IMPACT ANGLE CONTROL GUIDANCE CONSIDERING GRAVITY EFFECT

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Abstract

This paper proposes an impact angle control guidance law with the terminal acceleration constraint considering the gravity effect. To develop the law, the desired line-of-sight (LOS) angle to satisfy the terminal impact angle and acceleration constraints is defined. The guidance command is derived by letting the actual LOS angle converge to the desired LOS angle at the terminal stage of the homing. When designing the law, the gravity effect is considered so that the desired terminal constraints can be satisfied accurately under the realistic pitch-axis engagements. Furthermore, user-desired zero terminal acceleration can also be achieved directly because an additional gravity compensation is not required. For the performance evaluation, numerical simulation is conducted and the result demonstrates that the proposed guidance law achieves the desired impact angle and zero terminal acceleration under the gravity effect.

1 Introduction

In many engagement problems for guided weapon systems such as missiles or guided bombs, impact angle control is an important duty in virtue of its capability to modulate the terminal collision geometry. For anti-ground or anti-ship missiles, particularly, to control the impact angle enables a missile to attack a weak spot of a target so that the lethality of weapon systems could be maximized.

For these reasons, there has been much research for advanced guidance laws to achieve the predetermined impact angle using various approaches [1-10]. One of the initial efforts that impose the impact angle constraint to the terminal guidance problem is devised by [1]. In [2], an optimal guidance law with a terminal flight path angle constraint is proposed against a maneuvering target with a known trajectory. In [3], a time-varying bias is added to the conventional proportional navigation guidance (PNG) law in order to fulfill the homing and impact angle control simultaneously. The authors in [4] presents an impact angle constrained law with a generalized formation of minimizing the flight energy for an arbitrary system order. In [5], a PNG-based composite guidance law is developed for achieving an impact angle of any direction against a stationary target. The work in [6] suggests an accurate guidance law to satisfy the impact angle constraint against a maneuvering target based on Lyapunov theory.

Under the explained laws in [1-6], which only consider the terminal angle constraint, the desired impact angle can be achieved for ideal engagements. However, for some realistic engagements in which the maneuverable acceleration of the missile is limited, the fulfilment of the desired duty might be failed because of the possibility of command saturation. From this point of view, to minimize the terminal acceleration command of the missile is also important in order to ensure the stable interception. Additionally, in order to maximize the lethality of the missile warhead, it is required to make the terminal command converge to zero because the angle of attack (AOA) is generally proportional to the normal acceleration produced by lift force. In this respect, the impact angle control guidance laws considering the terminal acceleration constraint have been proposed [7-9].

The authors in [7] solve an optimal problem that assigns the impact angle and terminal acceleration constraints to the guidance problem against a fixed target. The law in [8] is derived by minimizing an energy cost function inversely weighted by time-to-go so that smooth convergence of the guidance command is achieved at the terminal stage of the homing. For the same purpose, an impact angle control guidance law composed of a polynomial function of time-to-go is proposed in [9].

In the pitch channel command, however, zero terminal acceleration cannot be expected under these laws because the generated command has to be combined with the gravity compensation term. Hence, a guidance law generates zero terminal command without the need of the gravity compensation is required for more effective terminal performance.

In this paper, we propose an impact angle control guidance law with the terminal acceleration constraint considering the gravity effect. To satisfy the terminal constraints accurately under the realistic engagement situations, two dimensional pitch-axis engagement dynamics involving the gravity effect are considered at the design stage. From the formulated dynamics, the desired line-of-sight (LOS) angle to satisfy the terminal impact angle and acceleration constraints is defined as a function of time-to-go. Then, the guidance command is derived in order to make the actual LOS angle converge to the desired LOS angle at the terminal stage of the homing. An estimation of the time-to-go required for the proposed law is calculated based on the trajectory solution of the overall closed loop.

The remainder of this paper consists of the following. In section 2, the guidance problem about impact angle control is formulated with considering the gravity effect. In section 3, the impact angle and terminal acceleration constrained guidance law is designed with considering the gravity effect on the pitch channel. In section 4, numerical simulations are conducted to demonstrate the performance of the proposed law. In section 5, the concluding remarks are provided.

2 Problem formulation

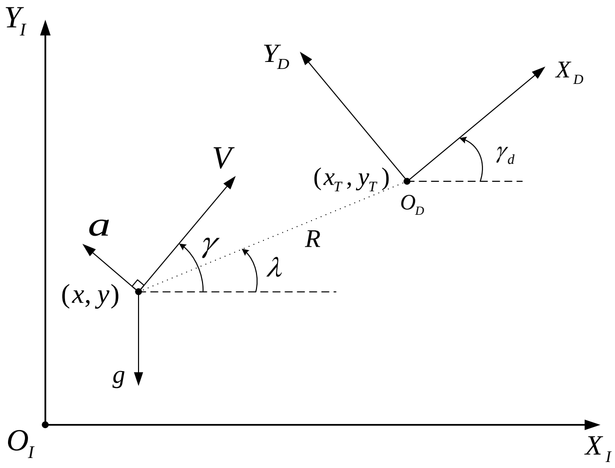

Figure 1. Engagement geometry

Consider the two dimensional engagement geometry about an interceptor and a stationary target in the inertial longitudinal frame $X_I O_I Y_I$ as shown in Fig. 1. The positions of the interceptor and the target are represented by (x, y) and (x_T, y_T) respectively. The relative range and the line-of-sight (LOS) angle between each other are denoted by R and λ, and the speed, normal acceleration and the flight path angle of the interceptor are represented by V, a and γ respectively. In addition, the desired impact angle and the acceleration of gravity are expressed as γ^d and g respectively. Then, the equations of motion about the interceptor in the inertial frame are given by

$$\dot{x} = V\cos\gamma \tag{1}$$

$$\dot{y} = V\sin\gamma \tag{2}$$

$$\dot{\gamma} = \frac{a - g\cos\gamma}{V}. \tag{3}$$

Because the variation of the speed within a short range such as terminal guiding phase can be negligible when the interceptor moves fast, the speed of the interceptor is assumed to be constant in this study.

In our problem, the guidance objective is to intercept the stationary target at the desired impact angle with zero terminal acceleration, which is expressed as

$$x(t_f) = x_T, \quad y(t_f) = y_T \tag{4}$$

$$\gamma(t_f) = \gamma_d, \quad a(t_f) = 0 \tag{5}$$

where t_f is the final time of the homing.

In order to derive and analyze the guidance law easily, the linearized engagement dynamics under the small angle assumption is required. In this regard, we can reformulate the dynamics of (1) ~ (3) in the desired impact angle frame $X_D O_D Y_D$ for the accurate linearization under the effective small angle assumption [9]. As shown in Fig. 1, the frame $X_D O_D Y_D$ has the origin on the target position and is rotated γ_d from the inertial frame $X_I O_I Y_I$. The redefined position and flight path angle with respect to the frame $X_D O_D Y_D$ are given by

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} \cos\gamma_d & -\sin\gamma_d \\ \sin\gamma_d & \cos\gamma_d \end{bmatrix} \left\{ \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_T \\ y_T \end{bmatrix} \right\} \tag{6}$$

$$\bar{\gamma} = \gamma - \gamma_d, \tag{7}$$

where the variables in the desired impact frame are denoted as $\overline{(\cdot)}$. Then, the linearized dynamics under the small angle assumption of redefined flight path angle is expressed as

$$\dot{\bar{y}} = V\bar{\gamma} \tag{8}$$

$$\dot{\bar{\gamma}} = \frac{a - g\cos\gamma_d}{V}. \tag{9}$$

The terminal constraints corresponding to the frame $X_D O_D Y_D$ is given by

$$\bar{y}(t_f) = 0, \quad \bar{\gamma}(t_f) = 0 \tag{10}$$

$$a(t_f) = 0. \tag{11}$$

Based on the linearized dynamics in (8) and (9), a guidance law to satisfy the desired terminal constraints in (10) and (11) is developed in next section.

3 Impact angle control guidance law considering gravity effect

In this section, the desired line-of-sight (LOS) angle is defined to satisfy the terminal impact angle and acceleration constraints. The guidance law is derived so that the actual LOS angle converges to the desired LOS angle at the terminal stage of the homing.

3.1 Line-of-sight angle shaping

In order to deal with the desired constraints in (10) under the zero terminal acceleration constraint in (11), we define a dynamics that assumes zero acceleration input as follows:

$$\dot{\bar{y}} = V\bar{\gamma} \tag{12}$$

$$\dot{\bar{\gamma}} = -\frac{g\cos\gamma_d}{V}. \tag{13}$$

From the terminal constraints in (10), the dynamics in (12) ~ (13) has a unique solution as

$$\bar{y}^*(t) = -\frac{1}{2} g\cos\gamma_d t_{go}^2 \tag{14}$$

where $t_{go} = t_f - t$ is the remaining time-to-go until the interception. From (14), the desired LOS angle in the desired impact frame can be defined as follows:

$$\bar{\lambda}_d(t) \simeq -\frac{\bar{y}^*(t)}{R(t)\left(=Vt_{go}\right)} = \frac{g}{2V}\cos\gamma_d t_{go}. \tag{15}$$

Now, it can be said that the desired terminal constraints in (10) can be satisfied under the zero acceleration if the missile moves with satisfying $\bar{\lambda} = \bar{\lambda}_d$ at the terminal stage of the homing. In order to fulfill this property about LOS, let us assume that the actual LOS angle is expressed as a following form:

$$\bar{\lambda}(t) = c_m t_{go}^m + c_n t_{go}^n + \bar{\lambda}_d(t) \tag{16}$$

where c_m and c_n are coefficients, and m and n are real numbers satisfying

$$m > n > 1. \tag{17}$$

Then, the actual LOS angle $\bar{\lambda}$ converges to the desired LOS angle $\bar{\lambda}_d$ at the terminal stage of the homing. Therefore, the desired terminal constraints in (10) and (11) can be achieved under the guidance law that makes the missile move with satisfying (16). In the next subsection, the values of coefficients c_m and c_n are determined based on boundary conditions, and the guidance law is derived to satisfy the LOS angle condition (16).

3.2 Design of the guidance law

From (16), the state variables and the guidance command are expressed as

$$\bar{y}(t) = -c_m V t_{go}^{m+1} - c_n V t_{go}^{n+1} - \frac{g}{2}\cos\gamma_d t_{go}^2 \quad (18)$$

$$\bar{\gamma}(t) = (m+1)c_m t_{go}^m + (n+1)c_n t_{go}^n + \frac{g}{V}\cos\gamma_d t_{go} \quad (19)$$

$$a(t) = -m(m+1)c_m V t_{go}^{m-1} - n(n+1)c_n V t_{go}^{n-1} . \quad (20)$$

The initial boundary conditions about state variables are given by

$$\bar{y}(t=0) = \bar{y}(0), \quad \bar{\gamma}(t=0) = \bar{\gamma}(0) . \quad (21)$$

Then, the coefficients c_m and c_n are determined as

$$c_m = \frac{(n+1)}{(m-n)Vt_f^{m+1}}\bar{y}(0) + \frac{1}{(m-n)t_f^m}\bar{\gamma}(0) + \frac{(n-1)\cos\gamma_d}{2(m-n)Vt_f^{m-1}} g \quad (22)$$

$$c_n = -\frac{(m+1)}{(m-n)Vt_f^{n+1}}\bar{y}(0) - \frac{1}{(m-n)t_f^n}\bar{\gamma}(0) - \frac{(m-1)\cos\gamma_d}{2(m-n)Vt_f^{n-1}} g . \quad (23)$$

Substituting (22) and (23) into (20) at the initial time $t=0$ leads to

$$a(0) = -(m+1)(n+1)\frac{\bar{y}(0)}{t_f^2} - (m+n+1)\frac{V\bar{\gamma}(0)}{t_f} - \frac{(mn-m-n-1)}{2} g\cos\gamma_d . \quad (24)$$

By recalculating the coefficients at each time, the feedback guidance command is obtained as

$$a(t) = -(m+1)(n+1)\frac{\bar{y}(t)}{t_{go}^2} - (m+n+1)\frac{V\bar{\gamma}(t)}{t_{go}} - \frac{(mn-m-n-1)}{2} g\cos\gamma_d . \quad (25)$$

Here, the state variables $\bar{y}$ and $\bar{\gamma}$ can be rewritten as

$$\bar{y} \simeq R\bar{\lambda} = Vt_{go}(\lambda - \gamma_d) \quad (26)$$

$$\bar{\gamma} = \gamma - \gamma_d . \quad (27)$$

As a result, from (25) ~ (27), we have

$$a(t) = \frac{V}{t_{go}}\left\{(m+1)(n+1)\lambda - (m+n+1)\gamma - mn\gamma_d\right\} - \frac{(mn-m-n-1)}{2} g\cos\gamma_d . \quad (28)$$

Note that the proposed command in (28) requires the information about V, λ, γ and t_{go}. Here, the time-to-go t_{go} cannot be directly measured by a sensor or seeker unlike V, λ and γ. In the next subsection, the time-to-go corresponding to the proposed law is estimated based on the trajectory solution.

3.3 Calculation of the time-to-go estimation

In common guidance problems, the time-to-go can be estimated by calculating the remaining length of curved path over the missile speed. To calculate the remaining length, it is required to obtain the closed-loop solution of the state variables. By yielding the command in (28) as a guidance input, the closed-loop solution of the state variables is given by (18) and (19). Then, from substituting

$t_{go} = \left((R_0 - s)/V\right)^m$ where $s \in [0, R_0]$ into (18) and (19), we obtain

$$\begin{aligned}\bar{\gamma}(s) &= (m+1)c_m\left(\frac{R_0 - s}{V}\right)^m + (n+1)c_n\left(\frac{R_0 - s}{V}\right)^n \\ &\quad + \frac{g}{V}\cos\gamma_d\left(\frac{R_0 - s}{V}\right) \\ \bar{\lambda}(s) &= c_m\left(\frac{R_0 - s}{V}\right)^m + c_n\left(\frac{R_0 - s}{V}\right)^n \\ &\quad + \frac{g}{2V}\cos\gamma_d\left(\frac{R_0 - s}{V}\right)\end{aligned} \quad .(29)$$

If the difference between the LOS and the missile velocity vector is assumed to be sufficiently small, the total length of missile path can be approximated as

$$\begin{aligned}L_{go}(0) &= \int_0^{R_0}\sqrt{1 + \left(\bar{\gamma}(s) - \bar{\lambda}(s)\right)^2}\,ds \\ &\approx \int_0^{R_0} 1 + \frac{1}{2}\left(\bar{\gamma}(s) - \bar{\lambda}(s)\right)^2 ds\end{aligned} \quad (30)$$

By substituting (29) into (30) and replacing the initial values with the current state variables for the continuous feedback, we can obtain the length of the remaining path $L_{go}(t)$. From $t_{go} = L_{go}(t)/V$, we can obtain the time-to-go estimation as follows:

$$\begin{aligned}t_{go} = \frac{R}{V}\Bigg\{1 + \frac{1}{2}\Bigg(&\frac{P_2}{P_1}\bar{\lambda}^2 + \frac{P_3}{P_1}\bar{\gamma}^2 + \frac{P_4}{P_1}\bar{\lambda}\bar{\gamma} \\ &+ \frac{P_6}{6P_5}\frac{R^2 g^2}{V^4}\cos^2\gamma_d - \frac{P_7}{P_5}\frac{Rg\bar{\gamma}}{V^2}\cos\gamma_d \\ &+ \frac{P_8}{P_5}\frac{Rg\bar{\lambda}}{V^2}\cos\gamma_d\Bigg)\Bigg\}\end{aligned} \quad (31)$$

where

$$\begin{aligned}P_1 &= (2m+1)(2n+1)(m+n+1) \\ P_2 &= 2m^2n^2 + m + n + 1 \\ P_3 &= 2mn + m + n + 1 \\ P_4 &= 2(m+1)(n+1) \\ P_5 &= (m+2)(n+2)(2m+1)(2n+1)(m+n+1) \\ P_6 &= (m-1)^2(n-1)^2(3mn + 2m + 2n + 2) \\ P_7 &= (m-1)(n-1)(3mn + 2m + 2n + 2) \\ P_8 &= 2(m-1)(m+1)(n-1)(n+1)(mn-1)\end{aligned} \quad .(32)$$

4 Simulation results

To evaluate the performance of the proposed law, numerical simulations are carried out in this section. For the simulations, an engagement against a fixed ground target located at the initial distance of 10 km is considered. Specific values of the simulation parameters are listed in Table 1. For the performance comparison, another law called TPG (time-to-go polynomial guidance) in [9] is also simulated. All the simulations are performed until the relative range R is less than 0.5 m.

Figures 2 and 3 show the simulation results for various impact angles under TPG. The results for impact angles of -30, -60, -90° are represented by the circle-marked line, square-marked line and triangle-marked line respectively. It can be seen that the interception at the desired impact angle is achieved for all cases as illustrated by Fig. 2. However, figure 3 shows that the acceleration command does not converge to zero except when $\gamma_d = -90°$. It is because the law requires the gravity compensation although the law is designed to achieve zero terminal acceleration for ideal engagements.

Table 1

Parameters	Values
Initial missile position $(x(0), y(0))$	(0,6) km
Stationary target position $(x_T(0), y_T(0))$	(8,0) km
Initial missile flight path angle $\gamma(0)$	0°
Missile speed V	250 m/s
Desired impact angle γ_d	-30, -60, -90°

The results under the proposed law are illustrated by Figs. 4 and 5. Similarly, the results for impact angles of -30, -60, -90° are denoted by the circle-marked line, square-marked line and triangle-marked line respectively. Like TPG, the proposed law also satisfies the interception at the desired impact angle for all cases as shown in Fig. 4. Unlike TPG, however, figure 5 shows that the proposed law generates zero command at the terminal stage of the homing for all cases. It is because the proposed law does not require the gravity compensation unlike TPG. Therefore, smaller terminal AOA can be expected under the proposed law in comparison with other laws.

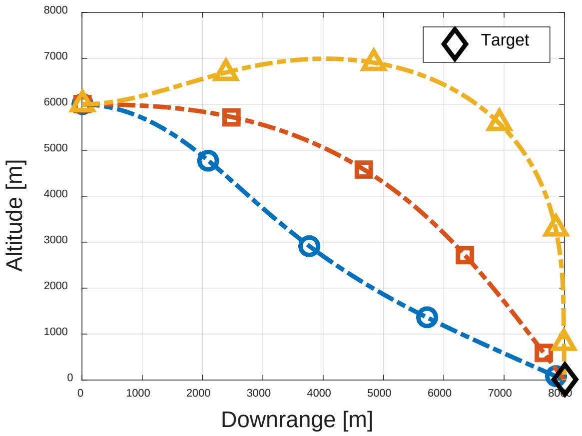

Figure 2. Missile trajectories for various desired impact angles under TPG

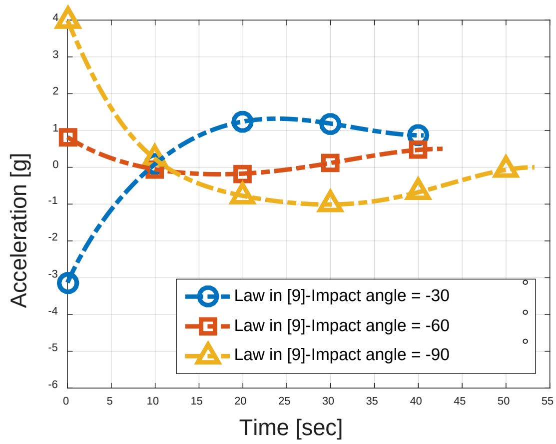

Figure 3. Acceleration command histories for various desired impact angles under TPG

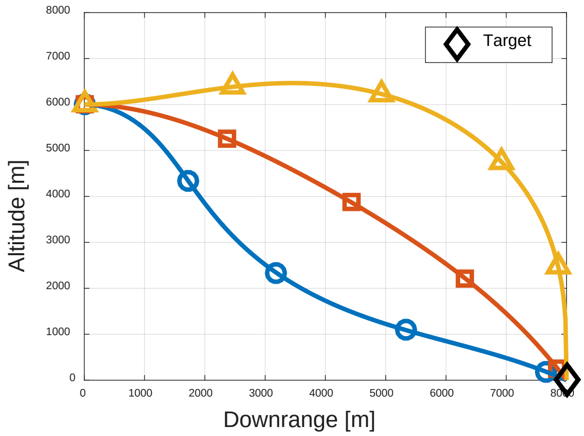

Figure 4. Missile trajectories for various desired impact angles under the proposed law

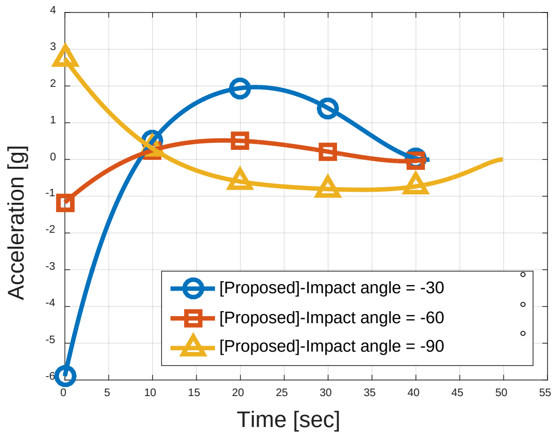

Figure 5. Acceleration command histories for various desired impact angles under the proposed law

5 Conclusion

This paper develops a terminal impact angle and acceleration constrained guidance law considering the gravity effect. The proposed law can satisfy the desired terminal constraints accurately under the realistic engagement because the gravity effect is considered at the design stage. In addition, the law does not require the gravity compensation when implemented to the pitch channel, which makes it possible to achieve user-desired zero terminal acceleration. Numerical simulation results also confirm this property about terminal acceleration, so the terminal performance of the guided weapon is expected to be increased under the proposed law.

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