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GENERAL APPROACH TO THE ANALYSIS OF THE ASYMMETRICALLY LOADED AIRPLANE FLIGHT LIMITATIONS

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Abstract

General approach to the analysis of asymmetrically loaded airplane limitations within the terminal flight phases is presented in this paper. The goal of the analysis is to supplement the flight envelope limitations of the symmetrically loaded airplane, providing the increase of the airplane use safety.

The analysis method is based upon the concept of the control surfaces authority, their capability to generate the aerodynamic controlling moments, and the definition of both, the asymmetric loads that need to be compensated and maneuvers that are to be performed. As the flight envelope limitations of the symmetrically loaded airplane positions the airplane lift coefficient function to the boundary of linear variation of the angle of attack, flight control deflections required to compensate the asymmetry and perform demanded maneuver can be considered as additive.

The factors defining available control surfaces authority are the dynamic pressure and structure limited deflections. Then, for the known structure limited deflections, required maneuver and form of asymmetric load, it is possible to define limiting flight parameters in the form of airspeed and lateral wind velocity component. Analysis method is applicable throughout whole airplane design and life cycle, from the initial design stages up to the end-of-life decommissioning.

The analysis problem can be solved numerically for any combination of asymmetric airplane loads. Two presented examples are the inherently asymmetric single propeller airplane (Figure 1.) and jet combat airplane (Figure 2.) asymmetric under wing store.

1 Introduction

The symmetric airplane is the basic form used in the analysis of the flight dynamics, but in practice this form is rather an exemption than the norm. There are numerous criteria for the classification of the airplane asymmetry, the general one being the asymmetry permitted in the airplane production.

Figure 1. LASTA-95 single propeller airplane.

However, in the airplane flight dynamics analysis, the asymmetries are classified from the flight control point of view. They are either intended flight control and/or airplane configuration or undesired disturbance. The airplane flight control is executed through airplane form modification by deflecting control surfaces. The control surface capability to generate the desired airplane motion can be defined as the control surface authority. The control surface authority depends upon dynamic pressure and control surface deflection and is of aerodynamic nature.

Figure 2. YUROM-Eagle jet combat airplane.

Then, the other significant classification type is the physical nature of the asymmetry, related to the capabilities of the control surface authority to compensate them. Therefore, regarding the physical nature of the compensation capability, the asymmetries can be classified as aerodynamic and non-aerodynamic ones.

Airplane asymmetries of the aerodynamic nature are scalable to the control surface authority throughout flight envelope, requiring straightforward approach in the analysis and compensation methods. The analysis of the asymmetries of the non-aerodynamic nature require particular attention within the terminal flight phases, the area of the flight envelope with the low dynamic pressure and reduced control surface authority.

The bulk of the non-aerodynamic nature asymmetries can be classified into two categories. The first comprise propulsive group generated ones, the typical example being the inherently asymmetric single propeller airplane (Figure 1.). The second comprise of combat airplane (Figure 2.) asymmetric under wing.

The requirement to develop the general method to analyze, within the terminal flight phases, flight limitations of the airplane with asymmetric loads of non-aerodynamic nature is generally solved in [1], with particular presentations in [2] and [3].

The aim of this paper is to emphasize the properties and applicability of airplane asymmetric load analysis method, not the flight limitations of the particular asymmetrically loaded airplanes.

2 Problem Definition

Within the terminal flight phases basic flight envelope limitations of the symmetrically loaded airplane are the paramount ones. MIL-F-8785C requirements defines the relations between stalling V_S, minimal V_m and landing V_l airspeed as $V_{\mathrm{m}} = 1.1V_S$ and $V_l = 1.15V_S$, with take-off wing load factor being practically the same as in the landing.

Terminal flight phases wing load positions the lift coefficient on the boundary of linear variation relative to the angle of attack. Therefore, any supplementing limitations must be positioned at the higher airspeed regime, resulting in the lower lift coefficient value.

The analysis method of the asymmetrically loaded airplane is based upon the assumption that lateral flight control surfaces must have sufficient authority for two basic functions, to compensate the asymmetry and perform the required maneuver.

The basic method requirement to supplement flight limitations of the symmetrically loaded airplane, positions its area of application at the values of the lift coefficient that are within the area of linear variation relative to the angle of attack. That fact permits that the deflections of the lateral flight controls demanded by the compensation of the asymmetric loads and requirement to perform maneuver can be treated as the additive.

Terminal flight phases demand of airplane flight path alignment with runway axis defines final requirement applied for the mathematical models used in the method. The method is to provide the approximate results, with exactness within the same range as the input data used in the airplane computational design. The basic idea in the method application is that appropriate approximation is significantly better than none approximation.

Maneuver flight path positions airplane velocity vector $\vec{V}_K$ in the plane vertical to the runway axis, with course angle Ψ_K and sideslip angle relative to inertial space β_K being $\psi_K = -\beta_K = 0$. Lateral wind v_{aw} is treated as the disturbance, defining disturbance sideslip angle $\beta_{aw} = \arctan\left(v_{aw}/V_K\right)$. Dynamic pres-sure is simply

$$Q = \frac{1}{2}\rho\left(V_K^2 + v_{aw}^2\right) \tag{1}$$

Maneuver function of the airplane lateral motion within the terminal flight phases is derived from the flying qualities criteria defined in MIL-F-8785C requirements [4]. Although not in the official use, document [4] is selected as it provides numerical values for the maneuver dynamics requirements. In the consideration of the demanded maneuver airplane roll motion is presented by the one-dimensional model in the form

$$\ddot{\varphi} I_x = QS_w b_w \left(\frac{b}{2V} C_{lp} p + C_{l\delta_l} \delta_l\right) \tag{2}$$

For each of the flight phases this specification defines, as from the initial conditions of the stationary trimmed flight, the required bank angle change φ_{rq} and time T_{ac} required to achieve it after the initiation of the abrupt input of the roll command.

The maneuver is performed by abrupt deflection of the roll control, reaching at the moment $T_{(m)}$ value $\delta_{l(m)}$ and holding it. The variation of roll control in time $\delta_l\left(t\right)$ is

$$\delta_l\left(t\right) = \begin{cases} \dfrac{\delta_{l(m)}}{2}\left(1 - \cos\dfrac{\pi}{T_{(m)}} t\right) & \Leftrightarrow \quad 0 \le t \le T_{(m)} \\ \delta_{l(m)} & \Leftrightarrow \quad t > T_{(m)} \end{cases} \tag{3}$$

The value of $\delta_{l(m)}$ that satisfies the condition $\varphi\left(T_{ac}\right) = \varphi_{rq}$ for $\varphi\left(t\right)$ in equation (2) is defined numerically. Integrating (2) as the set of two first order differential equation generates the function $F_\varphi = f\left(T_{ac}, T_{(m)}, \delta_{l(m)}\right) - \varphi_{rq} = 0$. The human operator properties define $T_{(m)}$, and shooting function F_φ is used to determine the maneuver roll command defection $\delta_{l(m)}$ by varying it.

Stationary lateral-directional equations are derived with contributions of the asymmetric loads due to propulsion group effects and under wing stores.

Propulsor generates lateral force $F_{ya(P)}$ and roll $L_{(P)}$ and yaw $N_{(P)}$ moment components, consisting of propulsor own and positional part. Single propeller airplane reactive moment M_{re} is the roll moment $L_{(P)}$ dominant part.

Under wing stores generate aerodynamic and gravitational loads. Aerodynamic loads of the under wing stores are the function of their form and position. They expressed as the increments to side force ΔC_{ya}, roll ΔC_{la} and yaw ΔC_{na} aerodynamic coefficient. Process of their calculation is explained in [1].

In stationary lateral-directional equations contribution of asymmetric gravitational load of under wing stores is simply given by

$$L_g = g\cos\varphi \sum_{i=1}^{n} m_{(i)} r_{y(i)} \,. \tag{4}$$

The stationary lateral-directional equations of the asymmetrically loaded airplane are

$$\begin{aligned} & QS\left(C_{y\beta}\beta_p + C_{y\delta_l}\delta_{l(c)} + C_{y\delta_n}\delta_{n(c)} + \Delta C_{ya}\right) + \\ & + mg\sin\varphi + F_{ya(P)} = 0 \\ & QSb\left(C_{l\beta}\beta_p + C_{l\delta_l}\delta_{l(c)} + C_{l\delta_n}\delta_{n(c)} + \Delta C_{la}\right) + \\ & + L_{a(P)} + L_g = 0 \\ & QSb\left(C_{n\beta}\beta_p + C_{n\delta_l}\delta_{l(c)} + C_{n\delta_n}\delta_{n(c)} + \Delta C_{na}\right) + \\ & + N_{a(P)} = 0 \end{aligned} \tag{5}$$

By introducing substitution

$$F = mg\sin\varphi \tag{6}$$

system (5) becomes linear one. The solution of linearized equation is the straightforward one, providing the values of roll $\delta_{l(c)}$ and yaw $\delta_{n(c)}$ control deflections and bank angle φ.

3 Problem solution presentation

Problem solution is given through the method based upon the comparison of available and required lateral flight control surface authority for given set of flight conditions. Required control surface authority is defined by maneuver $\delta_{l(m)}$ and compensation $\delta_{l(c)}$, $\delta_{n(c)}$ control deflections.

The correlation between lateral flight control angle deflections and control surface angle deflections is defined by NASA convention defining as positive the control surface deflection that generates positive increment of local airfoil lift. The relation between right δ_{a_r} and left δ_{a_l} aileron angle deflection and roll control angle deflection δ_l is

$$\delta_l = \frac{\delta_{a_r} - \delta_{a_l}}{2} \tag{7}$$

Likewise, relation between yaw control angle deflection δ_n and rudder deflection δ_r is

$$\delta_n = \delta_r \tag{8}$$

Ultimate control surface deflection is defined by hard stop at the surrounding airplane structure, whereas true deflection limits are determined by flight control system kinematics. Aileron and rudder deflection limits define through relations (7) and (8) structural limits of roll $\delta_{l(s)}$ and yaw $\delta_{n(s)}$ flight controls. Due to elasticity and non-linearity, true usable roll $\delta_{l(\lim)}$ and yaw $\delta_{n(\lim)}$ flight control limits applied in the analysis method are reduced by factors k_l and k_n

$$\delta_{l(\lim)} = k_l \delta_{l(s)}; \quad \delta_{n(\lim)} = k_n \delta_{n(s)} \tag{9}$$

Determination of safe flight boundary conditions for defined limiting lateral flight control $\delta_{l(\lim)}$ or $\delta_{n(\lim)}$ and known case of asymmetric load is made by the numerical analysis.

The required total roll control deflection is the sum of maneuver and compensation deflections

$$\begin{aligned} &\delta_l = \delta_{l(c)} + \left|\delta_{l(m)}\right| sign\left(\delta_{l(c)}\right) \\ &\delta_{l(c)} = \delta_{l(c)}\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) \\ &\delta_{l(m)ex} = \delta_{l(m)}\left(\rho\left(H\right), V_K, v_{aw}\right) \end{aligned} \tag{10}$$

The parameters vector contains asymmetric loads and air density $\rho(H)$, being of the form $\mathbf{p}_{asm} = \left[\rho(H), F_{ya(P)}, L_{a(P)}, N_{a(P)}, L_g, \Delta C_{ya}, \Delta C_{la}, \Delta C_{na}\right]^T$.

Then, the first equation in (10) is defined for unique set of flight conditions, airplane dynamics properties and asymmetric loads. For the given airplane, the boundaries of the safe flight on the basis of the roll command limitations are defined by

$$\begin{aligned} &\Delta\delta_l = \delta_l\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) - \delta_{l(\lim)} = 0 \\ &\Delta\delta_l\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) = \Delta\delta_l\left(x, \mathbf{p}'_{asm}\right) = 0 \end{aligned} \tag{11}$$

Either V_K or v_{aw} can be selected as the independent variable x used to numerically find the zero of $\Delta\delta_l$, the other one forming with $\mathbf{p}_{asm}$ parameter vector $\mathbf{p}'_{asm}$. The problem can be reversed in order to determine asymmetric load that can be held by roll control, like single propeller reactive torque M_{re} or under wing store weigh moment L_g.

The yaw control consists of compensating deflection only

$$\delta_n = \delta_{n(c)}\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) \tag{12}$$

As in previous case, the boundaries of the safe flight on the basis of yaw command limitations are

$$\begin{aligned} &\Delta\delta_n = \delta_{n(c)}\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) - \delta_{n(\lim)} = 0 \\ &\Delta\delta_n\left(V_K, v_{aw}, \mathbf{p}_{asm}\right) = \Delta\delta_n\left(x, \mathbf{p}'_{asm}\right) = 0 \end{aligned} \tag{13}$$

As the independent variable x used to numerically find the zero of $\Delta\delta_n$ either V_K or v_{aw} can be selected.

Equation (5) presents the special case of the airplane lateral-directional mode of motion, the motion system being multivariable one with roll δ_l and yaw control δ_n as inputs. Therefore, the roll control boundary of the safe flight envelope defined by the function (11) is satisfied if and only if compensating yaw control $\delta_{n(c)}$ that is solution of equations (5) fulfills the condition

$$\delta_{n(c)} \le \delta_{n(\lim)} \tag{14}$$

Likewise, the yaw command boundary of the safe flight envelope defined by the function (13) is satisfied if and only if the total roll command deflection consisting of maneuvering part $\delta_{l(m)}$ and compensating part $\delta_{l(c)}$ fulfills the condition

$$\delta_l = \delta_{l(c)} + \left|\delta_{l(m)}\right| sign\left(\delta_{l(c)}\right) \le \delta_{l(\lim)} \qquad (15)$$

In determination of the numerical solution of the safe flight boundaries is used either the pair of equations (11) and (14) or pair of equations (13) and (15).

Airplane asymmetric load analysis method is incorporated into the `FORTRAN` program software solution `'VMINLAT'`. Software language selection has been defined by the requirement that asymmetric load analysis program is included into the standard software package for airplane dynamics analysis and flight control system design that has been developed and is in use in the Military Technical Institute. `'VMINLAT'` program uses standard existing set of the subroutines for program initialization, input/output file manipulation and airplane data base handling. Zero finding and numerical integration subroutines are from "Numerical Recipes" package [5].

Program `'VMINLAT'` is designed to enable various variation of input variables.

4 Examples of analysis method

Two typical examples of the airplane asymmetric load analysis method are presented in this paper. The first is related to the asymmetrically loaded combat airplane (Figure 2.) and the second to the single propeller piston engine training airplane (Figure 1.). In both cases airplane landing flight phase is the objective of the analysis.

4.1 Combat airplane with asymmetric under wing store

Due to the shorter life-cycle and lesser development costs, the integration of new weaponry is the continuous process throughout the life-cycle of combat airplane. Among the obligatory requirements is the analysis of the airplane dynamics with asymmetric under wing stores, either as the regular or as the failure state of combat airplane use. The requirement for this analysis made significant contribution to the definition of the tasks and functions of the method presented herein.

The object of the analysis is the referent configuration of the YUROM-Eagle single seat combat airplane. The airplane mass is $m_a = 7245.6[kg]$ and X-axis moment of inertia $I_{xx} = 8142.1[kgm^2]$. Referent wing area is $S = 26.0[m^2]$ and wing span is $b_w = 9.3048[m]$.

The under wing store on the inner line (IL) is the generic one, with diameter $D = 0.400[m]$, length ahead of the wing leading edge $l_l = 0.9067[m]$ and mass $m = 465.0[kg]$, as defined in [1].

The calculation of aerodynamic load contribution is based upon the method given in [6], whereas their values are calculated in [1] and presented in [2].

Minimal $V_{\min} = 69.522[m/s]$ and landing speed $V_l = 72.682[m/s]$ for airplane with IL wing store are defined from pilot notes [7]. These two speeds are functions of dynamic pressure, depending of both V_K and v_{aw}, such as $V_l = \sqrt{V_{l(K)}^2 + v_{aw}^2}$, but for the simplicity sake, it considered as the constant in the analysis. Although the maximal landing speed is not defined in the pilots notes, practical pilot experience set maximal landing speed to $V_{l\max} = 320.0[km/h]$.

Roll and yaw control limiting deflections are $\delta_{l(\lim)} = \pm 12.27^{\circ}$ and $\delta_{n(\lim)} = \pm 18.75^{\circ}$ [8].

Airplane configuration corresponds to the failure state, therefore, the analysis is performed for the airplane being within the permissible flight enve-lope, as per definitions in [4]. Time to acquire required bank angle φ_{rq} is within Level II and Level III boundaries, with value $T_{ac} = 1.65[s]$. The analysis results are presented in the Figure 3.

Safe flight boundaries in Figure 3 are outlined by bold lines. It is evident that the safe flight boundaries are determined by the yaw control authority.

Furthermore, safe flight area is asymmetric and shifted to the lateral wind component from the loaded wing side, meaning that airplane landing is practically possible at high airspeeds with lateral wind component into the asymmetrically loaded wing.

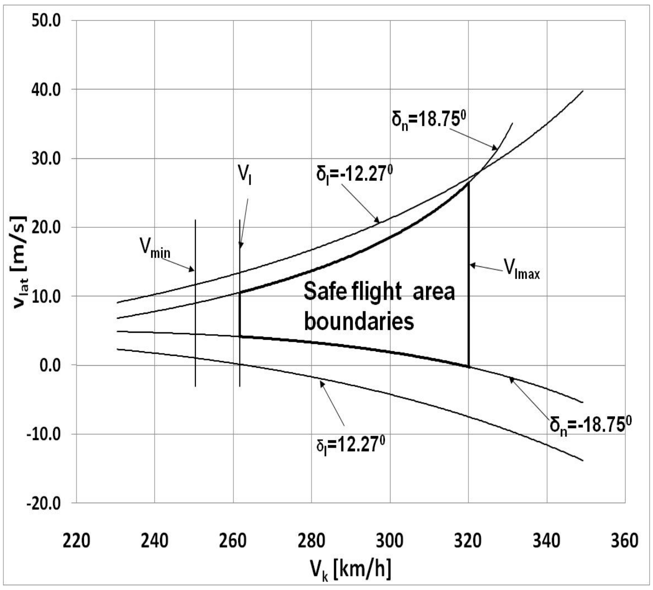

Figure 3. Safe flight boundaries of the combat airplane with asymmetric under wing load.

4.2 Single propeller piston engine airplane

Due to the propeller reactive torque, single propeller airplane is inherently asymmetric. Two factors are emphasizing this airplane property. The first is that in practically all cases initial student pilot training is performed on the single propeller airplanes (Figure 1.), the reference to the airplane-propeller interaction being the paramount one to the student. The second one is the general down-sizing tendency in aviation that reintroduced medium to high power single propeller airplane in advanced training and close support applications.

The object of the presented analysis is LASTA-95 light, single propeller piston-engine, training airplane in take-off configuration, with mass $m = 1064.0[kg]$, moment of inertia $I_x = 1323.7[kgm^2]$, wing area $S_w = 12.9[m^2]$ and span $b_w = 9.015[m]$.

Engine reactive torque at full power is $M_{re(l)} = = -778.09[Nm]$, pushing the left wing down.

The analysis has been done at the flight height $H = 200.0[\mathrm{m}]$, within the airspeed range from just below symmetric flight minimal airspeed to the terminal flight phase's upper limit of $V = 55.0[\mathrm{m/s}]$.

Permitted flight control stretch of 20%, as defined in [9] is used to determine reduction factors k_l and k_n. Yaw control available deflection is further reduced by 15% to take in the account wind gust compensation. The value of analyzed limiting roll flight control $\delta_{l(\lim)} = -11.36^{\mathrm{o}}$ is the reduced one appearing during the design phase only. The value of the limiting yaw control is $\delta_{n(\lim)} = \pm 19.5^{\mathrm{o}}$.

Analysis has been done for the airplane within the service flight envelope, corresponding to the Level II flying qualities and time to acquire required bank angle φ_{rq} of $T_{ac} = 1.8[s]$.

The analysis results are presented in the Figure 4.

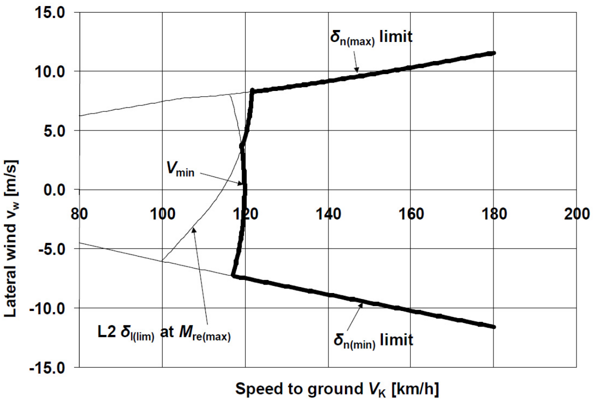

Figure 4. Safe flight boundaries of single propeller airplane in take-off.

Safe flight boundaries in figure 4 are outlined by bold lines. It is evident from figure 4 that safe flight boundaries by minimal airspeed $V_{\min}$ as defined by dynamic pressure, roll control authority limit determined by reactive moment M_{re} at maximal power and yaw control authority limit. Asymmetry of safe flight boundary due yaw control limit is not significant. There is evident and significant asymmetry of the boundary due to roll control authority limit, shifted to the left wing unloaded by the lateral wind v_{aw} blowing into it. Up to approximately $v_{aw} = 4.0[m/s]$ safe flight boundary defines dynamic pressure determined $V_{\min}$. Upward from $v_{aw} = 4.0[m/s]$ safe flight boundary is defined by roll control authority limit determined by requirement to compensate reactive moment M_{re} at maximal power.

4 Conclusions

Provision of the results for two different cases of non-aerodynamic related asymmetric loads confirmed the general applicability of presented method for the analysis flight limitations of the asymmetrically loaded airplane. Software implementation of the method enables various combinations of asymmetric loads, as well as the selection of various variables that are defined by the lateral-directional flight controls authority limitations. Analysis cases that have not been presented in this paper are dealing with the problem of determining optimal take-off power of the medium to high power single turbo-propeller airplane, task with increasing importance due to downsizing tendencies in the application of combat training airplanes.

Simplicity of use and integration of the method software implementation within the existing software package for airplane dynamics analysis and control system design is operational benefit. Furthermore, the analysis method is applicable throughout the airplane life cycle, from the initial conceptual design up to the end of the operational life and decommissioning.

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