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MINIMUM-TIME FORMATION USING OPTIMAL SOLUTIONS OF SINGLE FIXED-WING AIRCRAFT

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Abstract

This paper describes a procedure to obtain solutions of a minimum-time formation which has different start times for fixed-wing aircraft. Generally, this problem is formulated as an optimal control problem by treating all the aircraft as one group. However, it is solved by each aircraft with small communication because this method is based on optimal solutions of single aircraft. Numerical results show that the solutions by the general formulation correspond to those by this method.

1 Introduction

Formation flight has many advantages. For example, the time for search can be shortened and the continuity of a task can be brought against some troubles. The configuration depends on the given mission. Thus formation problem has an important role. In a mission, the aircraft has to shape the configuration as soon as possible, so the minimum-time formation problem is treated in this paper.

This problem is formulated as non-linear optimal control problem. Some researchers focus on optimal formation problem of aircraft [1-7] and also treat minimum-time formation problem [1-5]. Almost all of them treat the aircraft as one group i.e. one system. Robin et al [7] proposed a decentralized algorithm, however minimum-time problem dose not be treated.

Moreover, most researches do not deal with start time differences formation but simultaneous formation. On the other hand, in observation or other missions, the aircraft may pass through a given point at some interval and then shape the formation. In other words the minimum-time formation, which has start time differences, is considered. The aircraft which passes ahead has longer time. That is to say there are different control sections of the aircraft. To deal with this matter, the control sections are changed to [0, 1] by normalized time, and then all aircraft can be treated as on system as well as simultaneous formation problem. However, this general formulation means centralized control. The calculation costs increase rapidly with increasing the number of aircraft. When the loss of a part of the central function is considered, a part of the function needs to be deconcentrated.

In this paper, fixed-wing aircraft are considered. At the end of the formation, the relative distances between two aircraft about the final direction of forward movement are specified and the other states of aircraft are specified. This is a kind of problem with a free terminal constraint [1].

Thus, this paper introduces a method using optimal solutions of single aircraft for minimum-time formation with start time differences for fixed-wing aircraft. The problem can be solved by each aircraft with small communication. Numerical results show that the solutions by the general formulation correspond to those by this method.

The outline of this paper is as follows. In Section 2, the problem formulation is shown. Then, some numerical results are shown and analysis of the solutions is given in Section 3. The method is introduced in Section 4. In Section 5, verifications of this method are made. Finally, conclusions are given in Section 6.

2 Problem Formulation

2.1 Equations of Motion

The dimensionless point-mass equations of motion for the single aircraft are written as follows.

$$\dot{M} = Tw - SwM^2(C_{D0} + KC_L^2) - \sin\gamma \tag{1}$$

$$\dot{\gamma} = SwMC_L \cos\phi - \frac{1}{M}\cos\gamma \tag{2}$$

$$\dot{\psi} = SwMC_L \frac{\sin\phi}{\cos\gamma} \tag{3}$$

$$\dot{\xi} = M\cos\psi\cos\gamma \tag{4}$$

$$\dot{\eta} = M\sin\psi\cos\gamma \tag{5}$$

$$\dot{\zeta} = M\sin\gamma \tag{6}$$

where M, γ and ψ corresponds to Mach number, flight path angle and azimuth angle respectively. ξ, η and ζ are dimensionless x, y and reference height h. The definitions of states are shown in Fig. 1. Sw, C_{D0} and K are reciprocal of dimensionless wing, zero-lift drag coefficient and efficiency factor. Tw, C_L and ϕ are thrust to weight ratio, lift coefficient and bank angle. There are limits placed on these control variables and they are expressed as

$$Tw_{\min} \le Tw \le Tw_{\max} \tag{7}$$

$$C_{L\min} \le C_L \le C_{L\max} \tag{8}$$

$$\phi_{\min} \le \phi \le \phi_{\max} \tag{9}$$

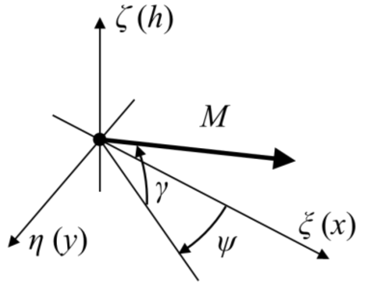

Fig. 1 Definitions of states

2.2 General Formulation as Optimal Control Problem

The criterion is given by

$$J = \tau_f \tag{10}$$

where τ_f is dimensionless terminal time. Then, eq. (10) is minimized.

Now, the states and inputs variable is expressed as

$$\boldsymbol{x}_i = [M_i,\ \gamma_i,\ \psi_i,\ \xi_i,\ \eta_i,\ \zeta_i]^{\mathrm{T}} \tag{11}$$

$$\boldsymbol{u}_i = [Tw_i,\ C_{Li},\ \phi_i]^{\mathrm{T}} \tag{12}$$

and Eqs. (1)-(6) are rewritten by

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \tag{13}$$

where $i = 1, 2, \ldots, N$.

To the beginning, the minimum-time simultaneous formation is formulated as an optimal control problem. The 'simultaneous formation' means that each aircraft starts a formation at the same time. In other word, the control sections of each aircraft are the same, i.e. it is [0, τ_f]. The dynamics of each aircraft are treated as one large system. Denoting the states as $\boldsymbol{x} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N]^{\mathrm{T}}$ and the inputs as $\boldsymbol{u} = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N]^{\mathrm{T}}$, the dynamics are rewritten by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{u}_1) \\ \boldsymbol{f}_2(\boldsymbol{x}_2, \boldsymbol{u}_2) \\ \vdots \\ \boldsymbol{f}_N(\boldsymbol{x}_N, \boldsymbol{u}_N) \end{bmatrix} \tag{14}$$

Secondly, the minimum-time formation with start time differences is formulated. The control sections are [τ_{DLi}, τ_f], i.e. they are different for each aircraft. To deal with this, normalized time is applied [8]. The new independent variable $\tilde{\tau} = \tau_f - \tau_{DLi}$ is applied, and then Eq. (14) can be rewritten by

$$\frac{d\boldsymbol{x}}{d\tilde{\tau}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \tau_f) = \begin{bmatrix} (\tau_f - \tau_{DL1})\boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{u}_1) \\ (\tau_f - \tau_{DL2})\boldsymbol{f}_2(\boldsymbol{x}_2, \boldsymbol{u}_2) \\ \vdots \\ (\tau_f - \tau_{DLN})\boldsymbol{f}_N(\boldsymbol{x}_N, \boldsymbol{u}_N) \end{bmatrix} \tag{15}$$

and the control sections are [0, 1]. Thus, dynamics can be treated as one system as well as simultaneous problem.

Boundary conditions are set as follows. The initial states specified and the conditions are given by

$$M_i(\tau_{DLi}) = M_{0i} \tag{16}$$

$$\gamma_i(\tau_{DLi}) = \gamma_{0i} \tag{17}$$

$$\psi_i(\tau_{DLi}) = \psi_{0i} \tag{18}$$

$$\xi_i(\tau_{DLi}) = x_{0i}\, g/a^2 (= \xi_{0i}) \tag{19}$$

$$\eta_i(\tau_{DLi}) = y_{0i}\, g/a^2 (= \eta_{0i}) \tag{20}$$

$$\zeta_i(\tau_{DLi}) = h_{0i}\, g/a^2 (= \zeta_{0i}) \tag{21}$$

where M_{0i}, γ_{0i}, ψ_{0i}, x_{0i}, y_{0i} and h_{0i} are states at beginning of the formation. g and a are acceleration of gravity and speed of sound respectively. As with terminal states, the states expect *x* axis are specified As for *x*, the relative distances between two aircraft are specified, because fixed-wing aircraft has the trust along the direction movement but does not have active force along the backward direction. The terminal conditions are given by

$$M_i(\tau_f) = M_f \tag{22}$$

$$\gamma_i(\tau_f) = \gamma_f \tag{23}$$

$$\psi_i(\tau_f) = \psi_f \tag{24}$$

$$\xi_1(\tau_f) - \xi_j(\tau_f) = Rx_{1j}\, g/a^2 \tag{25}$$

$$\eta_i(\tau_f) = y_{fi}\, g/a^2 (= \eta_{fi}) \tag{26}$$

$$\zeta_i(\tau_f) = h_{fi}\, g/a^2 (= \zeta_{fi}) \tag{27}$$

where M_f, γ_f, ψ_f, y_{fi} and h_{fi} are states at end of the formation. Rx_{1j} is relative distance of terminal *x* between 1st aircraft and *j*th aircraft. For simplicity, the constraints about relative distances among aircraft during the formation are not considered. The parameters in this paper are summarized in Table 1. This formulation means centralized control.

Table 1 Parameters of aircraft

Parameter	Value	Parameter	Value
$Tw_{\min}$	0	$Tw_{\max}$	0.5
$C_{L\min}$	0	$C_{L\max}$	1.0
$\phi_{\min}$	-180 [deg]	$\phi_{\max}$	180 [deg]
Sw	8.302	C_{D0}	0.020
K	0.20		

3 Analysis of Results

3.1 Numerical Results

Some numerical results of two aircraft from the general formulation are shown in this Section.

Firstly, Initial and terminal conditions are shown in Table 2 and 3 respectively ($T_{DLj} = \tau_{DLj} \cdot a/g$).

The numerical results are shown in Fig. 2 to 3. The red and blue lines are histories of 1st aircraft and 2nd aircraft respectively. Fig.2 shows the optimal trajectories. For convenience of showing, we assumed that 2nd aircraft fly at a constant speed (at the initial *M*) for T_{DL2}. Fig.3 shows inputs. As we can see, both *Tw* are bang-bang (maximum in this case) inputs.

Table 2 Initial Conditions

j	M	γ [deg]	ψ [deg]	x [ft]	y [ft]	h [ft]	T_{DLj} [s]
1	0.7	0	90	0	0	0	0
2	0.7	0	90	0	0	0	2

Table 3 Terminal Conditions

j	M_f	γ_f [deg]	ψ_f [deg]	y [ft]	h [ft]	Rx_{1j} [ft]
1	0.7	0	0	0	0	-
2	0.7	0	0	$1000\sqrt{3}$	0	2000

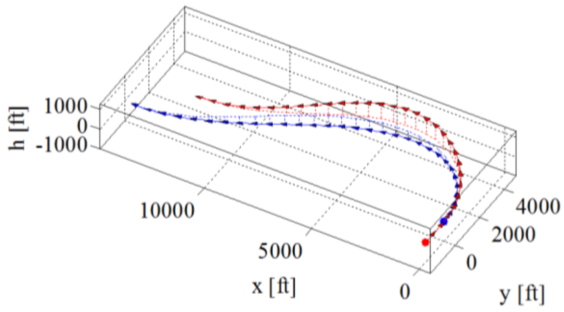

Fig. 2 Trajectories by general formulation (Table 2 and 3)

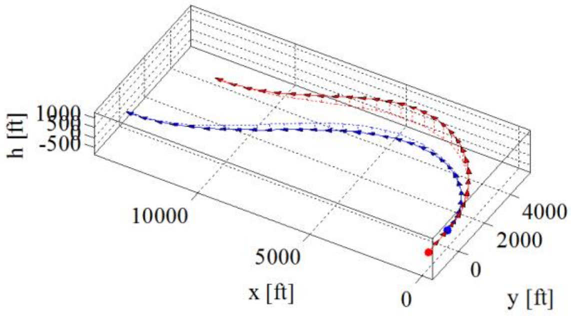

Fig. 4 Trajectories by general formulation (Table 2 and 4)

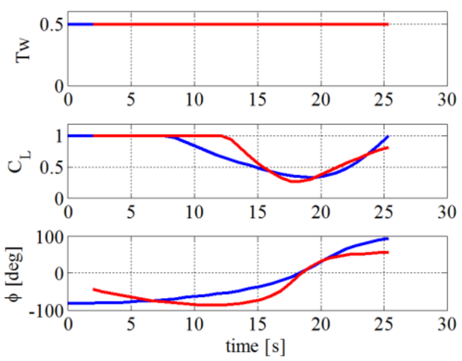

Fig. 3 Inputs by general formulation (Table 2 and 3)

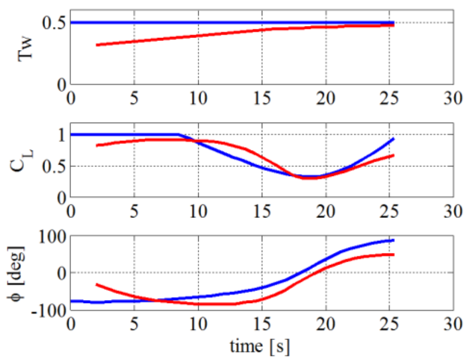

Fig. 5 Inputs by general formulation (Table 2 and 4)

Secondly, the terminal conditions are given in Table 4. The numerical results are shown in Fig. 4 to 5. Fig. 4 and Fig. 5 show optimal trajectories and inputs. As we can see, Tw about 1st aircraft is bang-bang (maximum) input. By contrast, Tw about 2nd aircraft is NOT bang-bang input.

Table 4 Terminal Conditions

j	M_f	γ_f [deg]	ψ_f [deg]	y [ft]	h [ft]	Rx_{1j} [ft]
1	0.7	0	0	0	0	-
2	0.7	0	0	$2000\sqrt{3}$	0	2000

3.2 Characteristics of Solutions

The solutions for minimum-time formation with start time differences can be divided by two features. It is the first one that the optimal solution depends on two aircraft's movement because relative distances of x about terminal conditions aforementioned in section 2.2 are specified. If all terminal x is specified, the solution depends on just only one aircraft. Moreover, the optimally condition [9] is considered about two aircraft. Another optimal condition is obtained by dividing optimal condition in this problem by the terminal adjoint about x. The obtained optimal condition is from an optimal control problem such that the relative terminal distance of x is maximized. That is to say, one aircraft has the optimal solution such that terminal x is minimized and the other aircraft has the optimal solution such that terminal x is maximized at the minimum-time for the formation. The second one is that the solution depends on single aircraft; the time for formation is as same as the minimum-time derived by solving an optimal control problem such that terminal x is not specified of one aircraft. We call the one or two aircraft 'time-controlling aircraft'.

From referred to above, the solutions for the minimum-time formation can be derived by the optimal solution of single aircraft as well as minimum-time simultaneous formation. We want to decide which aircraft is time-controlling aircraft in advance. However it is difficult, especially as for the case that time-controlling aircraft is two.

4 A Method Using Single Aircraft's Solution

As we aforementioned, the characteristics of the optimal solutions are equivalent to those of the simultaneous problem. It indicates that the method [10] proposed by the authors can be brought in. The method is modified in order to treat the start time difference formation.

The method can roughly be charactariscis as following 4 steps. The outline of this method is shown in Fig. 6.

Step 1: Select candidate of minimum-time

Solve the minimum-time to reach each given terminal condition expect terminal *x* about each aircraft. It means that following optimal control problem is solved. Take *j*th aircraft for example. The criterion is expressed as

$$J = \tau_f \tag{28}$$

and the boundary conditions are expressed as

$$\begin{aligned}&[M(\tau_{DLi}),\gamma(\tau_{DLi}),\psi(\tau_{DLi}),\xi(\tau_{DLi}),\eta(\tau_{DLi}),\zeta(\tau_{DLi})]\\&=[M_{0j},\gamma_{0j},\psi_{0j},\xi_{0j},\eta_{0j},\zeta_{0j}]\end{aligned} \tag{29}$$

$$\begin{aligned}&[M(\tau_f),\gamma(\tau_f),\psi(\tau_f),\eta(\tau_f),\zeta(\tau_f)]\\&=[M_{fj},\gamma_{fj},\psi_{fj},\eta_{fj},\zeta_{fj}]\end{aligned} \tag{30}$$

and then *J* is minimized.

Moreover, choose the maximum value τ_0 in the values of minimum-time (where *k*th aircraft has the value) and then the terminal *x* X_0 can be also driven.

Step 2: Derive reachable limits and check reachability (at τ_0)

Derive minimum terminal *x* and maximum terminal *x* at τ_0. It means that following optimal control problem is solved. For example, *j*th aircraft's terminal *x* $X_{j\min}$ and $X_{j\max}$ are derived. The criterion is expressed as

$$J = [\xi]_{\tau_0} \tag{31}$$

and initial conditions are expressed as eq. (29) and then terminal conditions are expressed as

$$\begin{aligned}&[M(\tau_0),\gamma(\tau_0),\psi(\tau_0),\eta(\tau_0),\zeta(\tau_0)]\\&=[M_{fj},\gamma_{fj},\psi_{fj},\eta_{fj},\zeta_{fj}]\end{aligned} \tag{32}$$

and then *J* is minimized and maximized. $X_{j\min}$ and $X_{j\max}$ mean the limits that *j*th aircraft can reach at τ_0.

Moreover, check whether following equation is satisfied or not.

$$X_{j\min} \le X_0 - Rx_{kj} \le X_{j\max} \tag{33}$$

If eq. (33) is satisfied in respect of all aircraft, the time for formation is τ_0 and we finish this method. The optimal solution depends on that of *k*th aircraft. That is to say, *k*th aircraft is the time-controlling aircraft. If eq. (33) is NOT satisfied in terms of any aircraft, go to next step.

Step 3: Derive reachable limits and check reachability (at $\tau_1 = \tau_0 + \Delta\tau$)

Derive minimum terminal *x* $X_{j\min}$ and maximum terminal *x* $X_{j\max}$ at τ_1 as well as those of previous step. Next, check whether following equation is satisfied or not.

$$X_{j\min} \le X_1 - Rx_{1j} \le X_{j\max} \tag{34}$$

where X_1 is maximum terminal value of 1st aircraft. If eq. (34) is satisfied in respect of all aircraft, the time for formation is shorter than τ_1. If eq. (34) is NOT satisfied in respect of all aircraft, X_1 and Rx_{1j} in eq. (34) are converted to those of 2nd aircraft (i.e. $X_1 \to X_2$ and $Rx_{2j} \to Rx_{2j}$). Then check whether eq. (34) is satisfied or not. If eq. (34) is satisfied, go to step 4. If eq. (34) is NOT satisfied in respect of all aircraft, X_2 is changed to that of another aircraft. If all aircraft cannot arrive with respect to all $X_i (i = 1,\ldots,N)$, the time for formation is longer than τ_1, $\tau_0 \to \tau_1$ and repeat step 3.

Step 4: Detect time for formation

Narrow the range of $\Delta\tau$. The bisection method is used in this paper.

After Step 4, the time-controlling aircraft can be detective and their states and controls are also derived. Each aircraft's states and controls except the time-controlling aircraft can be derived by solving an optimal control problem. Of cause, the problem is an optimal control problem of single aircraft. Moreover, if all τ_{DLi} are set 0, the minimum-time simultaneous formation problem can be solved.

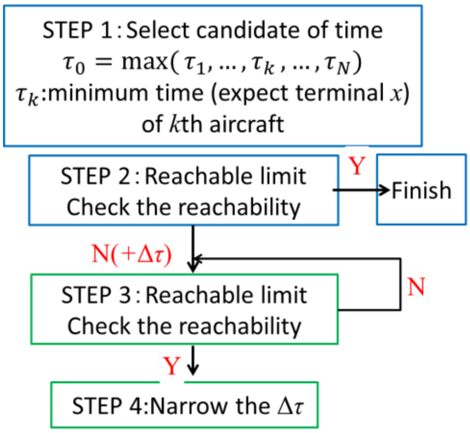

Fig. 6 Outline of method

5 Verifications

Firstly, the results of two aircraft are shown. When boundary conditions are given in Table 2 and 3, the results from proposed method are shown in Fig. 7 to 8. Compare to those form general formulation shown in Fig. 2 and 3, the results are consistent.

When boundary conditions are given in Table 2 and 4, the results from proposed method are shown in Fig. 9 to 10. In this case, this method ends at step 2 in Section 4 i.e. the time-controlling aircraft is one aircraft, which is 1st aircraft in this case. Compare to those by general formulation which are shown in Fig. 4 and 5, it indicates that the results of 1st aircraft are consistent. The results of 2nd aircraft are different because optimal control problem such that *Tw* is minimized, is solved as for the proposed method. Of cause, the times for formation are consistent.

Secondly, the results of multi-aircraft are shown. Boundary conditions are given in Table 5 and 6. The results by proposed method are shown in Fig. 11 and 12. Fig. 11 shows optimal trajectories and Fig. 12 shows histories of inputs about 4th aircraft and 9th aircraft, which are time-controlling aircraft in this case. Moreover Fig. 13 shows inputs from general formulation about 4th aircraft and 9th aircraft. As we can see, the inputs about time-controlling aircraft are consistent. The times for formation are 36.09 [s]. As shown previously, numerical results show that the solutions of minimum-time formation with start time differences can be derived by proposed method.

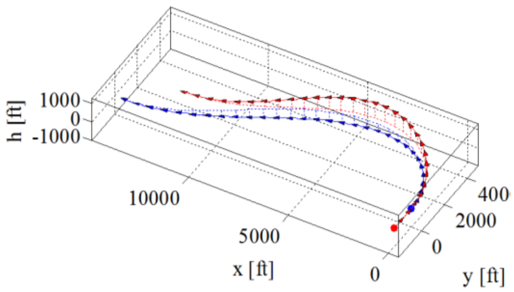

Fig. 7 Trajectories by proposed method (Table 2 and 3)

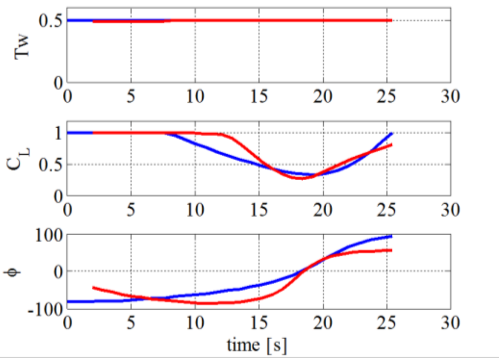

Fig. 8 Inputs by proposed method (Table 2 and 3)

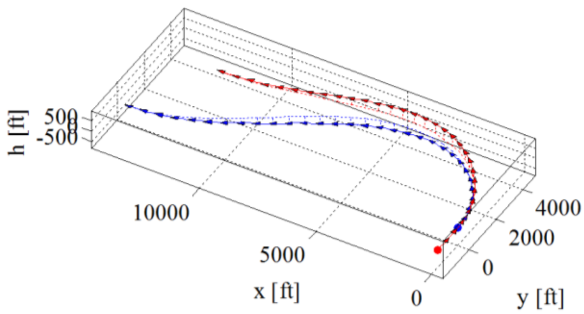

Fig. 9 Trajectories by proposed method (Table 2 and 4)

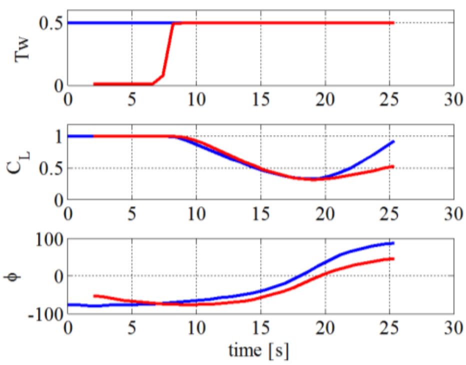

Fig. 10 Inputs by proposed method (Table 2 and 4)

Table 5 Initial Conditions

j	M	γ [deg]	ψ [deg]	x [ft]	y [ft]	h [ft]	T_{DLj} [s]
1	0.7	0	90	0	0	0	0
2	0.7	0	90	0	0	0	2
3	0.7	0	90	0	0	0	4
4	0.7	0	90	0	0	0	6
5	0.7	0	90	0	0	0	8
6	0.7	0	90	0	0	0	10
7	0.7	0	90	0	0	0	12
8	0.7	0	90	0	0	0	14
9	0.7	0	90	0	0	0	16

Table 6 Terminal Conditions

j	M_f	γ_f [deg]	ψ_f [deg]	y [ft]	h [ft]	Rx_{1j} [ft]
1	0.7	0	0	0	0	-
2	0.7	0	0	$1000\sqrt{3}$	0	1000
3	0.7	0	0	$2000\sqrt{3}$	0	2000
4	0.7	0	0	$3000\sqrt{3}$	0	3000
5	0.7	0	0	$4000\sqrt{3}$	0	4000
6	0.7	0	0	$5000\sqrt{3}$	0	5000
7	0.7	0	0	$6000\sqrt{3}$	0	6000
8	0.7	0	0	$7000\sqrt{3}$	0	7000
9	0.7	0	0	$8000\sqrt{3}$	0	8000

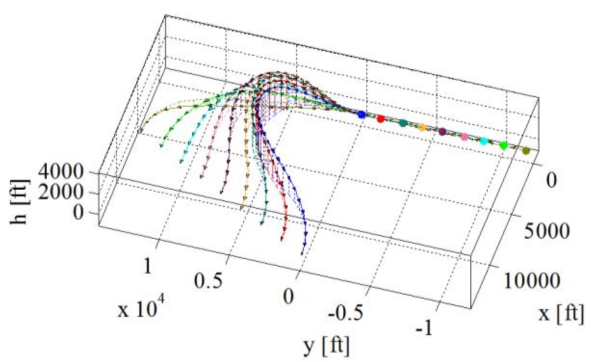

Fig. 11 Trajectories by proposed method (Table 5and 6)

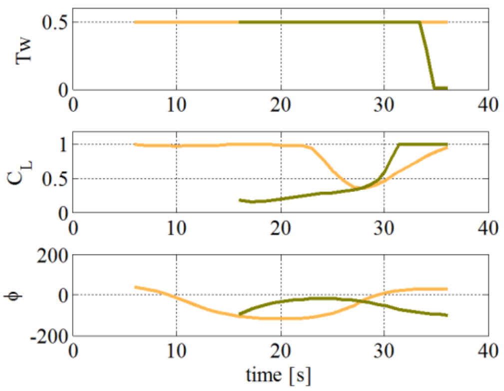

Fig. 12 Inputs about 4th aircraft and 9th aircraft by proposed method (Table 5 and 6)

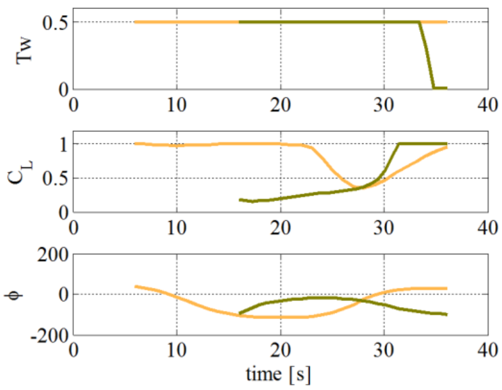

Fig. 13 Inputs about 4th aircraft and 9th aircraft by general formulation (Table 5 and 6)

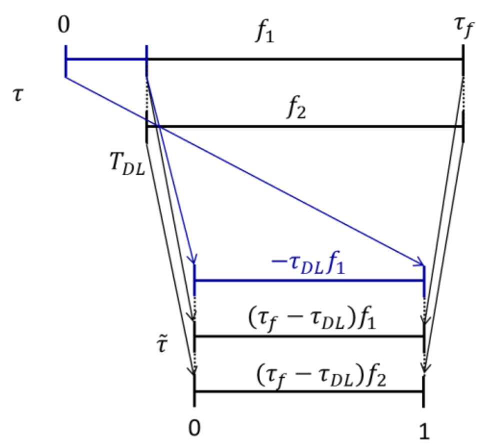

Fig. 14 A schema for formulation taking into relative distances of each aircraft

6 Conclusions and Future Works

For minimum-time formation with start time differences, this paper introduced a method, based on optimal solutions of single aircraft. Numerical results showed the concordances.

This method is only applicable to the problems that do not take into account relative distances among aircraft during the formation. It is also said as for the general formulation that has different control sections. This problem can be formulated by a method which for two aircraft is shown in Fig. 14. The control section from 0 to its different start time is replicated, normalized time is used and then the problem can be formulated as two point boundary value problem by dealing with the motions as one motion. Thus we can treat the relative distances, which are expressed as inequality constraints. However there is also a disadvantage that the states and inputs are increased rapidly. The future works are improving the proposed method in order to treat the problem such that relative distances among aircraft during the formation are considered.

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