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RESEARCH ON VIBRATION CHARACTERISTIC FOR VARIABLE ROTATING VELOCITY BLADES

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Abstract

As engine speed increase, centrifugal stiffening causes a blade's natural frequencies to increase. Because the frequencies of bending modes tend to increase at a faster rate than those of torsion mode, the frequencies of bending and torsion modes are disposed to cross. Aimed at the configuration and operation characteristics of the blades of the aero engine, based on sheet bending theory, considering the centrifugal load, the frequency and mode shape of the blades are studied by the beam function combination method. Considering the different engine speed, aero elastic differential integration method for differential equation are researched. The theoretical foundation of the frequency and mode shape research on cantilever sheet are obtained. The new methods of investigation on frequencies and mode shape have been studied. The given frequency and mode shape solutions are rather practical for the blade fatigue detecting, what's more, the theoretical base of flutter designing for engine blade are supplied.

1 Introduction

The blade is the key dynamic part in aeroengine, whose employment circumstance of high speed rotating and strong impact gas flow is so severe, which bears centrifugal load, air-actuated load, vibration load and so on. Therefore, strength problem of blade is rather predominant. Especially, thevibration characteristic of blade affects the aero engine characteristic directly. Resonance vibration can induce rupture and failure of the blade, the percent of accident , which lead by rupture and failure of the blade mentioned above, is 30%-40% for accident of part in aero engine[1-4]. As well, blade flutter is significant factor affecting reliability and life for aero engine.

As engine speed increase, centrifugal stiffening causes a blade's natural frequencies to increase. Because the frequencies of bending modes tend to increase at a faster rate than those of torsion mode, the frequencies of bending and torsion modes are disposed to cross. Aimed at the demand of high ratio of push-quality, the measure which is adopted conclude increasing boost ratio of compressor/blast fan and applying new-style and lightweight material, in order to enhance air-powered performance and to live weight loss. In general, the measures mentioned above can obviously lead the air-actuated load to increase, and cause the stiffness descending , make the problem of self excitation vibration for the blade (flutter) grow predominant particularly. Consequently, fracture failure because of resonance vibration for blade is one of major accident in the aero engine systemcommonly[5-7]. Therefore, researching vibration characteristics for variable rotating velocity blades is vital problem terribly during the investigation, manufacture and employment for the aero engine.

Aimed at the configuration and operation characteristics of the blades of the aero engine, based on sheet bending theory, considering the centrifugal load, the frequency and mode shape of the blades are studied by the beam function combination method. Considering the different engine speed, aero elastic differential integration method for differential equation are researched. The theoretical foundation of the frequency and mode shape research on cantilever sheet are obtained. The new methods of investigation on frequencies and mode shape have been studied. The given frequency and mode shape solutions are rather practical for the blade fatigue detecting, what's more, the theoretical base of flutter designing for engine blade are supplied.

2 Variation equation for blade vibration

According to assumption of plate theory[8], the stress and strain are listed as follow:

1) strain component

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y} \tag{1}$$

2) stress component

$$\begin{cases} \sigma_x = -\dfrac{E}{1-\mu^2} z\left[\dfrac{\partial^2 w}{\partial x^2} + \mu\dfrac{\partial^2 w}{\partial y^2}\right] \\ \sigma_y = -\dfrac{E}{1-\mu^2} z\left[\dfrac{\partial^2 w}{\partial y^2} + \mu\dfrac{\partial^2 w}{\partial x^2}\right] \\ \tau_{xy} = -2Gz\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{2}$$

3) Considering centrifugal force

The blade vibrated in rotating plane, the bending plane is shown in fig.1, the expression of centrifugal force N' for unit area is given as follow.

$$N' = \rho h\omega^2\left(\sqrt{(x+r0)^2 + (y\sin\alpha)^2}\right) \tag{3a}$$

$$N_x' = \frac{\partial N'}{\partial x} = \rho h\omega^2 \frac{r_0 + x}{\sqrt{(r_0+x)^2 + y^2\sin^2\alpha}} \tag{3b}$$

$$N_y' = \frac{\partial N'}{\partial y} = \rho h\omega^2 \frac{y\sin^2(\alpha)}{\sqrt{(r_0+x)^2 + y^2\sin^2\alpha}} \tag{3c}$$

$$\sigma_x' = \frac{N_x'}{h}, \sigma_z' = \frac{N_z'}{h}, \tau_{xz}' = 0 \tag{4}$$

$$\begin{aligned} \varepsilon_x' &= \frac{1}{E}(\sigma_x' - \mu\sigma_z') = \frac{1}{E}\left(\frac{N_x'}{h} - \mu\frac{N_z'}{h}\right), \\ \varepsilon_z' &= \frac{1}{E}(\sigma_z' - \mu\sigma_x') = \frac{1}{E}\left(\frac{N_z'}{h} - \mu\frac{N_x'}{h}\right), \\ \varepsilon_{xz}' &= 0 \end{aligned} \tag{5}$$

where ρh and is unit area mass density and α is blade setting angle, r_0 is disk radius. ω is rotating angular velocity of blade.

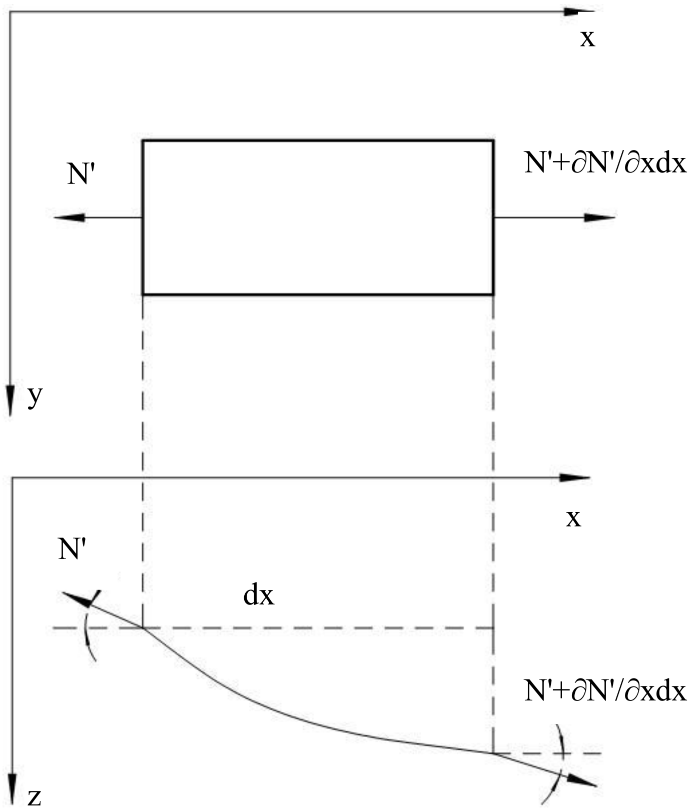


fig.1

4) Variation equation

Besides kinetic energy T and deformation energy U ,the energy of blade unit include potential energy, which lead by centrifugal force. According to energy method principle

$$T = U + U_\omega \tag{6}$$

In accordance with plate theory assumption, specific energy is

$$\tilde{W} = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy}) \tag{7}$$

Substituting Eq.(1)and Eq.(2) into Eq.(7), the expression of deformation energy U is

$$U=\iiint_V \tilde{W}dV=\iint_A\left(\int_{-\frac{h}{2}}^{\frac{h}{2}}\tilde{W}dz\right)dxdy$$
$$=\iint_A\left\{\begin{aligned}&\frac{D}{2}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]\\&+D_{xy}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2D_k\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\end{aligned}\right\}dxdy \tag{8}$$

Kinetic energy

$$T=\iiint_V\frac{\rho}{2}\left(\frac{\partial^2 w}{\partial t^2}\right)^2dV=\iint_A\left(\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho dz\right)\left(\frac{\partial^2 w}{\partial t^2}\right)^2dxdy=\frac{\rho h}{2}\iint_A\left(\frac{\partial^2 w}{\partial t^2}\right)^2dxdy \tag{9}$$

Analogously, assuming $w(x,y,t)=W(x,y)\sin(ft+\varphi)$, the potential energy U_ω becomes

$$U_\omega=\iint_A\frac{1}{2Eh}\left[\begin{aligned}&\left(N'\frac{\partial^2 W}{\partial y^2}+\frac{\partial N'}{\partial y}\frac{\partial W}{\partial y}\right)^2\\&-2\mu\left(N'\frac{\partial^2 W}{\partial y^2}+\frac{\partial N'}{\partial y}\frac{\partial W}{\partial y}\right)\left(N'\frac{\partial^2 W}{\partial x^2}+\frac{\partial N'}{\partial x}\frac{\partial W}{\partial x}\right)\\&+\left(N'\frac{\partial^2 W}{\partial x^2}+\frac{\partial N'}{\partial x}\frac{\partial W}{\partial x}\right)^2\end{aligned}\right]dxdy \tag{10}$$

Free vibration variation equation is

$$\delta\int_{t_0}^{t_1}(T-U-U_\omega)dt=0 \tag{11}$$

Substituting Eq.(8)- Eq.(10) into Eq.(11), range of integration is $ft=2\pi$, mode of vibration variation equation becomes

$$\delta\iint_A\left\{\begin{aligned}&\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2+\left(\frac{\partial^2 W}{\partial y^2}\right)^2\right]+2\frac{D_{xy}}{D}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right)+4\frac{D_k}{D}\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2+\frac{1}{DEh}\\&\left[\left(N'\frac{\partial^2 W}{\partial y^2}+\frac{\partial N'}{\partial y}\frac{\partial W}{\partial y}\right)^2-2\mu\left(N'\frac{\partial^2 W}{\partial y^2}+\frac{\partial N'}{\partial y}\frac{\partial W}{\partial y}\right)\left(N'\frac{\partial^2 W}{\partial x^2}+\frac{\partial N'}{\partial x}\frac{\partial W}{\partial x}\right)+\left(N'\frac{\partial^2 W}{\partial x^2}+\frac{\partial N'}{\partial x}\frac{\partial W}{\partial x}\right)^2\right]\\&-f^2\frac{\rho h}{D}W^2\end{aligned}\right\}dxdy=0 \tag{12}$$

flexural rigidity and torsional stiffness is expressed as

$$\begin{cases}D=\dfrac{Eh^3}{12(1-\mu^2)}\\D_{xy}=\dfrac{E\mu h^3}{12(1-\mu^2)}\\D_k=\dfrac{Gh^3}{12}\end{cases} \tag{13}$$

3 Blade dynamic characteristics solution

Flexibility mode function is

$$W_{ij}(x,y)=A_{ij}X_i(x)Y_j(y) \tag{14}$$

where $X_m(x)$ and $Y_n(y)$ are i order and j order mode of vibration function[9], which correspond to x direction and y direction boundary condition for girder respectively, the boundary condition are fixing one side and making the other three sides free. A_{ij} is undetermined factor.

Substituting Eq.(14) into Eq.(12), after variation operation, the linear algebra equations which satisfy A_{ij} are available

$$\sum_{i=0}^{p}\sum_{j=0}^{q}\left(C_{ij}-\lambda^4\right)A_{ij}=0 \qquad (i=0,1,2,3,\cdots p; j=0,1,\text{.} \tag{15}$$

In Eq.(15), frequency coefficient for plate is $\lambda^4 = f^2 A_{ii} B_{jj} \dfrac{\rho h}{D}$ (16a)

$$C_{ij}=G_{ii}B_{jj}+A_{ii}L_{jj}+2\frac{D_{xy}}{D}E_{mm}F_{nn}+4\frac{D_k}{D}H_{mm}K_{nn}$$
$$+\frac{\left(\rho h\omega^2-\dfrac{Eh\alpha_T t}{1-\mu}\right)^2}{hDE}\Big[M_{ii}L_{jj}+S_{ii}B_{jj}+2U_{ii}B_{jj}$$
$$-2\mu\left(O_{ii}F_{jj}+Q_{ii}F_{jj}\right)+Z_{ij}+$$
$$\left(A_{ii}T_{jj}+G_{ii}N_{jj}+2A_{ii}V_{jj}-2\mu E_{ii}P_{jj}-2\mu E_{ii}R_{jj}\right)\sin^2\alpha\Big] \tag{16b}$$

$$\left\{\begin{array}{ll}
A_{ii}=\int_0^a X_i^2 dx & B_{jj}=\int_0^b Y_j^2 dy \\
G_{ii}=\int_0^a\left(\dfrac{d^2X_i}{dx^2}\right)^2 dx & L_{jj}=\int_0^b\left(\dfrac{d^2Y_j}{dy^2}\right)^2 dy \\
E_{ii}=\int_0^a X_i\dfrac{d^2X_i}{dx^2}dx & F_{jj}=\int_0^b Y_j\dfrac{d^2Y_j}{dy^2}dy \\
H_{ii}=\int_0^a\left(\dfrac{dX_i}{dx}\right)^2 dx & K_{jj}=\int_0^b\left(\dfrac{dY_j}{dy}\right)^2 dy \\
M_{ii}=\int_0^a X_i^2\left(r_0+x\right)^2 dx & N_{jj}=\int_0^b Y_j^2 y^2 dy \\
O_{ii}=\int_0^a X_i\left(r_0+x\right)^2\dfrac{d^2X_i}{dx^2}dx & P_{jj}=\int_0^b Y_j y^2\dfrac{d^2Y_j}{dy^2}dy \\
Q_{ii}=\int_0^a X_i\left(r_0+x\right)\dfrac{dX_i}{dx}dx & R_{jj}=\int_0^b Y_j y\dfrac{dY_j}{dy}dy \\
S_{ii}=\int_0^a\left(r_0+x\right)^2\left(\dfrac{d^2X_i}{dx^2}\right)^2 dx & T_{jj}=\int_0^b y^2\left(\dfrac{d^2Y_j}{dy^2}\right)^2 dy \\
U_{ii}=\int_0^a\left(r_0+x\right)\dfrac{d^2X_i}{dx^2}\dfrac{dX_i}{dx}dx & V_{jj}=\int_0^b y\dfrac{d^2Y_j}{dy^2}\dfrac{dY_j}{dy}dy \\
\multicolumn{2}{l}{Z_{ij}=\iint_A\dfrac{\left[\begin{array}{l}X_i^2y^2\sin^4\alpha\left(\dfrac{dY_j}{dy}\right)^2-2\mu X_iY_j\left(r_0+x\right)\\ y\sin^2\alpha\dfrac{dX_i}{dx}\dfrac{dY_j}{dy}+Y_j^2\left(r_0+x\right)^2\left(\dfrac{dX_i}{dx}\right)^2\end{array}\right]}{\left(r_0+x\right)^2+y^2\sin^2\alpha}}
\end{array}\right. \tag{16c}$$

Eq.(16c) are integral value concerned beam function, which refer to literature[9]. Determinant of coefficient for Eq.(15) is zero ,in order to make solution non-vanishing. Available frequency equation e is

$$\mathrm{Det}\left(\left[\mathbf{C}\right]-\lambda^4\left[\mathbf{I}\right]\right)=0 \tag{17}$$

Generally, Eq. （14） and Eq. （16） are expressions for mode shape and natural frequency of blade.

4 Aeroelastic differential equation of blade considering centrifugal force

The cantilever plate shown in fig.2:length a , width b , depth h ,plate density ρ . x direction air flow act on upper face, velocity U_∞ ,mach number M_∞ ,air density ρ_∞ , N' is centrifugal force acting in the middle of plate.

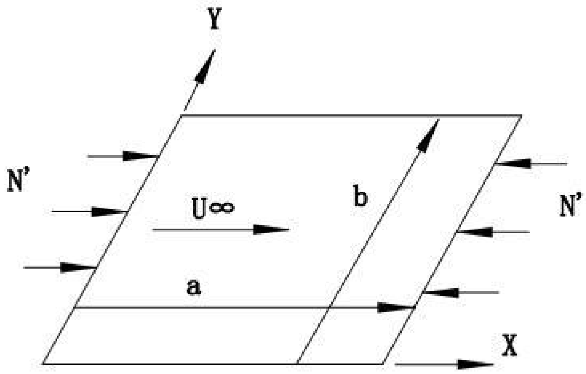


fig.2

Substituting dimensionless parameter:

$\bar{x}=x/a$, $\bar{y}=y/b$, $\alpha=a/b$, $\bar{\upsilon}=\upsilon/h$, $\bar{u}=u/h$, $\bar{\omega}=\omega/h$, $N=N'a^2/D$, $\tau=t\sqrt{D/\rho h a^4}$, $\lambda=2qa^3/\beta D$, $\theta=\rho_\infty a/\rho h M_\infty$

on the effect of aerodynamic load, the cantilever plate equilibrium equation is

$$\beta^{-1}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+\frac{1-\mu}{2}\alpha^2\beta^{-1}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}+\frac{1+\mu}{2}\alpha^2\beta^{-1}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}=\frac{1}{12}\beta^{-2}\frac{\partial^2 u}{\partial t^2} \tag{18a}$$

$$\alpha^2\frac{\partial^2 v}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+\mu}{2}\alpha\frac{\partial^2 u}{\partial x\partial y}+\alpha^3\beta^{-1}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\frac{1-\upsilon}{2}\alpha\beta^{-1}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}+\frac{1+\mu}{2}\alpha\beta^{-1}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}=\frac{1}{12}\beta^{-2}\frac{\partial^2 v}{\partial t^2} \tag{18b}$$

$$\frac{\partial^4 w}{\partial x^4}+2\alpha^2\frac{\partial^4 w}{\partial x^2\partial y^2}+\alpha^4\frac{\partial^4 w}{\partial y^4}-12\beta^2\frac{\partial^2 w}{\partial x^2}\left[\beta^{-1}\frac{\partial u}{\partial x}+\frac{1}{2}\beta^{-2}\left(\frac{\partial w}{\partial x}\right)^2+\mu\left(\begin{array}{c}\alpha\beta^{-1}\frac{\partial \upsilon}{\partial y}\\+\frac{1}{2}\alpha^2\beta^{-2}\left(\frac{\partial w}{\partial y}\right)^2\end{array}\right)\right]-12\alpha^2\beta^2\frac{\partial^2 w}{\partial y^2}\left[\begin{array}{c}\alpha\beta^{-1}\frac{\partial \upsilon}{\partial y}+\frac{1}{2}\alpha^2\beta^{-2}\left(\frac{\partial w}{\partial y}\right)^2\\+\mu\left(\beta^{-1}\frac{\partial u}{\partial x}+\frac{1}{2}\beta^{-2}\left(\frac{\partial w}{\partial x}\right)^2\right)\end{array}\right]-12\left(1-\mu^2\right)\alpha\beta^2\frac{\partial^2 w}{\partial x\partial y}\left(\begin{array}{c}\alpha\beta^{-1}\frac{\partial u}{\partial y}+\beta^{-1}\frac{\partial \upsilon}{\partial x}\\+\alpha\beta^{-2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\end{array}\right)+\frac{\partial^2 w}{\partial t^2}-\lambda\frac{\partial w}{\partial x}-N\frac{\partial^2 w}{\partial x^2}-\sqrt{\lambda\theta}\frac{\partial w}{\partial t}=0 \tag{18c}$$

the boundary condition for cantilever plate are listed as below:

fixed boundary

$$w=0,\frac{\partial w}{\partial x}=0\ (x=0) \tag{19a}$$

free boundary

$$\begin{cases}M_x=0,\\ Q_x+\dfrac{\partial M_{xy}}{\partial y}=0\end{cases}\quad (x=a) \tag{19b}$$

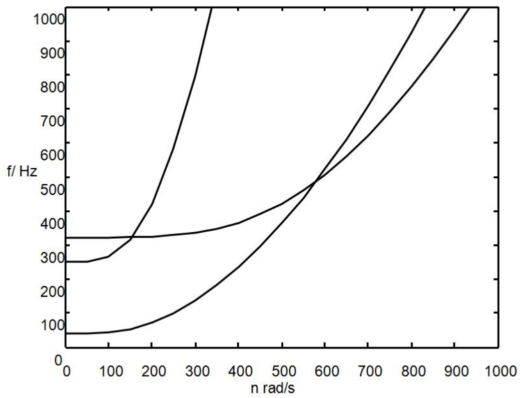
$$\begin{cases}M_y=0,\\ Q_y+\dfrac{\partial M_{xy}}{\partial x}=0\end{cases}\quad (y=0,y=b) \tag{19c}$$

Differential quadrature method（DQM）is weighted approximation of some function, Adopting DQM, mixing partial derivative of Eq.(18)and Eq.(19) are written to matrix formulations.

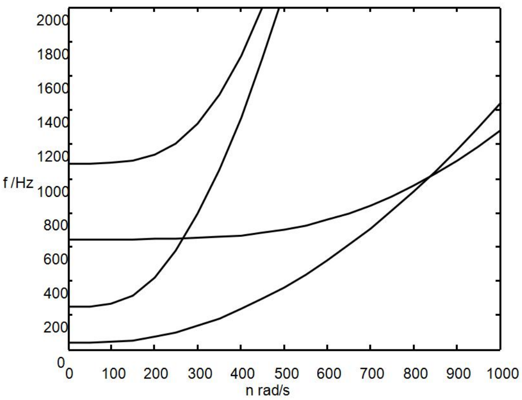
Substituting $w_{ij}=\xi_{ij}\exp(i\omega\tau)$ into linear part of Eq.(18)and Eq.(19), systematic eigenvalue equation are received, ω is plural, $\omega=\omega_R+i\omega_I$,real part ω_R is systematic damp, imaginary part ω_I is systematic frequency, based on Lyapunov indirect method, when $\omega_R<0$, system is stable; when $\omega_R>0$, system is unstable; while $\omega_R=0$,systematic flutter generate， and corresponding dynamic pressure is systematic flutter critical dynamic pressure. On condition that given centrifugal force N and θ ,when dimensionless dynamic pressure $\lambda=\lambda_c$, systematic eigenvalue is zero, λ_c is desired flutter boundary.

5 Vibration frequency numerical study for blade considering centrifugal force

One side of blade is fixed, the other three sides are free. blade parameters are supposed as follow: the length $a=0.8\text{m}$, the depth $h=0.006\text{m}$, modulus of elasticity $E=1.4\times10^{11}\text{Pa}$ material density $\rho=8.8\times10^3\text{kg/m}^3$,installing angle $\alpha=30^\circ$, the blade is fixed on the disk, whose radius is $r_0=0.5\text{m}$. Ignoring the blade mass, when aspect ratio are 2:1 and 4:1,the campbell diagrams are obtained respectively, which are shown in Fig.3.



(a) the curves of frequency for aspect ratio=2:1



(b) the curves of frequency for aspect ratio=4:1

Fig.3 Campbell diagram

The diagram describe that :with plate thickness and length kept constant, coupling between 1order and 3 order vibration mode exist, coupling between 2 order and 3 order vibration mode appear, while coupling between 1order and 2 order vibration mode is nonexistent; the intersection point correspond to rotating speed and frequency higher respectively than those corresponding to intersection point.

6 Conclusion

Based on beam function combination method, the expression of frequency and mode shape for blade considering centrifugal force are received. The phenomena of frequency veering for blade is exampled by analyzing aspect ratio influence on blade frequency. Considering centrifugal force and aerodynamic force cooperation , aero elastic differential integration method for three-dimensional differential equation are received. The theoretical base of flutter designing for engine blade are supplied. The main conclusions are listed as follow:

1) The dynamic character of the cantilever sheet is studied by the beam function combination method. Founded on sheet bending theory , the analytical solutions of frequency and mode shape are deduced in variable rotating speed state .

2) Considering geometry large deformation nonlinearity, based on plate theory and linear elastic relation of stress and strain , the differential quadrature method was applied to deal with the three-dimensional aeroelastic dynamic equation using piston theory. The theory foundation of analyzing the influence of system's parameters on flutter boundary are applied.

3)The simulation result indicate that the single bending mode shape or the single torsion mode shape, which will never couple with the other mode shape, leads to mode coupling; considering the blades of the same thickness and length, the vector and frequency subjected to the lager aspect ratio blades are bigger than those subjected to the smaller aspect ratio blades.

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