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Fairing Optimization Design of Wing Structure

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Abstract

This paper introduces a method to realize the fairing design of wing structure. Through the definition of energy function of structure, the structure fairing degree is defined quantitatively. At the same time, analytical derivative formula of energy to design variables has been derived. The quick sensitivity analysis of objective function to design variables and genetic algorithm are combined to realize the optimization. A composite structure application finally proves the method's efficiency.

1 General Introduction

The performance of civil aircraft mainly depends on the accurate aerodynamic design of wing. It is desired that the actual wing shape keep consistent with the ideal shape. In order to obtain the best aerodynamic shape after wing deformation at cruise state, it is necessary to design the jig shape accurately. The jig shape generally is calculated from the ideal cruise shape iteratively. However, for the concentrate load from the engine, which causes the revulsion of shear distribution, the bending moment and torsion moment, it induces the local unfairness of the wing structure. Therefore, it is necessary to optimize the wing structure to keep the wing shape fairing.

Fig. 1.Concentrate load from the engine

2 Optimization Method and Process

"Fairing" is a concept from engineering, which is different from "smooth" of mathematics. If the wing shape is not fairing, it can neither fulfill the aerodynamic design nor the manufacture demand. Smooth often refers to the continuity or geometry continuity of curve or surface which is defined mathematically and has rigid mathematical definition. Fairing includes the property of continuity, but emphasizes more on functional demand, such as aerodynamic, manufacture and dynamic. In the overall fairing, the widely used method is energy method[1-2] that is to make the energy of the line minimized or optimized with the proper constraints.

The wing is usually designed based on the rectilinear generator method. Fairness of the wing can be evaluated by the structure along the spanwise (approximately along the rectilinear generator). The chordwise stiffness is provided by the ribs and the length chordwise is obviously shorter than the spanwise, so the fairing problem along the chordwise aroused by the engine could be ignored.

This paper adopts the energy method to evaluate the fairness of the structure. The energy method of dispersed points has been well studied. However, these methods are often applied in ship, automobile and other products. The application in wing structure is rarely reported.

The method is given by formula 1[3]. The points marked with 1.1,1.2….m.n are type value points and a set of points along the spanwise constitute a type value line marked with 1,2 …m in Fig. 2.

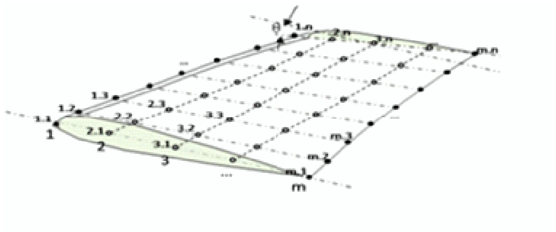

Fig.2.Definition of type value points of the wing

$$E_c = \sum_{i=1}^{n-1} \frac{1}{l_i + l_{i+1}} \| e_{i+1} - e_i \|^2 \quad (1)$$

In equation one, E_c is the energy of the spanwise structure.

$l_i = \|p_i - p_{i-1}\|$ is the length of spanwise vector formed by the adjacent points.

$e_i = \frac{\|p_i - p_{i-1}\|}{l_i}$ is the unit vector of the adjacent points of type value point set.

The more closer to zero of E_c, the more fairing the structure is.

The fairing optimization of the wing structure at cruise state is to minimize the energy function with the minimum weight increment. The question can been described as:

Min E_C

St. $w \leq [w]$

For the composite wing, the design variables are the thickness increment Δx_k of each ply orientation.

This energy optimization is often used in fairing of curve and surface in which the genetic algorithm(GA)[4] is directly applied. This paper derives the analytical derivative form of this equation. The sensitivity of the energy functionis acquired with the analytical equation and with the sensitivity the optimization is carried out based on the genetic algorithm[5].

2.1 Sensitivity Analysis of Energy Function to Design Variables

For the design variables are the increment Δx_k of thickness of each ply orientation of the whole wing skin, the number of the design variables mostly could reach to several thousand. Before the optimization, the problem of the quick analysis of the sensitivity of energy to the design variables should be settled.

To calculate the sensitivity of energy to design variables is to calculate the derivative of formula 2.

$$\frac{\partial E_{cj}}{\partial x_i} = \frac{\partial \sum_{i=1}^{n-1} \frac{\|e_{i+1} - e_i\|^2}{l_i + l_{i+1}}}{\partial x_k} \quad (2)$$

The value of energy function is determined by the location of the type value points. The points will generate displacement with the wing deformation, therefore, the energy function could be transformed to the function of displacement of type value points.

$$E_{cj} = f(\mathrm{u}_i)$$

The displacement of type value points could be taken as the intermediate variable, so the formula 2 can be expanded to formula 3. The problem is decomposed to the derivative of energy function to displacement $\frac{\partial f}{\partial u_i}$ and the derivative of displacement to design variables $\frac{\partial u_i}{\partial x_k}$.

$$\frac{\partial E_{cj}}{\partial x_k} = \frac{\partial f(u_i)}{\partial x_k} = \frac{\partial f}{\partial u_i} \frac{\partial u_i}{\partial x_k} \quad (3)$$

2.2 Derivative of Displacement to Design Variables

Based on the finite element model, the derivative of displacement to design variables could be settled with two methods: finite difference method and direct derivative method.

For those structure of which design variables are smaller than 1000, the finite difference method could be used. The large scale parallel computation is suggested to be applied to accelerate the efficiency.

For the problem with large scale design variables, the direct derivative method has higher efficiency. In the static problem，the displacement to design variables derivative could be stated with formula 4.

$$\frac{\partial U}{\partial x_i} = -K^{-1} \frac{\partial K}{\partial x_i} U \quad (4)$$

Virtual load methodis used to solve this formula so as to avoid the calculation of inverse matrix of stiffness matrix K.

$$\frac{\partial U_r}{\partial x_i} = -U^T \frac{\partial K}{\partial x_i} U_{pse} \quad (5)$$

Here, $U_{pse}$is given by $K^{-1}P_r$.

Equation 5 is the analytical expression of the displacement derivative to design variables. The derivative of displacement to design variables can be calculated by solving the stiffness derivative matrix, displacement vector and virtual displacement vector.

2.3 Derivative of Energy to Displacement

The load causes the bending and torsion deformation of the structure. Therefore, the location of each type value point on the curve will change correspondingly.

The energy of the type value line is determined by the location of the point that it composes of. Taking three points for example, the coordinates of the points are assumed to be $P_{i-1}(x_{i-1}, y_{i-1}, z_{i-1})$, $P_i(x_i, y_i, z_i)$, $P_{i+1}(x_{i+1}, y_{i+1}, z_{i+1})$, so the energy of this type value line can be calculated by this formula.

$$E_c = \frac{1}{l_i + l_{i+1}} \| e_{i+1} - e_i \|^2$$
$$= \frac{1}{l_i + l_{i+1}} \left[\left(\frac{x_{i+1} - x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i} \right)^2 + \left(\frac{y_{i+1} - y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i} \right)^2 + \left(\frac{z_{i+1} - z_i}{l_{i+1}} - \frac{z_i - z_{i-1}}{l_i} \right)^2 \right] \tag{6}$$

The displacement vector of the type value point could be decomposed into three directions. The derivative of energy to displacement equals to the calculation of derivatives in three directions respectively.

$$\begin{cases} \vec{u}_{i-1} = (u_x^{i-1}, u_y^{i-1}, u_z^{i-1}) \\ \vec{u}_i = (u_x^i, u_y^i, u_z^i) \\ \vec{u}_{i+1} = (u_x^{i+1}, u_y^{i+1}, u_z^{i+1}) \end{cases} \tag{7}$$

Taking the above equation into (6), the derivatives can be calculated. For the convenient of expression, the function of energy can be transformed to the following form. The characters A to D are defined in equation 9.

$$E_c = A(B + C + D) \tag{8}$$

$$\begin{cases} A = \frac{1}{l_i + l_{i+1}} \\ B = \left(\frac{x_{i+1} - x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i} \right)^2 \\ C = \left(\frac{y_{i+1} - y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i} \right)^2 \\ D = \left(\frac{z_{i+1} - z_i}{l_{i+1}} - \frac{z_i - z_{i-1}}{l_i} \right)^2 \end{cases} \tag{9}$$

The derivative of energy to each type value point can be further simplified with formula 10 to 13 .

$$\begin{cases} \frac{\partial E_c}{\partial x_{i-1}} = \frac{\partial A}{\partial x_{i-1}}(B + C + D) + A\left(\frac{\partial B}{\partial x_{i-1}} + \frac{\partial C}{\partial x_{i-1}} + \frac{\partial D}{\partial x_{i-1}}\right) \\ \frac{\partial E_c}{\partial y_{i-1}} = \frac{\partial A}{\partial y_{i-1}}(B + C + D) + A\left(\frac{\partial B}{\partial y_{i-1}} + \frac{\partial C}{\partial y_{i-1}} + \frac{\partial D}{\partial y_{i-1}}\right) \\ \frac{\partial E_c}{\partial z_{i-1}} = \frac{\partial A}{\partial z_{i-1}}(B + C + D) + A\left(\frac{\partial B}{\partial z_{i-1}} + \frac{\partial C}{\partial z_{i-1}} + \frac{\partial D}{\partial z_{i-1}}\right) \end{cases} \tag{10}$$

$$\begin{cases} \frac{\partial A}{\partial x_{i-1}} = \frac{1}{(l_i + l_{i+1})^2} \left[\frac{1}{l_i} (x_i - x_{i-1}) \right] \\ \frac{\partial B}{\partial x_{i-1}} = 2 \left(\frac{x_{i+1} - x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i} \right) \left[\frac{l_i^2 - (x_i - x_{i-1})^2}{l_i^3} \right] \\ \frac{\partial C}{\partial x_{i-1}} = 2 \left(\frac{y_{i+1} - y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i} \right) \left[\frac{-(y_i - y_{i-1})(x_i - x_{i-1})}{l_i^3} \right] \\ \frac{\partial D}{\partial x_{i-1}} = 2 \left(\frac{z_{i+1} - z_i}{l_{i+1}} - \frac{z_i - z_{i-1}}{l_i} \right) \left[\frac{-(z_i - z_{i-1})(x_i - x_{i-1})}{l_i^3} \right] \end{cases} \tag{11}$$

$$\begin{cases} \frac{\partial A}{\partial y_{i-1}} = \frac{1}{(l_i + l_{i+1})^2} \left[\frac{1}{l_i} (y_i - y_{i-1}) \right] \\ \frac{\partial B}{\partial y_{i-1}} = 2 \left(\frac{x_{i+1} - x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i} \right) \left[\frac{-(y_i - y_{i-1})(x_i - x_{i-1})}{l_i^3} \right] \\ \frac{\partial C}{\partial y_{i-1}} = 2 \left(\frac{y_{i+1} - y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i} \right) \left[\frac{l_i^2 - (y_i - y_{i-1})^2}{l_i^3} \right] \\ \frac{\partial D}{\partial y_{i-1}} = 2 \left(\frac{z_{i+1} - z_i}{l_{i+1}} - \frac{z_i - z_{i-1}}{l_i} \right) \left[\frac{-(z_i - z_{i-1})(y_i - y_{i-1})}{l_i^3} \right] \end{cases} \tag{12}$$

$$\begin{cases} \frac{\partial A}{\partial z_{i-1}} = \frac{1}{(l_i + l_{i+1})^2} \left[\frac{1}{l_i} (z_i - z_{i-1}) \right] \\ \frac{\partial B}{\partial z_{i-1}} = 2 \left(\frac{x_{i+1} - x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i} \right) \left[\frac{-(z_i - z_{i-1})(x_i - x_{i-1})}{l_i^3} \right] \\ \frac{\partial C}{\partial z_{i-1}} = 2 \left(\frac{y_{i+1} - y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i} \right) \left[\frac{-(y_i - y_{i-1})(z_i - z_{i-1})}{l_i^3} \right] \\ \frac{\partial D}{\partial z_{i-1}} = 2 \left(\frac{z_{i+1} - z_i}{l_{i+1}} - \frac{z_i - z_{i-1}}{l_i} \right) \left[\frac{l_i^2 - (z_i - z_{i-1})^2}{l_i^3} \right] \end{cases} \tag{13}$$

In the same way, $\frac{\partial E_c}{\partial m}, \frac{\partial E_c}{\partial m_{i+1}} (m = x, y, z)$ can also be expressed with the form of equation 10 to 13. It can be found that after the derivative, the energy function include the displacement of each type value point, which needs once static analysis to get the current displacement of each type value point. This set of equation is derived with three points and obviously, the derivative of P_{i-1}, P_i,P_{i+1} are different from each other. For those lines that include more than three type value points, the Ec derivative function remains the same form. The value of Ec can be calculated by summation of the derivative of all sets of type value point that each set includes the adjacent three points.

For each type value line $\{P_1, P_2, \ldots, P_i, \ldots P_{n-1}, P_n\}$, the first point P_1 comes up only once in the Ec derivative formula. The corresponding values of the second point P_2 obviously occur twice in Ec derivative formula. Similarly，the last point P_n will be counted once and point Pn-1 twice. Except for these four points, the left points will be counted three times. The derivative of different type value points are listed in table one. According to the location of each point in the line, the derivative of each point can be calculated.

Table 1. Derivative formula of type value points

point	the corresponding derivative formation
P_1	$\left(\frac{\partial E_c}{\partial x_{i-1}}, \frac{\partial E_c}{\partial y_{i-1}}, \frac{\partial E_c}{\partial z_{i-1}}\right)$
P_2	$\left(\frac{\partial E_c}{\partial x_{i-1}}, \frac{\partial E_c}{\partial y_{i-1}}, \frac{\partial E_c}{\partial z_{i-1}}\right) + \left(\frac{\partial E_c}{\partial x_i}, \frac{\partial E_c}{\partial y_i}, \frac{\partial E_c}{\partial z_i}\right)$
P_i	$\left(\frac{\partial E_c}{\partial x_{i-1}}, \frac{\partial E_c}{\partial y_{i-1}}, \frac{\partial E_c}{\partial z_{i-1}}\right) + \left(\frac{\partial E_c}{\partial x_i}, \frac{\partial E_c}{\partial y_i}, \frac{\partial E_c}{\partial z_i}\right) + \left(\frac{\partial E_c}{\partial x_{i+1}}, \frac{\partial E_c}{\partial y_{i+1}}, \frac{\partial E_c}{\partial z_{i+1}}\right)$
P_{n-1}	$\left(\frac{\partial E_c}{\partial x_i}, \frac{\partial E_c}{\partial y_i}, \frac{\partial E_c}{\partial z_i}\right) + \left(\frac{\partial E_c}{\partial x_{i+1}}, \frac{\partial E_c}{\partial y_{i+1}}, \frac{\partial E_c}{\partial z_{i+1}}\right)$
P_n	$\left(\frac{\partial E_c}{\partial x_{i+1}}, \frac{\partial E_c}{\partial y_{i+1}}, \frac{\partial E_c}{\partial z_{i+1}}\right)$

A set of sample points are used to identify the derivative stated above. These points are showed in Fig.3. The transverse axis stands for the coordinate x and the longitudinal for y. From the distribution of the points, some primary and qualitative conclusions can be gotten.

Firstly, the longitudinal value of point 1(points are sequenced from left to right) keeps invariable or decreases will contribute to the fairness of the curve. Secondly, appropriate increment on y value of the third point will be good for the fairness. Thirdly, the fifth point has the maximum y value. The increment on the y value of the fifth point will make the fairness of this curve getting worse.

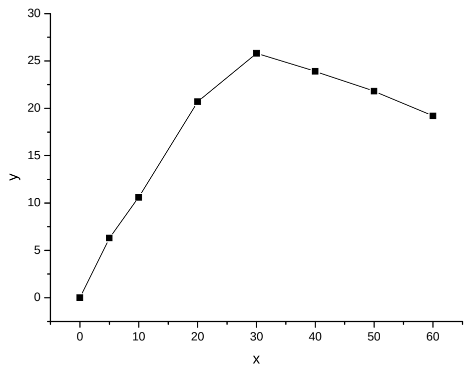

Fig.3.Distribution of sample points

Fig.4 displays the result of sensitivity calculated through derivative and finite difference. It shows that the sensitivity calculated from the derivatives equation are nearly the same with the finite difference. It should be mentioned that the less the energy is, the more fairness of the curve becomes. Therefore, the more negative of the derivative becomes, the more contribution it will make to the decrease of the energy with the same increment on its value.

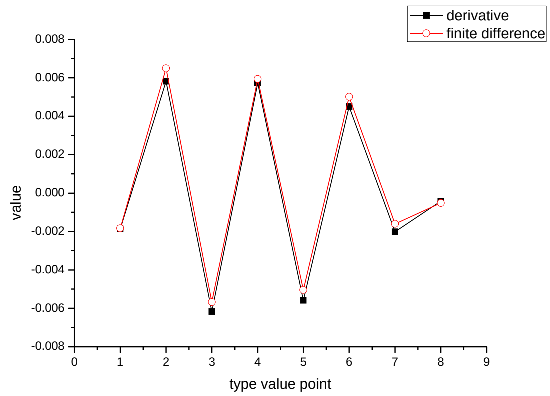

Fig.4. Sensitivity calculated by different methods

4 Application

Based on the method described above, a composite wing is optimized. The objective is to minimize the fairing energy function with the minimum weight increment.

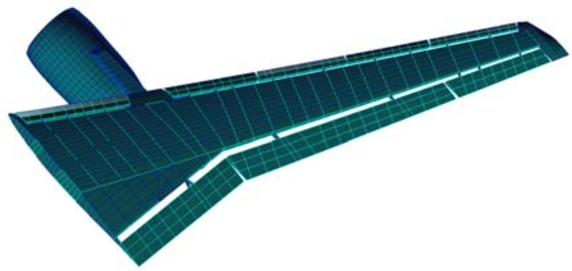

Fig. 5.Finite element of wing model

According to the finite element model, 15 type value lines are selected. The type value points are the crossover points of the ribs and stringers showed in Fig. 6. From the trailing edge to leading edge, the stringers are cut off for the geometry changes. The number of type value point decreases accordingly.

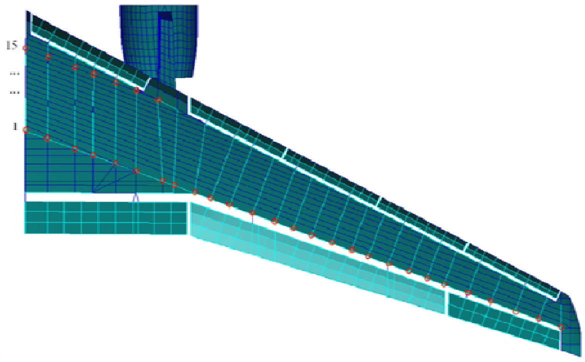

Fig. 6.Part of the type value points

From formula 1, the Ec value quantizes the variation of the direction of the adjacent vector constituted by the type value points. The less direction of the adjacent vector changes, the more fairing of the line is. Through primary analysis, it is found that line 14 and 15 has the highest energy. Fig.7 shows the first three points of line 14 and 15. It shows that the second type value point of both lines occurs negative deformation, which causes changes on the direction of these two vectors. The first three points are located near the wing root. For the negative load from the engine, it generates greater displacement than the aerodynamic load which causes the local unfairness. Considering the line 15 has higher energy, the energy of line 15 is chosen as the objective function in the optimization and the energy of line 14 is only checked after the optimization.

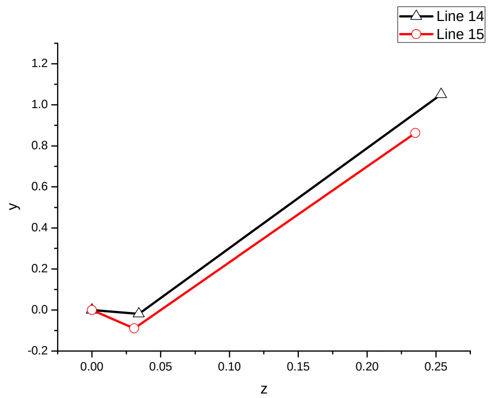

Fig.7.Displacement of part type value points of line 14&15

The skin elements nearby the engine are chosen as the design elements. The corresponding thickness of each ply orientation of the skin is the design variable. The total number of design variables of the model showed in Fig. 5 sums up to 1040. Combining the optimization method and the specific model, the analysis process is formed.

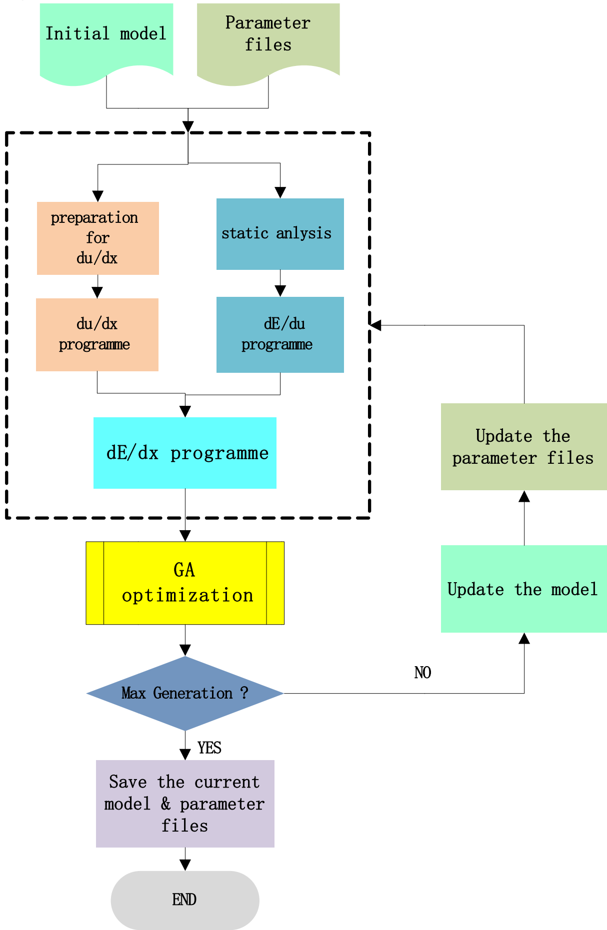

Fig.8.Process of optimization

The result of sensitivity analysis of the first step is showed in Fig. 9. It manifests that the most sensitive design variables are -45 degree ply orientation which are located in the wing root.

With the optimization processes, the sensitivity of the design variables changes accordingly. Fig.10 shows the comparison of different steps. It is the minimum optimization, so the less the sensitivity is, the better the design variables is to the objective value. It can be seen that the minimum of sensitivity becomes larger and the difference between the design variables comes conspicuous.

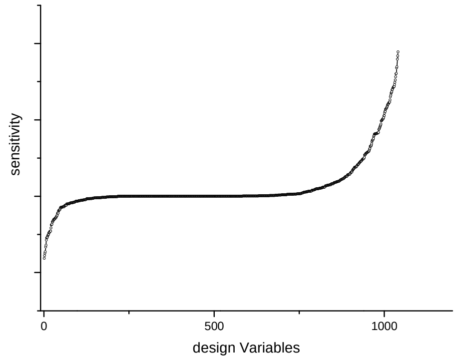

Fig. 9.Result of sensitivity analysis of the first step

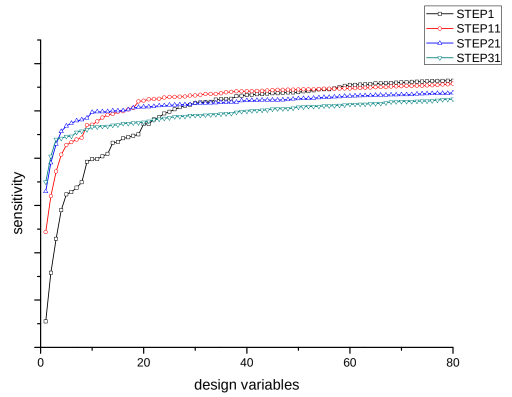

Fig.10. Part result of sensitivity analysis of different step

According to the result of sensitivity analysis, the genetic algorithm is adopted to optimize. In order to carry out the process automatically, c programming language is used to realize the relative mode showed in Fig. 8 and script is used to run the whole process. The curve in Fig. 11 shows the optimization results. It can be derived that per unit increment on weight could reduce 0.7 percent value of objective function.

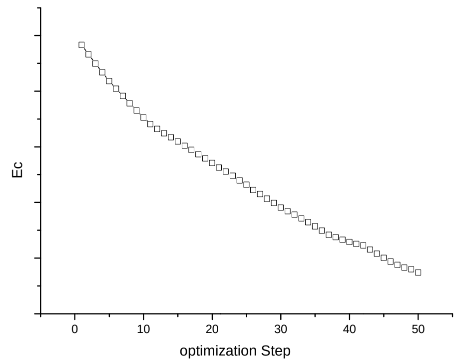

Fig.11. Objective result of optimization

The following picture shows the skin thickness of the wing before and after optimization. It has been discussed that the high energy is caused by the negative displacement near the root. Through the optimization, the thickness of the skin located between the root and engine are increased obviously which strengthen the stiffness of the skin so as to decrease the displacement. At the same time, the skin between engine and the tip rarely changes which indicates that the structure weight is added on the most efficient area.

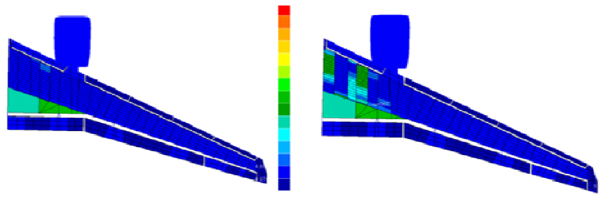

Fig.12. Skin thickness before and after optimization

The energy decrease of line 15 is caused by the changes of the two vector's direction constituted by the first three points. The original and optimized locations of the first three points are displayed in Fig.13. The transverse axis stands for the deformation along axis of z (chordwise) and the longitudinal axis for the deformation along the axis of y(spanwise).

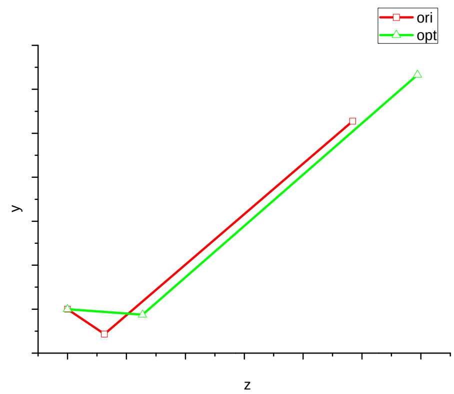

Fig.13. Displacement of part type value points before and after optimization

5 Conclusion

Through the specific wing structure optimization, it is proved that the method and process stated in chapter 2 is efficient. The optimization design makes the energy become lower than the initial state.

The analytical derivative formula of energy to design variables has been derived so as to realize quick sensitivity analysis.

In this application, the fairing problem caused by the engine load is limited. The structure fairing of the wing for manufacture and aerodynamic is not so obviously. However, this research has a new try in fairing design. It is a method which can be used for relative research and application.

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