraints a non-linear, constrained optimization problem results with the Lagrangian

$$\begin{aligned} & L\left(\mathbf{x}, \mathbf{u}, \lambda\right) \\ = \; & \varphi\left(\mathbf{x}_1\right) + \sigma^T \psi\left(\mathbf{x}_0, \mathbf{x}_1\right) \\ + \; & \textstyle\sum_{i=0}^{N-1} \lambda_{i+1}^T \left(\mathbf{x}_i + h\Phi\left(t_i, \mathbf{x}_i, \mathbf{u}_i, h\right) - \mathbf{x}_{i+1}\right) \\ + \; & \textstyle\sum_{i=0}^{N} \mu_i^T c\left(t_i, \mathbf{x}_i, \mathbf{u}_i\right), \end{aligned} \tag{17}$$

where $\sigma$, $\lambda$ and $\mu$ are the Lagrange multipliers to the boundary conditions $\psi$, the state defects and the constraints $c$. Comprehensive theory can be found in [7, 20, 22]. As described in section 3.1, the inequality constraint

$$c_{t_f} = t_{Slot,l} - t_1 \leq 0 \tag{18}$$

is implemented within each optimal control problem according to equation (16), where the corresponding Lagrange multiplier $\mu_{t,Slot}$, to the first order, expresses the sensitivity of the objective w.r.t the mentioned constraint [20]:

$$\mu_{t,Slot} \approx -\frac{df}{dc} \tag{19}$$

This Lagrange multiplier is zero, if the constraint is inactive, i.e. if the optimal solution's final time is larger than the assigned first slot time.

In optimal control theory, the collocation method is applied to transform the continuous optimal control problem into a discrete, nonlinear program (NLP). This is achieved by utilizing the FSD Optimal Control Tool for MATLAB (*FALCON.m*) [1], which has been developed at the Institute of Flight System Dynamics of the TU München. The tool offers methods for automatic differentiation to obtain local gradient and Hessian matrices of the objective function, the system dynamics and all constraints [23]. The local gradients and Hessians are subsequently assembled to represent the overall problem. All functions are evaluated and results are passed to an NLP solver in each iteration until convergence. Within this work the interior point NLP solver IPOPT has been used [2].

## 4 Upper Level Optimization

The upper level optimization problem represents the air traffic control officer's (ATCO) task to determine a sequence of arriving aircraft such that a specific objective function is minimized. The sequence is enforced by imposing slot times on each aircraft (see. equation 2). Additionally, path constraints are imposed on each aircraft to maintain a safe separation (see section 2.2.2).

Similarly, the upper level control problem's objective function depends on the solution of all lower level problems. E.g., if the capacity of the considered runway is to be maximized, the objective function can be modeled as the time of reaching the final approach fix of the last aircraft within the sequence. This results in a combinatorial problem, which has been in the focus of mathematicians for several years. [24]

### 4.1 Combinatorial Problem

The combinatorial problem of scheduling $n$ arriving aircraft can be described as finding the optimal permutation

$$a^* = (a_1, a_2, a_3, \dots, a_n) \in \mathbb{A}_n \qquad (20)$$

that represents the global extremal of a specific objective function, where $\mathbb{A}_n$ represents the set of all feasible permutations $(a)_i$, that each represent a specific sequence of arriving aircraft.

Within this paper, no constraints are imposed on the sequence $(a)_i$, which results in a search space of $n!$ possible permutations. Additionally, the computational cost of an upper level objective function evaluation for each candidate sequence $(a)_i$ increases linearly with $n$. Finally, the objective function may be discontinuous over the search space. Hence, an efficient algorithm is needed to generate solutions at a reasonable computational cost.

### 4.2 Genetic Algorithm

Genetic algorithms are well researched methods of numerical optimization, which are designed to resemble the process of natural selection. These are especially suitable for large, noisy and discontinuous search spaces, which are difficult to be handled by traditional optimization techniques [24]. Within every iteration of the algorithm, a population consisting of candidate solutions is updated in a way, that the overall fitness is increased. In most cases, the fitness can be defined by a specific objective function. The algorithm consists of several steps, which are explained in more detail below.

#### *4.2.1 Initialization*

An intelligent initialization of the population is crucial to increase overall performance of the algorithm. This is achieved by choosing initial candidate solutions from the part of the search space, which will most likely contain the optimal solution.

To generate sequences with a high likelihood of optimality, efficient arrival times $\tilde{t}_{i,1}$ for all aircraft are estimated by the efficient cruise speed published in BADA 3 [15]:

$$\tilde{t}_{i,1} = t_{i,0} + \frac{1}{V_{Cr}} \sqrt{(\mathbf{x}_1 - \mathbf{x}_0)^T (\mathbf{x}_1 - \mathbf{x}_0)}. \qquad (21)$$

Subsequently, the aircraft are sorted by their estimated arrival time.

Initial sequences are generated using a Markov Transition matrix as displayed in Fig. 2, where the state is defined as the aircraft landing in the current slot. It is assumed that the most likely transition from a leading aircraft to a following respects the order of efficient arrival times. A normal distribution is assigned around this aircraft to create transition probabilities to different aircraft and normalized over the respective column. An example of the Markov Transition matrix for four aircraft is shown in Tab. 4, where columns represent leading aircraft and rows following. The first column (0) contains the probability that a specific aircraft is the first in leading the sequence.

**Tab. 4** Markov Transition Matrix

| Aircraft | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | 0,46 | 0 | 0,41 | 0,18 | 0,07 |
| 2 | 0,35 | 0,65 | 0 | 0,41 | 0,28 |
| 3 | 0,15 | 0,28 | 0,41 | 0 | 0,65 |
| 4 | 0,04 | 0,07 | 0,18 | 0,41 | 0 |

A sequence is generated be creating a set of random numbers containing $n + 1$ scalars between zero and one, where $n$ represents the number of aircraft. The first element of this set is compared to the cumulated first column of the Markov Transition Matrix. When the cumulated values exceed the random number for the first time, the respective aircraft is chosen. E.g., if a random number of 0.6 is generated, aircraft 2 is assigned to the first slot. Subsequently, the values in the second row are set to zero, all columns are normalized and the process is repeated for the second slot, i.e. column two.

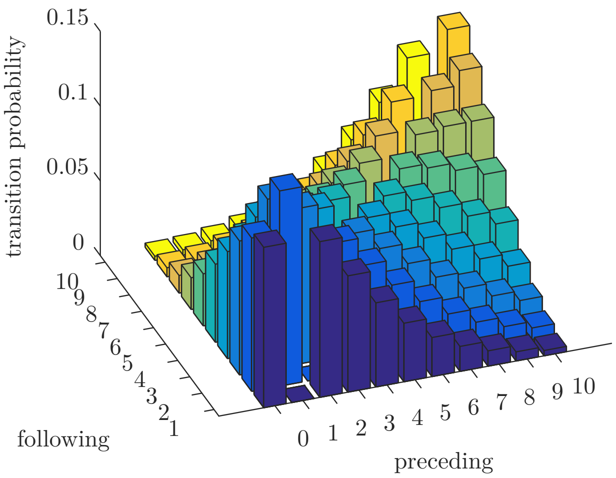


**Fig. 2** Normalized Markov transition matrix for an example including ten arriving aircraft

### *4.2.2 Objective Evaluation*

In every iteration, the objective function is evaluated for each chromosome, i.e. each sequence. The overall objective of the upper level problem is a function of the solutions of all lower level problems. Furthermore, the solution of each lower level problem depends on the trajectory of the preceding aircraft. The objective can thus only be evaluated by solving all optimal control problems subsequently. For efficiency it is thus crucial to avoid any unnecessary computations.

This computational overhead is mitigated by storing common results of the solution process, e.g. if the sequences

$$a_k = (1, 2, 3, \dots)$$

and

$$a_l = (1, 2, 4, \dots)$$

are elements of the population, the optimal control problems for the subsequence

$$a_{kl} = (1, 2)$$

are only calculated once and stored along with any relevant data, such as the optimal trajectories for aircraft one and two. It has to be noted that common subsequences only exist for the first part of a sequence, because the constraint for the first terminal time depends on the aircraft preceding the subsequence, in general. However, the landing time for the first aircraft is unconstrained.

### *4.2.3 Selection*

The selection step is performed by sorting all candidate sequences by the corresponding objective function. Subsequently, the best solutions are selected, while all other candidate sequences are discarded from the current population. The number of selected candidate sequences is determined by a specific fraction, which is a parameter of the algorithm.

### *4.2.4 Mutation*

Within a genetic algorithm, one or multiple genetic operators are applied to generate new chromosomes. Efficient operators are subject to several requirements. Firstly, only feasible chromosomes can be evaluated in the next generation. Moreover, operators must ensure genetic diversity to prevent the algorithm from quickly converging to local minimums, that may not be globally optimal. Finally, the operator's efficiency has a great influence on the overall performance of the algorithm and thus is to be maximized.

Mutation operators can be considered as a straight-forward means of generation feasible chromosomes. For instance, swapping two slots within the sequence preserves unique assignments of aircraft to slots. To ensure basic efficiency by respecting the estimate for the initial sequence, swaps are only allowed between aircraft with adjacent slots. However, the swaps will occur with a uniform probability. Hence, a large number of unreasonable chromosomes will be generated.

As described in section 3.2, the Lagrange multipliers can be seen as a sensitivity of the objective function with respect to small changes in a constraint. Hence, the Lagrange multiplier corresponding to the lower bound of the final time can be considered to augment the mutation. This way, additional intelligence is added to the operator to increase the algorithm's overall performance by applying the following steps:

- sort aircraft by Lagrange multiplier
- compare a random number to a cumulated normal distribution as described in section 4.2.1
- interchange selected aircraft with predecessor

The probability to select the first aircraft is assigned to zero to avoid infeasible swaps. Anyway, since landing time of the first aircraft is unconstrained, the Lagrange multiplier is identically zero and thus receives a low probability of selection in any case.

#### *4.2.5 Termination*

The genetic algorithm can theoretically be terminated by applying several different conditions independently or in combination. These have to be chosen with care to prevent the algorithm from performing unnecessary computational overhead, especially when the fitness evaluations are costly.

However, due to the efficient storing of all required properties as described in section 4.2.2, the computational overhead is minimized in this case once the genetic algorithm has converged to a specific sequence. Hence, it is sufficient to set a terminal condition for the maximum number of iterations.

## 5 Results

The algorithm is tested for a scenario of five aircraft entering the TMA of Munich airport within under four minutes at different way points, as can be seen in Tab. 5. The algorithm is executed on a desktop computer equipped with a Core i7 CPU and 16 GB of RAM running Windows 10. The optimal sequence is found within 31 min and 56.4 s. A total number of 47 optimal control problem are solved due to the efficient storage and reuse of solutions for subsequence. Each optimal control problem is discretized on a grid of 501 points in time, resulting in a overall problem size of 5011 optimization variables and 4503 constraints.

**Tab. 5** Arriving aircraft during the test case

| Number | Type | initial way point | initial time / s |
|---|---|---|---|
| 1 | A320 | *ABGAS* | 45 |
| 2 | A380 | *ANORA* | 90 |
| 3 | B737 | *AKANU* | 135 |
| 4 | B747 | *RIXED* | 180 |
| 5 | A330 | *ABGAS* | 225 |

The trajectories within the optimal sequence are displayed in Fig. 3. The trajectories for aircraft one and 5 are almost identical, indicating a fuel efficient operation. The maximum norm of the Lagrange multipliers to the slot time constraints yields a value of $0.23 \cdot 10^{-6}\,\frac{kg}{s}$, i.e. by moving the slot time, the objective can only be increased by a very small amount.

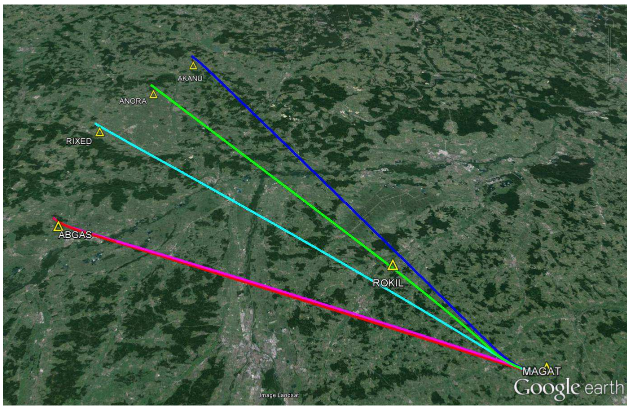


**Fig. 3** Optimal trajectories for a five aircraft scenario on runway 08L of Munich Airport

In comparison, Fig. 4 displays all feasible solutions obtained by the algorithm. It can be clearly seen that several sequences force individual aircraft to create considerable detours to fulfill the separation constraints. However, these solutions have a low fitness value and will quickly be excluded by the algorithm.

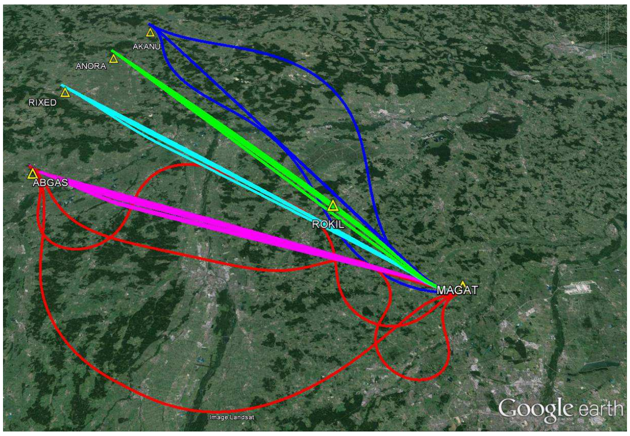


**Fig. 4** All feasible trajectories evaluated during runtime of the algorithm

## 6 Conclusion

A bi-level algorithm has been introduced, which combines direct optimal control methods with a genetic algorithm, resembling aircraft operators (lower level) and ATCO (upper level). An optimal sequence has been found under the assumption that all aircraft are operated in a fuel-optimal way individually and non-cooperatively, however respecting constraints imposed by the upper level algorithm.

The utilization of the analytic Hessian matrix of the problem within the numerical interior point solver (IPOPT), as well as the development of an efficient genetic mutation operator, which is based on the Lagrange multiplier of the slot time constraints, has significantly improved the algorithms performance. The computation time for a limited number of aircraft can be considered reasonable, if the algorithm is utilized within pre-tactical planning. Moreover, the bi-level approach shows a robust behavior and quickly outputs feasible, suboptimal solutions.

The bi-level structure of the algorithm allows for several extensions and modifications. Firstly, the existing combination may be enhanced by additional or alternative genetic operators, such as a cross-over. The functionality may be extended by an optimal runway allocation for multi-runway scenarios.

Secondly, the modular structure allows for the integration of alternative methods, such as the cross entropy method proposed by Rubinstein [25]. Finally, convergence properties may be investigated when additional aircraft are added during the runtime of the algorithm, resembling the fixed horizon of an optimal A-MAN.

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